International Institute of Seismology
and Earthquake Engineering (IISEE)
Seismology Course Lecture Notes

TRAINING COURSE IN
SEISMOLOGY AND EARTHQUAKE ENGINEERING

Introduction to Digital Data Processing

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1. Introduction

In this lecture note, several basic and essential topics that are useful for understanding digital data processing techniques are explained. These topics are directly related to measurements using digital equipment.

1.1. Linear System

Almost all measuring equipment is “linear systems.” This is because “linear systems” make it easier to reproduce the measured value of the physical parameters from the results of the measurement.

1.1.1. Signal, Input, and Output

Since it could be difficult to define the terms “signal”, “input” and “output” exactly, we employ their practical definitions.

A signal is any quantity that we measure or input. For example, mechanical signals may be displacement, velocity, acceleration, or force, whereas electric signals may be charge, voltage, or current. Signals often depend on time. In seismology, we typically treat time-dependent signals. Such signals can have many relations. A system can be defined either as a relation between more than two signals itself, or an equipment (or algorithm) that yields such relations. The input can be defined as the conditions given to a system. The signal generated in the system that corresponds to the input is called the output (Fig. 1).

For example, consider ground motion. Ground motion will be the input to a seismograph (which is the “system”). The output is the recorded seismogram. If we consider a seismometer to be the system, the input will be the ground motion again while the voltage imbalance between the two terminals of the seismometer will be the output.

Fig. 1 Block diagram of a system.

1.1.2. Linear System

Almost all the systems used in the measurement of physical quantities have “linear” characteristics. This can imply the following. Suppose that the output of a system that corresponds to the input $x_1(t)$ is $y_1(t)$ while the output that corresponds to the input $x_2(t)$ is $y_2(t)$. The first requirement of a linear system is that the output corresponding to the input $x_1(t) + x_2(t)$ is $y_1(t) + y_2(t)$. The second is that the output corresponding to $\alpha x_1(t)$ is $\alpha y_1(t)$, where α is a constant. These requirements lead to a proportional relation between the input and the output. Such a relation is called “linearity.” Namely,

$$\text{If } \begin{cases} x_1(t) \to y_1(t) \\ x_2(t) \to y_2(t) \end{cases}, \text{ then } \begin{cases} x_1(t) + x_2(t) \to y_1(t) + y_2(t) \\ \alpha x_1(t) \to \alpha y_1(t) \end{cases}$$

Such “linear” characteristics of the relation between an input and an output ensure that the input signal can be reconstructed by using the output signal. This is why almost all measuring equipment is linear systems. However, in reality, linear characteristics can be usually obtained only for a limited range of the input signal. Clipping of a seismometer and saturation of an amplifier are simple examples. When “linearity” is lost, the system becomes “non-linear” (Fig. 2).

There are several important “linear” transformations in mathematics that can be applied to physics, for example, the Fourier transform.

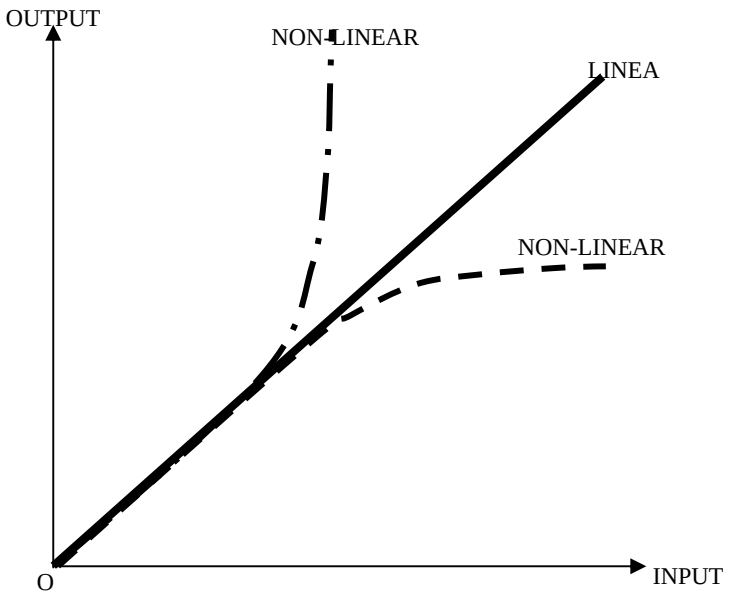


Fig. 2 Schematic figure showing both linearity and non-linearity.

1.1.3. Response Characteristics of Linear Systems

Suppose that *g(t)* is the output of a system when the input signal is a unit impulse *δ(t)*. An arbitrary function *x(t)* can be expressed by using the technique of "convolution" as follows.

$$x(t) = \int_{-\infty}^{\infty} x(u)\delta(t-u)du = \int_{-\infty}^{\infty} x(t-u)\delta(u)du = \lim_{\Delta u \to 0} \sum_{m=-\infty}^{\infty} x(t-m\Delta u)\delta(m\Delta u).$$

The last term shows that *x(t)* is the weighted sum of the delta functions at an infinitively small *Δu*. Each delta function *δ(mΔu)* gives the output *g(mΔu)*. The second requirement (mentioned previously in 1.1.2) suggests that the input *x(t – mΔu)δ(mΔu)* gives *x(t – mΔu)g(mΔu)*, since *x(t – mΔu)* is a constant. The first requirement suggests that the input, that is, the sum of *x(t – mΔu)δ(mΔu),* gives the sum of the output for each weighted delta function *x(t – mΔu)g(mΔu)*.

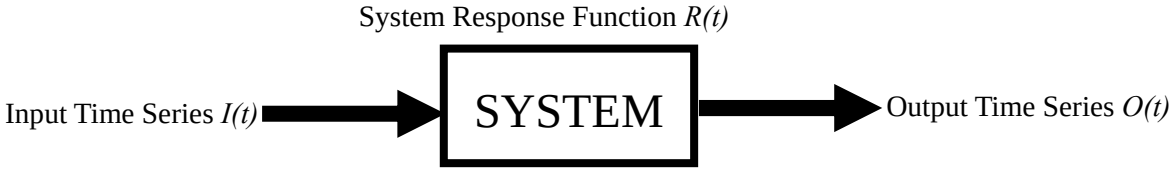
Thus, the output signal that corresponds to the input signal *x(t)* is given by the following.

$$\lim_{\Delta u \to 0} \sum_{m=-\infty}^{\infty} x(t-m\Delta u)g(m\Delta u) = \int_{-\infty}^{\infty} x(t-u)g(u)du = \int_{-\infty}^{\infty} x(u)g(t-u)du = y(t).$$

For a seismic signal, it is reasonable to suppose that *g(t) = 0* at t < *0* ; then

$$y(t) = \int_{0}^{\infty} x(u)g(t-u)du \quad .$$

These formulas show that the output that corresponds to an arbitrary input signal can be defined by *g(t)*. In other words, the response to the unit impulse *g(t)* contains all information on the characteristics of a linear system. We call such a response "impulse response" or "system characteristics."



$$O(t) = I(t) * R(t)$$

Fig. 3 System response, input, and output signals.

1.2. Digitization of Time-Dependent Functions

Today, digital data acquisition and processing systems are so dominant in physical measurements that we can expect analog or continuous systems to become obsolete by the first half of the twenty-first century. We can convert every analog signal to a digital once they are converted to the form of a voltage imbalance. Some basic knowledge is required to prevent difficulties that might occur during such analog-to-digital conversions, which are referred to as “discretization.”

1.2.1. Dirac Comb Function

First, let us understand the discretization process. Consider a box car function *b(t)*:

$$b(t) = \begin{cases} B, & |t| \le t_0 / 2, \\ 0, & |t| > t_0 / 2. \end{cases}$$

The Fourier expansion in [*–T/2, T/2*] (*T*>*t*$_0$) gives

$$b(t) = \frac{Bt_0}{T}\left[1 + 2\sum_{n=1}^{\infty} \frac{\sin(\omega_n t_0 / 2)}{\omega_n t_0 / 2} \cos \omega_n t\right] \ , \ \omega_n = 2n\pi / T \, .$$

Note that the time window of the length *T* is used implicitly. *T* has to be longer than *t*$_0$ and no other constraint is placed on it.

Let *t*$_0$ tend to zero under the condition *Bt*$_0$ = *1*; in this manner, the Fourier expansion of *δ(t)* is obtained.

$$\delta(t) = \frac{1}{T}\left[1 + 2\sum_{n=1}^{\infty} \cos \omega_n t\right] = \frac{1}{T} \sum_{n=-\infty}^{\infty} e^{i\omega_n t} \ , \ \omega_n = 2n\pi / T.$$

The Fourier expansion of the signal in a limited time window implicitly assumes the periodicity of the signal. This formula, when written in exact terms, represents an infinite series of the impulse and it is called as the Dirac comb function. Note that the interval between the delta functions is *T*.

Let us change the notation from *T* to *Δt* for the convenience of the following description (Fig. 4).

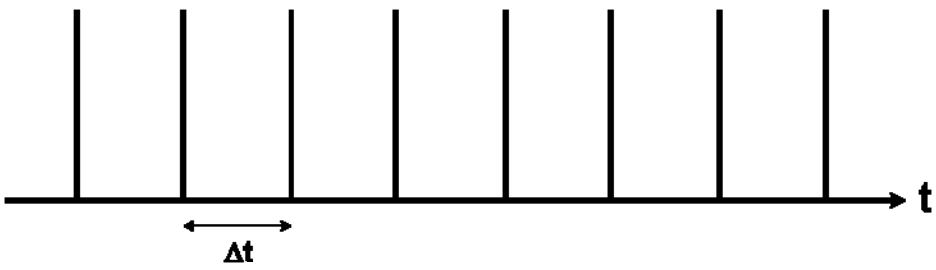


Fig. 4 Dirac comb function.

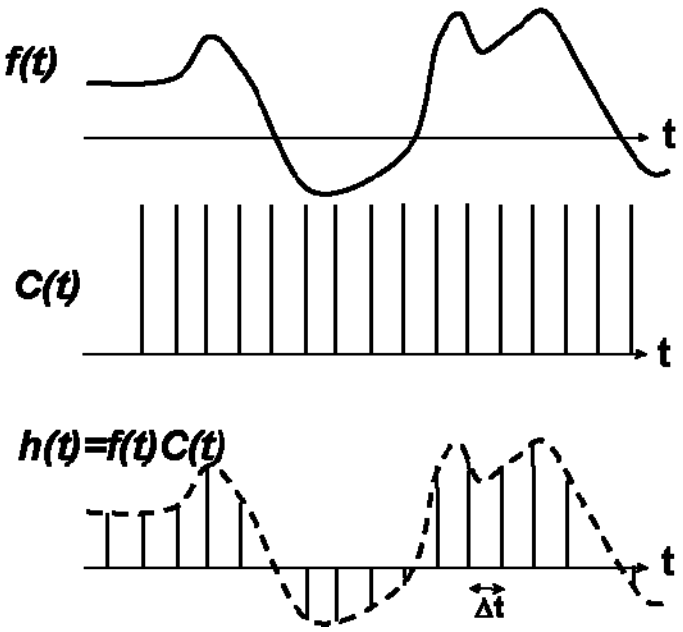


Fig. 5 Continuous function, Dirac comb function, and time series.

1.2.2. Time Series Discretization

Today, it is popular and convenient to handle time-dependent data by using computers. For doing this, it is necessary that recorded data is digital or discrete. The process for converting an continuous analog signal *f(t)* to its digital equivalent is called "analog-to-digital conversion," "digitization," or "discretization." This process is expressed mathematically as the multiplication of a continuous function *f(t)* with the Dirac comb function *C(t)*, in which the interval of the neighboring delta function is Δt (Fig. 5).

$$h(t) = f(t) \cdot C(t).$$

1.2.3. Folding and Aliasing

From the above, we obtain the Fourier expansion as follows:

$$C(t) = \frac{1}{\Delta t} \sum_{n=-\infty}^{\infty} e^{i(2n\pi/\Delta t)t},$$

where Δt is the sampling interval.

The Fourier transform of the discrete function *h(t)* is given by

$$h(\omega) = \int_{-\infty}^{\infty} f(t) \cdot C(t) e^{-i\omega t} dt = \frac{1}{\Delta t} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f(t) e^{-i(\omega - 2n\pi/\Delta t)t} dt = \sum_{n=-\infty}^{\infty} F\left(\omega - \frac{2n\pi}{\Delta t}\right).$$

This shows that $h(\omega)$ is a repetition of $F(\omega)$ with an interval $1/\Delta t$ and that a spectra outside the frequency range $[-1/2\Delta t, 1/2\Delta t]$ does make any sense (Fig. 6). This border frequency $1/2\Delta t$ is called the folding frequency or the Nyquist frequency.

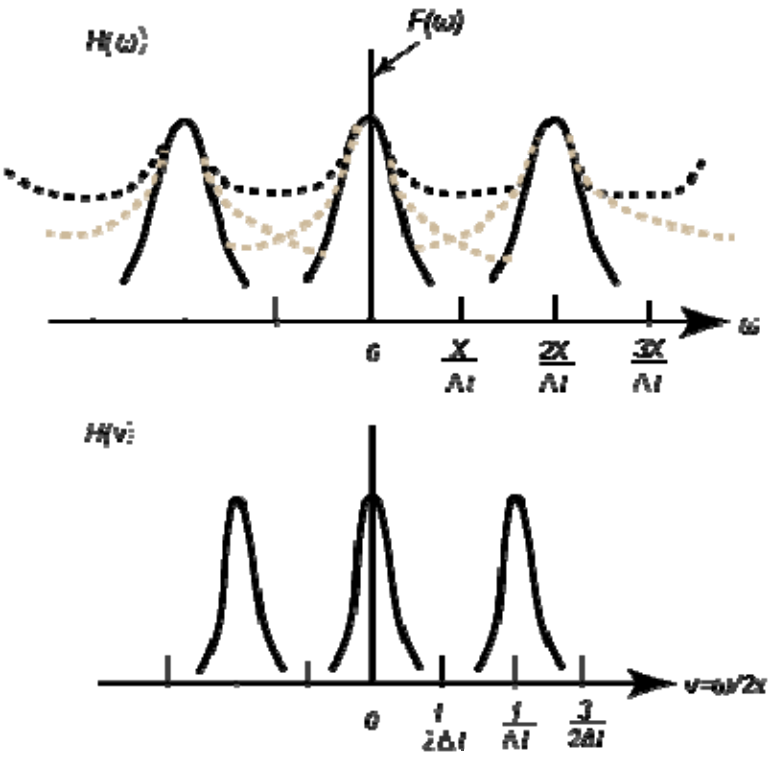


Fig. 6 Spectra of a time series.

The influence of digitization can be minimized easily when the Fourier spectra of the original continuous function have a negligible value at the Nyquist frequency. Otherwise, the foot of the neighboring spectral peak invades in the range [$-1/2\Delta t,\ 1/2\Delta t$] and contaminates the signal. This phenomenon in the frequency domain is referred to as folding. The above consideration suggests that the sampling interval Δt must be shorter than half of the shortest period that is included in the original continuous function. In other words, the frequency components of the period that is shorter than twice the sampling interval must be eliminated before digitization.

The disturbance is more clearly shown in the time domain. Fig. 7 clearly shows that coarse discretization cannot identify fine peaks of the original continuous function, and the result is completely different from the original one.

Folding in the frequency domain and aliasing in the time domain represent the same phenomenon. The relationship between aliasing and folding is schematically shown in Fig. 8.

An analog filter that is applied to the original analog signal in order to prevent the aliasing or folding is called an anti-alias filter. Re-sampling of the digital data also requires the application of an anti-alias filter.

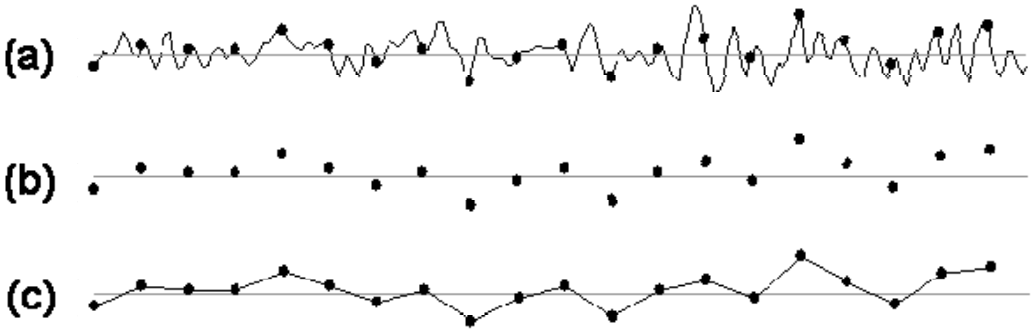


Fig. 7 Example of Aliasing in the time domain. (a) Original analog signal where dots denote the sampling, (b) digitized signal, and (c) Re-constructed analog signal by the linear interpolation of the digitized data. Produced newly based on the concept of Yilmaz(1994).

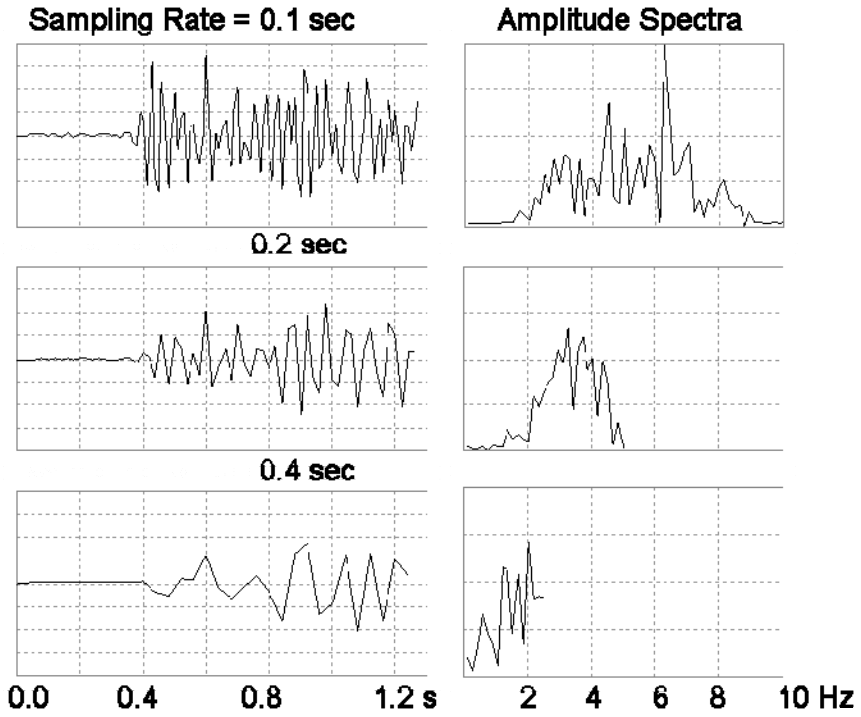


Fig. 8 Example of Folding in the frequency domain. A waveform sampled at 0.1sec has Nyquist frequency of 5.0 Hz. Resampling to 0.2 and 0.4 sec confines the frequency band to 2.5 and 1.25 Hz, respectively. Note the loss of high frequencies at larger sampling intervals. Produced newly based on the concept of Yilmaz(1994).

2. Fast Fourier Transform (FFT)

The Fourier transform is one of the basic mathematical tools used for data processing. A signal in the time domain can be converted to one in the frequency domain by applying the Fourier transform and in this manner, different features of the converted data can be obtained. The Fourier transform of digital data is defined in this chapter.

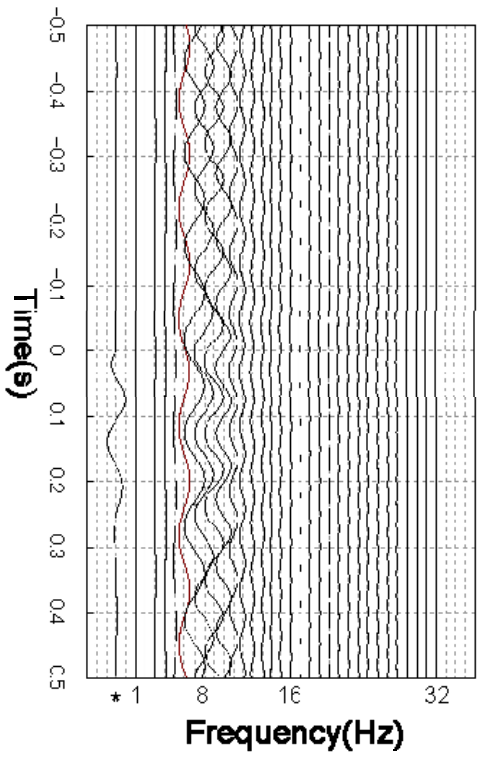
There are several published subroutines of FFT in BASIC, FORTRAN, and C, which are very useful and simplify our task. However, we must focus on the definitions of the Fourier transform and its inverse transform that are given in different books. There are several possible definitions and mathematically they are all equivalent. When we use a subroutine given in a textbook, it is important to carefully read the main text. This lecture note employs the definition of Papoulis (1962, 1984) and Ohsaki (1976).

$$\begin{cases} X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-i\omega t} dt, \\ x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \cdot e^{i\omega t} d\omega. \end{cases}$$

2.1. Fourier Expansion of a Finite Time Series

The Fourier expansion of a time series x_m ($m = -N/2 + 1,\ldots -1, 0, 1, \ldots N/2$) is given as follows. The coefficient of expansion is given by

$$C_k = \frac{1}{N} \sum_{m=-N/2+1}^{N/2} x_m e^{-i(2\pi km/N)} \quad , k = -N/2+1, \cdots, -1, 0, 1, \cdots, N/2. \tag{1}$$



By using these coefficients, the original time series x_m is expanded as

$$x_m = \sum_{k=-N/2+1}^{N/2} C_k e^{i2\pi km/N} \quad , m = -N/2+1, \cdots N/2. \tag{2}$$

Naturally, these formulas imply that any limited time series can be expanded into a finite number of sinusoidal waves of which the frequency is discrete, as shown in Fig. 9. Note that the periodicity in the time domain is implicitly introduced.

Fig. 9 A series of sinusoidal curves with different frequencies, peak amplitudes, and phase lags can be superimposed to synthesize a waveform on the left-most curve as indicated by the asterisk. The sampling frequency of this waveform is 512Hz. The sinusoidal curves of frequencies higher than 33 Hz are omitted because their amplitudes are negligible. Produced newly based on the concept of Yilmaz(1994).

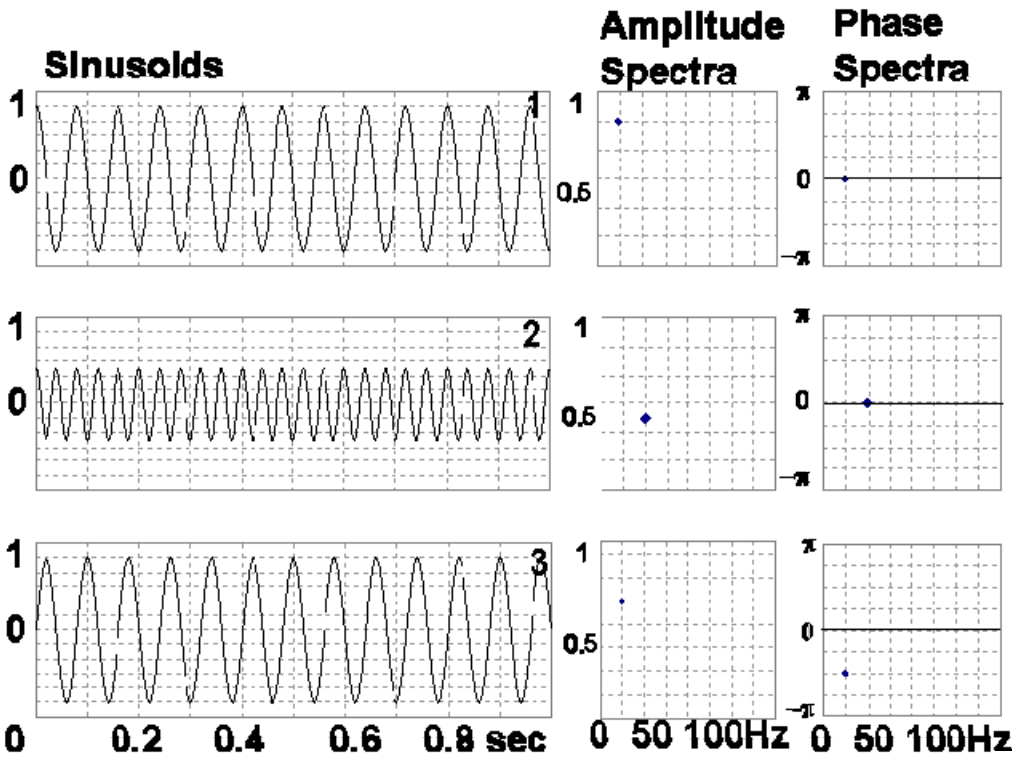


Fig. 10 Three sinusoids (*left*) and their amplitude (*center*) and phase spectra (*right*). The time between two consecutive peaks is the period of the sinusoid, the inverse of which is called frequency. Finally, the time delay of the onset is defined as phase lag. Produced newly based on the concept of Yilmaz(1994).

2.2. Sinusoidal Functions and Fourier Spectra of Amplitude and Phase

Yilmaz(1994) shows a persuasive way of explaining Fourier Spectra of Amplitude and Phase. A sinusoidal function is defined by its frequency, amplitude, and time shift, as shown by an example given in Fig. 10. A phase lag, that is, a time shift normalized by the period is usually used. Assume that the phase lag of the signals in the *top* panels is zero and its amplitude is unity. The frequency of signals in the *top* panels is *12.5 Hz*. The *middle* panels have a half amplitude, frequency of *25.0 Hz* and the phase lag is zero. The *bottom* panels have unit amplitude, a frequency of *12.5 Hz*, and phase lag of $-\pi/2$.

Every sinusoid drawn in Fig. 9 has a frequency, amplitude, and phase lag. The latter two variables can be plotted against the frequency (Fig. 11). Each point along the amplitude spectrum curve (Fig. 11 *Top*) corresponds to the peak amplitude of the sinusoid at that frequency, as shown in Fig. 9. Note the correspondence of the peak in the amplitude spectra with the high-amplitude frequency range in Fig. 9. Each point along the phase spectrum (Fig. 11 *Bottom right*) corresponds to the phase delay of a peak or trough along the sinusoid at that frequency with respect to the timing line at $t = 0$ in Fig. 9. Note the correspondence of the phase curve with the trend of a positive peak from trace to trace (Fig. 12).

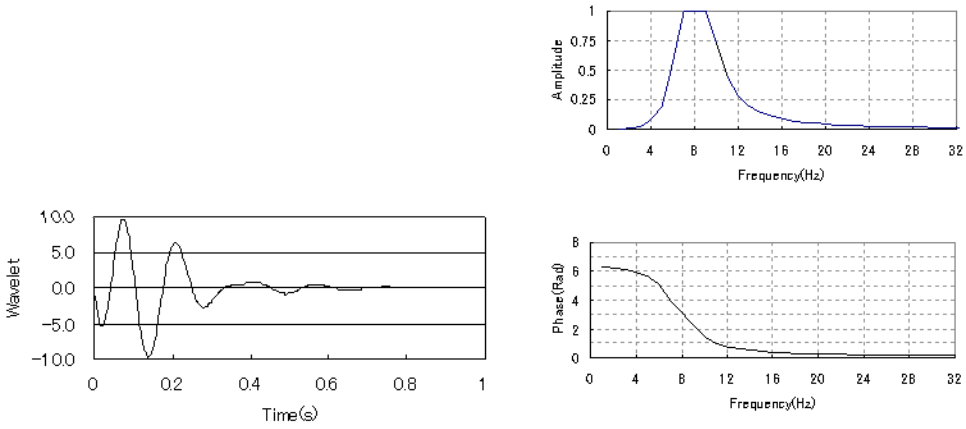


Fig. 11 *Bottom left*: The waveform in Fig.9, its amplitude and phase spectra (top and *bottom* right panels). Produced newly based on the concept of Yilmaz(1994).

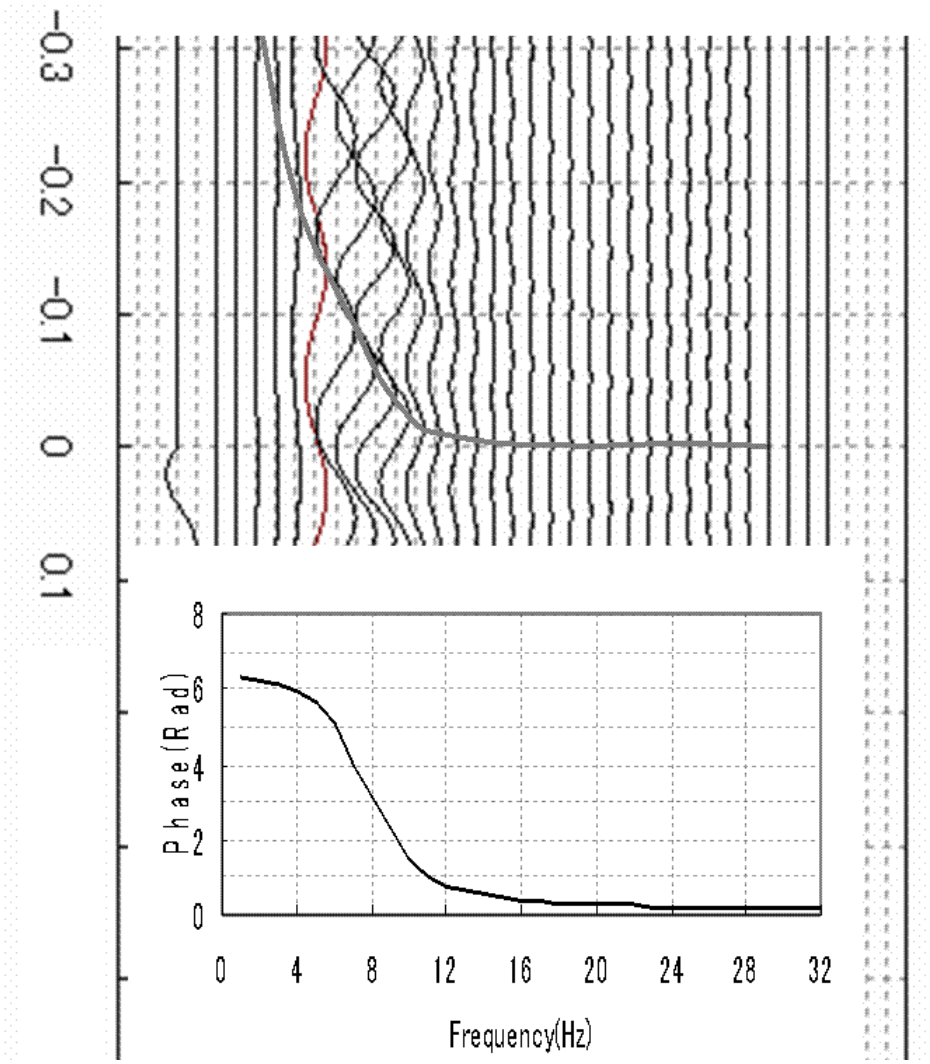


Fig. 12 An enlarged view of Fig. 9 that delineate the trend of the phase curve from curve to curve in comparison of the phase spectra in Fig. 11 *bottom right* panel. Produced newly based on the concept of Yilmaz(1994).

2.3. Discrete Fourier Transform

Define the time window length of the time series x_m as $T = N\Delta t$. Eq. (1) can be written as

$$C_k = \frac{1}{N\Delta t}\sum_{m=-N/2+1}^{N/2} x_m \Delta t e^{-i(2\pi km\Delta t/N\Delta t)} \quad , k = -N/2+1, \cdots, -1, 0, 1, \cdots, N/2.$$

Let Δt tend to zero while T is kept constant ($m\Delta t \to t$). This gives a continuous function in a limited time window.

$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{-i(2\pi kt/T)} dt, \quad -\infty < k \le \infty, k : discrete. \tag{3}$$

Similarly,

$$x(t) = \sum_{-\infty}^{\infty} C_k e^{i(2\pi kt/T)}, \qquad -T/2 < t \le T/2. \tag{4}$$

The frequency f in these formulas is given by $f = k/T$. Since k is an integer, the frequency f takes a discrete value with an interval $\Delta f = 1/T$. Eq. (3) and Eq. (4) show the Fourier expansion of a continuous time-windowed time function. Note again that Eq. (4) shows the repetitive nature of the re-constructed time function.

Change Eq. (4) by using $\Delta f = 1/T$.

$$x(t) = \sum_{-\infty}^{\infty} (TC_k) e^{i(2\pi k\Delta ft)} \Delta f, \qquad -1/2\Delta f < t \le 1/2\Delta f.$$

Let Δf tend to zero, i. e., let T tend to infinity ($k\Delta f \to f$). This means that the periodicity in Eq. (3) and Eq. (4) becomes eliminated. Eq. (3) gives

$$TC_k = \int_{-\infty}^{\infty} x(t) e^{-i(2\pi ft)} dt, \qquad -\infty < k \le \infty, \; k: continuous. \tag{5}$$

$$x(t) = \int_{-\infty}^{\infty} (TC_k) e^{i(2\pi ft)} df = \frac{1}{2\pi}\int_{-\infty}^{\infty} (TC_k) e^{i(2\pi\omega t)} d\omega, \qquad -\infty < t \le \infty. \tag{6}$$

A comparison with the definition of Fourier transform shows that (TC_k) corresponds to the Fourier transform. Thus, the discrete Fourier transform is given by

$$F(f) = TC_k = \frac{T}{N}\sum_{m=-N/2+1}^{N/2} x_m e^{-i(2\pi km/N)}, \tag{7}$$
$$f = k/T, \; k = -N/2+1, \cdots, -1, 0, 1, \cdots, N/2.$$

If the time series is defined in [$0, T = N\Delta t$], the limit of the summation has to be changed to [from $m = 0$ to $m = N$].

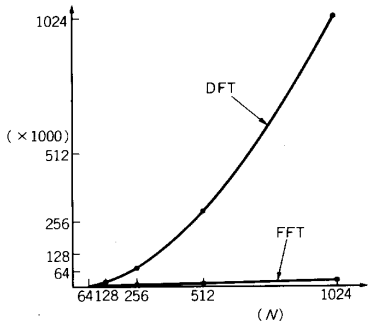


Fig. 13 Comparison of the calculation time of DFT and FFT.

2.4. Algorithm for FFT

The calculation of discrete Fourier transform by using Eq. (7) (denoted by DFT) uses considerable time. The time that is necessary for the calculation increases proportionally with N^2, where N is the number of samples. For example, the time *is 40* s for $N = 1024$ on a *486 DX2 50 MHz PC*.

Fast Fourier transform (FFT) is a technique to compute the Fourier transform of a time series efficiently; this was invented by J. W. Cooley and J. W. Tukey. The calculation time increases proportionally with $(N/2)\log_2 N$ (Fig. 13).

2.4.1. Removal of DC Component and Linear Trend

When the time series has a DC component or a linear trend, the Fourier spectrum cannot be estimated correctly because of the assumption of periodicity. Therefore, it is necessary to remove them before the application of the FFT. It is usually sufficient to remove the straight line connecting the first and last data; however, least square fitting is a recommended method.

Let the discrete time variable $t_n = n\Delta t$, and the objective time series $x_n = x(t_n)$. The misfit function S is defined as

$$S = \sum_{n=1}^{N} (r_n)^2 = \sum_{n=1}^{N} \{x_n - (at_n + b)\}^2,$$

where r_n denotes the residual of the fitting; a, the linear trend; and b, the DC component. The minimum value of this misfit function is given at

$$\frac{\partial S}{\partial a} = 0, \frac{\partial S}{\partial b} = 0.$$

Thus,

$$\begin{cases} \left(\sum_{n=1}^{N} n^2\right)\Delta t \cdot a + \left(\sum_{n=1}^{N} n\right) \cdot b = \sum_{n=1}^{N} (nx_n), \\ \left(\sum_{n=1}^{N} n\right)\Delta t \cdot a + \quad N \cdot b = \sum_{n=1}^{N} x_n . \end{cases} \tag{8}$$

where the formulas

$$\sum_{n=1}^{N} n = \frac{N(N+1)}{2}, \text{ and } \sum_{n=1}^{N} n^2 = \frac{N(N+1)(2N+1)}{6},$$

can make the calculation easier.

The coefficients of these linear simultaneous equations a and b can be easily obtained.

2.4.2. Tapering or Windowing

After removing the DC offset and linear trend, the processed time series begins with one value and ends with another. This causes an unexpected jump or step due to the implicit assumption of periodicity by FFT and results in a bad influence on the estimation of the Fourier transform. A method of preventing such an artificial effect is tapering or windowing, which causes the time series to start with zero and end with zero.

The processing comprises a multiplication with the window function *w(t)* in the time domain. The box car window is the simplest solution, but it also has the abovementioned problem.
The tapered window is given as follows:

$$w(t)=\begin{cases} t/t_{taper} & for\ 0 \le t < t_{taper}, \\ 1 & for\ t_{taper} \le t \le t_{win}-t_{taper}, \\ (t_{win}-t)/t_{taper} & for\ t_{win}-t_{taper} < t \le t_{win}, \\ 0 & for\ t_{win} < t. \end{cases} \tag{9.1}$$

The sine tapered window is given as follows:

$$w(t)=\begin{cases} \sin(\pi t/2t_{taper}) & for\ 0 \le t < t_{taper}, \\ 1 & for\ t_{taper} \le t \le t_{win}-t_{taper}, \\ \sin\{\pi(t_{win}-t)/2t_{taper}\} & for\ t_{win}-t_{taper} < t \le t_{win}, \\ 0 & for\ t_{win} < t. \end{cases} \tag{9.2}$$

The tapering time length t_{taper} is usually approximately one tenth of the time window length t_{win}.
The following two windowing functions are also used popularly.
Hanning Window:

$$w(t)=\begin{cases} 0.5-0.5\cos(\pi t/t_{taper}) & for\ t < t_{taper}, \\ 1 & for\ t_{taper} \le t \le t_{win}-t_{taper}, \\ 0.5-0.5\cos\{\pi(t_{win}-t)/t_{taper}\} & for\ t_{win}-t_{taper} < t \le t_{win}, \\ 0 & for\ t_{win} < t. \end{cases} \tag{9.3}$$

Hamming Window:

$$w(t)=\begin{cases} 0.54-0.46\cos(\pi t/t_{taper}) & for\ t < t_{taper}, \\ 1 & for\ t_{taper} \le t \le t_{win}-t_{taper}, \\ 0.54-0.46\cos\{\pi(t_{win}-t)/t_{taper}\} & for\ t_{win}-t_{taper} < t \le t_{win}, \\ 0 & for\ t_{win} < t. \end{cases} \tag{9.4}$$

2.4.3. Zero padding

The FFT can be performed efficiency when the number of data N is 2^n where n is an integer. Otherwise, zeros must be padded up to the nearest 2^n. Usually, the zeros are padded at the end of the time series. They are not padded at the beginning of the time series, even though this is acceptable theoretically. This is because padding them at the beginning apparently changes the arrival time and causes confusion.

2.4.4. Algorithm

Ohsaki(1976) explained the Algorithm for performing the FFT as a disassembling process.

First, the coefficients of the Fourier expansion of the original time series are given by

$$C_k = \frac{1}{N}\sum_{m=0}^{N} x_m e^{-i(2\pi km/N)},$$

Disassemble the time series x_m into two time series in the following manner:

$$\begin{cases} y_m = x_{2m}, \\ z_m = x_{2m+1}, \end{cases} \quad m = 0,1,2,\cdots,\frac{N}{2}-1.$$

The coefficients of the Fourier expansion of the disassembled time series are

$$\begin{cases} Y_k^{\langle N/2\rangle} = \dfrac{2}{N}\displaystyle\sum_{m=0}^{N/2-1} y_m e^{-i[2\pi km/(N/2)]}, \\ Z_k^{\langle N/2\rangle} = \dfrac{2}{N}\displaystyle\sum_{m=0}^{N/2-1} z_m e^{-i[2\pi km/(N/2)]}, \end{cases} \quad k = 0,1,2,\cdots,\frac{N}{2}-1. \tag{10.1}$$

The original definition becomes

$$\begin{aligned} C_k^{\langle N\rangle} &= \frac{1}{N}\sum_{m=0}^{N} x_m e^{-i(2\pi km/N)} \\ &= \frac{1}{N}\left\{\sum_{m=0}^{N/2-1} y_m e^{-i(2\pi k(2m)/N)} + \sum_{m=0}^{N/2-1} z_m e^{-i(2\pi k(2m+1)/N)}\right\} \\ &= \frac{1}{N}\sum_{m=0}^{N/2-1} y_m e^{-i(2\pi k(2m)/N)} + e^{-i[\pi k/(N/2)]}\frac{1}{N}\sum_{m=0}^{N/2-1} z_m e^{-i(2\pi km/(N/2))} \\ &= \frac{1}{2}Y_k^{\langle N/2\rangle} + \frac{1}{2}e^{-i[\pi k/(N/2)]}Z_k^{\langle N/2\rangle}, \end{aligned} \tag{10.2}$$

for $k = 0, 1, 2,...N/2 - 1$.

Replace k in Eq. (10.1) with $k + N/2$; then

$$\begin{cases} Y_{k+N/2}^{\langle N/2\rangle} = \dfrac{2}{N}\displaystyle\sum_{m=0}^{N/2-1} y_m e^{-i[2\pi(k+N/2)m/(N/2)]} = \dfrac{2}{N}\sum_{m=0}^{N/2-1} y_m e^{-i[2\pi km/(N/2)+2\pi m]} \\ \qquad = \dfrac{2}{N}\displaystyle\sum_{m=0}^{N/2-1} y_m e^{-i[2\pi km/(N/2)]} = Y_k^{\langle N/2\rangle}, \\ Z_{k+N/2}^{\langle N/2\rangle} = \dfrac{2}{N}\displaystyle\sum_{m=0}^{N/2-1} z_m e^{-i[2\pi(k+N/2)m/(N/2)]} = \dfrac{2}{N}\sum_{m=0}^{N/2-1} z_m e^{-i[2\pi km/(N/2)+2\pi m]} \\ \qquad = \dfrac{2}{N}\displaystyle\sum_{m=0}^{N/2-1} z_m e^{-i[2\pi km/(N/2)]} = Z_k^{\langle N/2\rangle}, \end{cases}$$

$$for\ k = 0,1,2,\cdots,\frac{N}{2}-1.$$

Eq. (10.2) gives

$$
\begin{aligned}
C_{k+N/2}^{\langle N\rangle} &= \frac{1}{2}Y_{k+N/2}^{\langle N/2\rangle} + \frac{1}{2}e^{-i[\pi(k+N/2)/(N/2)]}Z_{k+N/2}^{\langle N/2\rangle} \\
&= \frac{1}{2}Y_{k+N/2}^{\langle N/2\rangle} + \frac{1}{2}e^{-i[\pi k/(N/2)]}e^{-i\pi}Z_{k+N/2}^{\langle N/2\rangle} \\
&= \frac{1}{2}Y_{k}^{\langle N/2\rangle} - \frac{1}{2}e^{-i[\pi k/(N/2)]}Z_{k}^{\langle N/2\rangle}.
\end{aligned}
$$

This last change is due to Euler's formula. Therefore,

$$
\begin{cases}
2C_{k}^{\langle N\rangle} = Y_{k}^{\langle N/2\rangle} + e^{-i[\pi k/(N/2)]}Z_{k}^{\langle N/2\rangle}, \\
2C_{k+N/2}^{\langle N\rangle} = Y_{k}^{\langle N/2\rangle} - e^{-i[\pi k/(N/2)]}Z_{k}^{\langle N/2\rangle}.
\end{cases}
\quad k = 0,1,2,\cdots,\frac{N}{2}-1 \qquad (10.3)
$$

This process clearly shows that the coefficients of the Fourier expansion of the original time series x_m can be easily given by the coefficients of the Fourier expansion of the two time series y_m and z_m obtained by disassembling.

By applying the same process repetitively, N time series, each with only one sample, are obtained. The coefficient of the Fourier expansion of the time series having only one sample is the sample itself, as shown by

$$
C_0^{\langle 1\rangle} = \frac{1}{1}\sum_{m=0}^{0} x_m e^{-i(2\pi km/1)} = x_0. \qquad (10.4)
$$

Thus, the coefficients of the Fourier expansion of the original time series x_m can be obtained by using Eq. (10.3) repetitively. Let us check the process by using an example of a time series of 8 samples.

Example

Consider a time series of 8 samples, as shown in Table 1.

Table 1 (After Ohsaki(1976))

m	0	1	2	3	4	5	6	7
x_m	5	32	38	–33	–19	–10	1	–8

The first disassembling gives two series of 4 samples, as shown in Table 2. The second disassembling gives four time series of 2 samples, as shown in Table 3. The third disassembling gives eight series that has only one sample, as shown in Table 4.

Table 2 (After Ohsaki(1976))

m	0	1	2	3
y_m	5	38	–19	1
z_m	32	–33	–10	–8

Table 3 (After Ohsaki(1976))

m	0	1
y_m'	5	–19
Z_m'	38	1
y_m''	32	–10
z_m''	–33	–8

Table 4 (After Ohsaki(1976))

m	0
y_m'''	5
Z_m'''	–19
y_m''''	38
z_m''''	1
y_m'''''	32
Z_m'''''	–10
y_m''''''	–33
Z_m''''''	–8

Table 5. (After Ohsaki(1976))

k	0
y_k'''	5
Z_k'''	-19
y_k''''	38
z_k''''	1
y_k'''''	32
Z_k'''''	-10
y_k''''''	-33
Z_k''''''	-8

Table 6 (After Ohsaki(1976))

k	0	1
Y_k'	-7.0	12.0
Z_k'	19.5	18.5
Y_k''	11.0	21.0
Z_k''	-20.0	-12.5

The coefficients of the Fourier expansion of these 8 series of only one sample are given in Table 5. These are the same series as shown in Table 4.

By using relations such that

$$e^{-i0} = 1.0,$$
$$e^{-i[\pi/4]} = 0.7071 - 0.7071i,$$
$$e^{-i[\pi/2]} = -1.0i,$$
$$e^{-i[3\pi/4]} = -0.7071 - 0.7071i,$$

the coefficients of the Fourier expansion of the four time series of two samples in Table 3 are given in Table 6.

The coefficients of the Fourier expansion of the two time series of four samples in Table 2 are given in Table 7.

Table 7 (After Ohsaki(1976))

k	0	1	2	3
y_k	6.25	6.00 – 9.25i	–13.25	6.00 + 9.25i
Z_k	–4.75	10.50 + 6.25i	15.75	10.50 – 6.25i

Finally, the coefficients of the Fourier expansion of the original time series in Table 1 are given in Table 8.

Table 8 (After Ohsaki(1976))

k	0	1	2	3	4	5	6	7
C_k	0.75	8.922 – 6.128i	6.625 – 7.875i	–2.922 + 3.122i	5.5	–2.922 – 3.122i	–6.625 + 7.875i	8.922 + 6.128i

For the time series having a large number of samples, disassembling process will take time. The special feature, however, can shorten the disassembling process drastically. A comparison of Table 1 and Table 4 is shown in Table 9. The order numbers m' (binary) in Table 4 are completely *bit-reverse*d ones of those in Table1. This provides an efficient strategy to obtain pivoted time series like those in Table 4.

Table 9 (After Ohsaki(1976))

Table 1				Table 4			
x_m	m	m (binary)		m' (binary)	m'	(X_m)	m
5	0	000		000	0	5	0
32	1	001		100	4	–19	1
38	2	010		010	2	38	2
–33	3	011		110	6	1	3
–19	4	100		001	1	32	4
–10	5	101		101	5	–10	5
1	6	110		011	3	-33	6
-8	7	111		111	7	-8	7

Express the index m of x_m in binary, and then reverse their bit order to obtain the pivoted index m'. The pivoted time series X_m is given by $X_m = x_{m'}$. Begin the process to obtain the coefficients of a Fourier expansion by using Eq. (10.3).

2.4.5. Time Window and Periodicity

As mentioned above, the FFT can be applied only to a time series within a limited range of time. This time window of finite length can affect the estimation of the Fourier spectra.

Consider the following time window defined in *[–T/2, T/2]*:

$$B(t) = \begin{cases} 1, & |t| \le T/2, \\ 0, & |t| > T/2, \end{cases}$$

the Fourier transform of which is given by

$$B(\omega) = \int_{-\infty}^{\infty} B(t) e^{-i\omega t} dt = \int_{-T/2}^{T/2} B(t) e^{-i\omega t} dt = \left[\frac{1}{-i\omega} e^{-i\omega t} \right]_{-T/2}^{T/2} = \frac{2}{\omega} \sin\left(\frac{\omega T}{2} \right),$$

as shown in Fig. 14. Note that this depends on the window length *T*.

Applying the time window *B(t)* to the original function $x_0(t)$ implies the product of these two time functions:

$$x(t) = B(t) \cdot x_0(t).$$

The mathematically defined Fourier transform of the time windowed function *x(t)* is given by

$$x(\omega) = B(\omega) * x_0(\omega) = \frac{2}{\omega} \sin\left(\frac{\omega T}{2} \right) * x_0(\omega),$$

$$x(\omega) = B(\omega) * x_0(\omega) = \frac{2}{\omega} \sin\left(\frac{\omega T}{2} \right) * x_0(\omega),$$

where ω is the angular frequency and * denotes convolution. Since *B(ω)* depends on *T*, *x(ω)*also depends on *T*. The length of the time window can affect the result of the estimation of the Fourier transform for a time windowed function.

As mentioned previously, the coefficients of the Fourier expansion for a time series of a finite length C_k implicitly satisfy the assumption of periodicity outside the time window, whereas the discrete Fourier transform TC_k assumes zeros outside of the time window.

For a sinusoid that is time windowed by the same time length as its period multiplied by an integer, the coefficients of the Fourier expansion C_k is not affected by the length of the time window, whereas the discrete Fourier transform TC_k changes its value depending on *T*.

In contrast, for an impulse function, the coefficients of the Fourier expansion

$C_k = 1/N\Delta t = 1/T,$

changes, whereas

$TC_k = 1,$

does not depend on *T*.

Since the effect of the time window length is an artifact, it is better to select a measure that is not influenced by the window length in order to estimate the Fourier spectra of a given time series. The examples explained above suggest that the coefficients of the Fourier expansion C_k is a good measure for time series assumed to be a digitized part of a periodic function, because the feature of the original time

function coincides with the assumption accompanying C_k. Further, it is suggested that the discrete Fourier transform TC_k is a better measure of the estimation of the Fourier spectra of an impulse function.

However, actual seismic signals, are transient and neither periodic nor impulsive. Thus, there is an ambiguity with respect to the selection of a measure for estimating the frequency components of time series, that is, a time windowed and discretized function. It is important to recognize these characteristics of a discrete Fourier transform and the coefficients of Fourier expansion and to select an appropriate one for each problem.

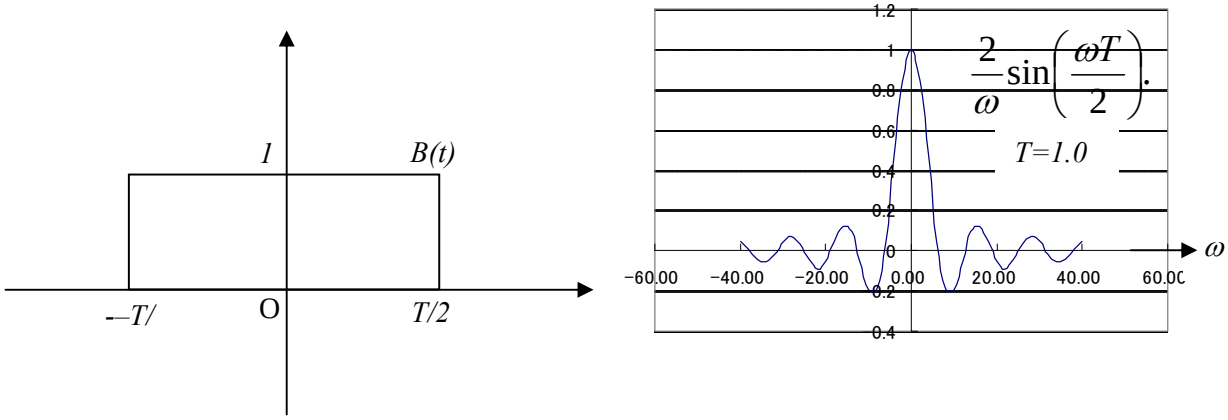


Fig. 14 Boxcar function and its amplitude spectra.

2.4.6 Spectral Smoothing

The Fourier spectra of real seismograms deviate considerably. Since the result of the FFT analysis are obtained for constantly sampled frequencies, the deviation is emphasized at higher frequency ranges. If plotted on a full logarithmic chart, the high frequency portion is almost completely painted black. This makes it difficult to observe a general tendency. Moreover, they occasionally take very small values. This causes instability of the spectral ratio, simply when required. Therefore, the spectral smoothing techniques are applied widely.

In order to plot them on a linear logarithmic chart, a simple moving average over the frequency works well typically.

$$Y(f_i) = \sum_{j>i-u}^{j<i+u} y(f_j),$$

where u denotes half bandwidth.

The following are examples for the weighted moving average that are applied repeatedly until the processed spectra become sufficiently smooth.

$Y(f_i) = 0.25y(f_{i-1}) + 0.5y(f_i) + 0.25y(f_{i+1})$,

$Y(f_i) = 0.23y(f_{i-1}) + 0.54y(f_i) + 0.23y(f_{i+1})$.

The weight coefficients can be given in the form of specially selected functions *w(f)*. For example,

$$w(f) = u \cdot \left(\sin \tfrac{\pi u f}{2} \Big/ \tfrac{\pi u f}{2}\right)^2 \quad \text{: Bartlett window}$$

$$w(f) = 0.75u \cdot \left(\sin \tfrac{\pi u f}{2} \Big/ \tfrac{\pi u f}{2}\right)^4 \quad \text{: Parzen window}$$

$$w(f) = a\left[\sin\left\{b \log_{10}(f / f_c)\right\} / b \log_{10}(f / f_c)\right]^4 \quad \text{: Logarithmic window}$$

There is a trade-off relation between the capacity of smoothing techniques to a stabilizing spectra and the resolution of the processed spectra. Thin peaks may be smoothed out and diminished by efficient smoothing. Insufficient smoothing cannot be used to show the general features of spectra. The objectives of smoothing are not achieved in the both extreme cases. The only way to find an appropriate smoothing technique is the trial-and-error approach.

2.5. Practice for FFT

The topics in this chapter can be understood more easily if the distributed programs are used for practice. In the following pages, the coefficients of the Fourier expansion obtained by FFT, amplitude spectra, and phase spectra are calculated for various test signals. *Ghost View* and *Ghost Script* can draw G.PS on the computer.

The following six programs have been prepared for practice:

TESTSIG.EXE prepares test signals such as cosine or sine function, impulse, etc.
PTIME.EXE plots the signal.
FFT.EXE calculates Fourier coefficients by using FFT.
PCFFT.EXE plots raw coefficients of FFT.
PSPEC.EXE plots discrete Fourier transforms (Fourier spectra),
IFFT.EXE calculate the inverse Fourier transform from data given in the frequency domain.

(1) Assume that *UT* is a filename for the time series *u(t)* and *UF* its FFT coefficients *U(f)*. First make the file *UT* by using *TESTSIG*. The output from *TESTSIG* is *UT*.
(2) Draw *UT* in the file *G.PS* by using *PTIME*. The input file name for *PTIME* is *UT* and the output file name is *G.PS*.
(3) Calculate the coefficients of the Fourier expansion for the time series stored in the file *UT* by using *FFT*. The input file is *UT* for *FFT* and the output file is *UF*.
(4) Draw the coefficients of the Fourier expansion stored in the file *UF* by using *PCFFT*. The input file is *UF* for *PCFFT* and the output is *G.PS*.
(5) Draw the Fourier spectra for the data stored in the file *UF* by using *PSPEC*. The input file is *UF* for *PSPEC* and the output is *G.PS*.

The data in the file *UT* consist of number of data *N* and the sampling interval Δt or the frequency interval Δf in the header, followed by data in “one-data-a-line” format. The programs are prepared separately so that you could use each of them as a basic tool of data processing.

2.5.1. Cosine and Sine Wave

Exercise: Cosine wave

Compute the FFT of a cosine wave with Δt = 1.0 s, N = 32, period = 16.0 s, amplitude = 10.0, phase = 0.0, and damping = 0.0.

Draw the time series, the FFT coefficients, and Fourier spectra.

Note that the original cosine wave is decomposed into two cosine waves of a half amplitude having positive and negative frequencies as

$$u(t) = 5.0e^{i2\pi ft} + 5.0e^{-i2\pi ft} = 10.0\cos 2\pi ft. \qquad (11.1)$$

Note that the calculated FFT coefficients have these values, and the Fourier spectra has the value of 5.0 × 32.0 = 160.0

Repeat the same procedure but with $N = 64$ and check the amplitude of the Fourier spectra and FFT coefficients.

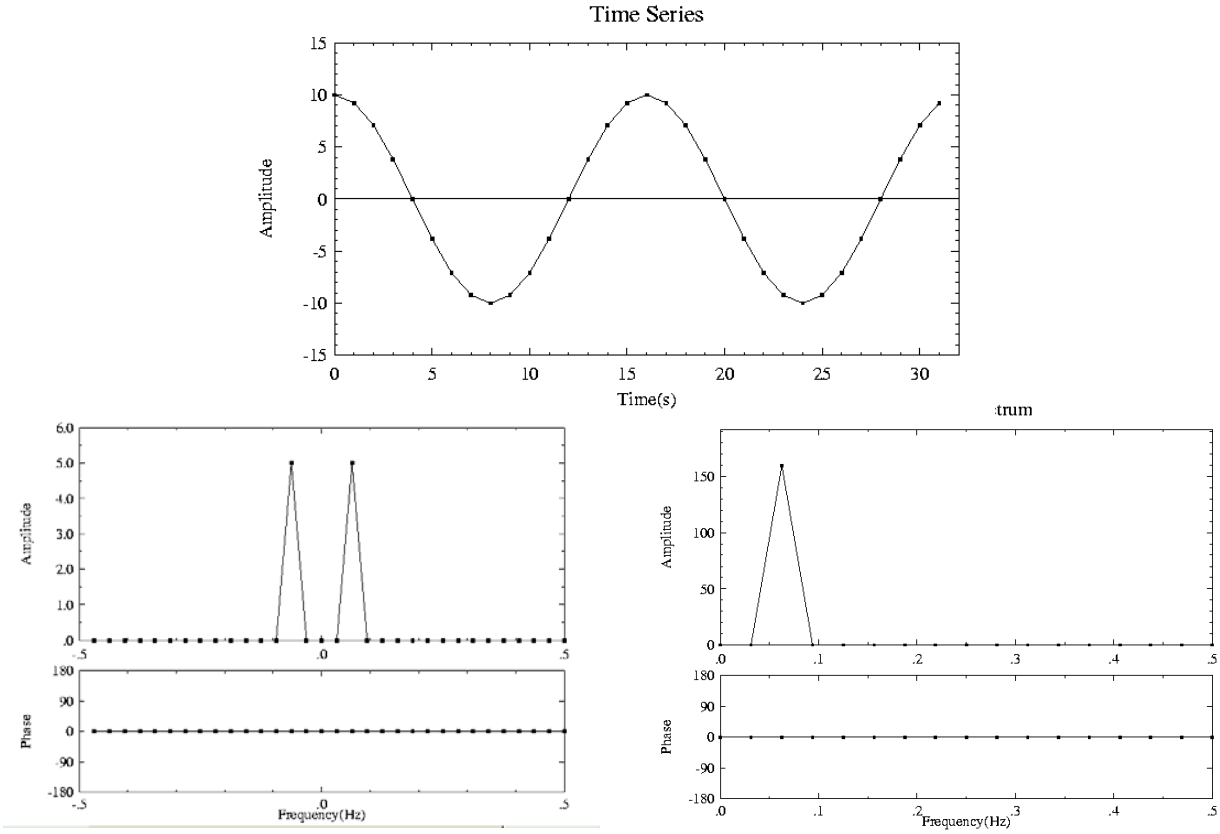


Fig. 15 Time series (*top*), coefficients of FFT (*left bottom*), and discrete Fourier spectra (*right bottom*) of the given time series, i.e., a cosine function.

Exercise: Sine wave

Calculate the FFT of a sine wave by making phase = 90.0 (that is, the progress of a phase in deg.) with Δt = 1.0 s, N = 32, period = 16.0 s, amplitude = 10.0, and damping = 0.0.

Draw the time series, FFT coefficients, and Fourier spectra.

Note that the phase ϕ for the positive and negative frequencies have opposite signs, i. e.,

$$u(t) = 5.0e^{i\phi}e^{i2\pi ft} + 5.0e^{-i\phi}e^{-i2\pi ft} = 5.0e^{i(2\pi ft+\phi)} + 5.0e^{-i(2\pi ft+\phi)}$$
$$= 10.0\cos(2\pi ft + \phi) = -10.0\sin 2\pi ft. \qquad (11.2)$$

The last change is due to ϕ = 90.0 degrees. A complex conjugate relation of the FFT coefficients for the negative frequencies with those for the corresponding positive ones are required to ensure that the original time series is real.

Note again that the calculated FFT coefficients have such values, the Fourier spectra has the value of 5.0 × 32.0 = 160.0, and the phase is 90.0 degrees.

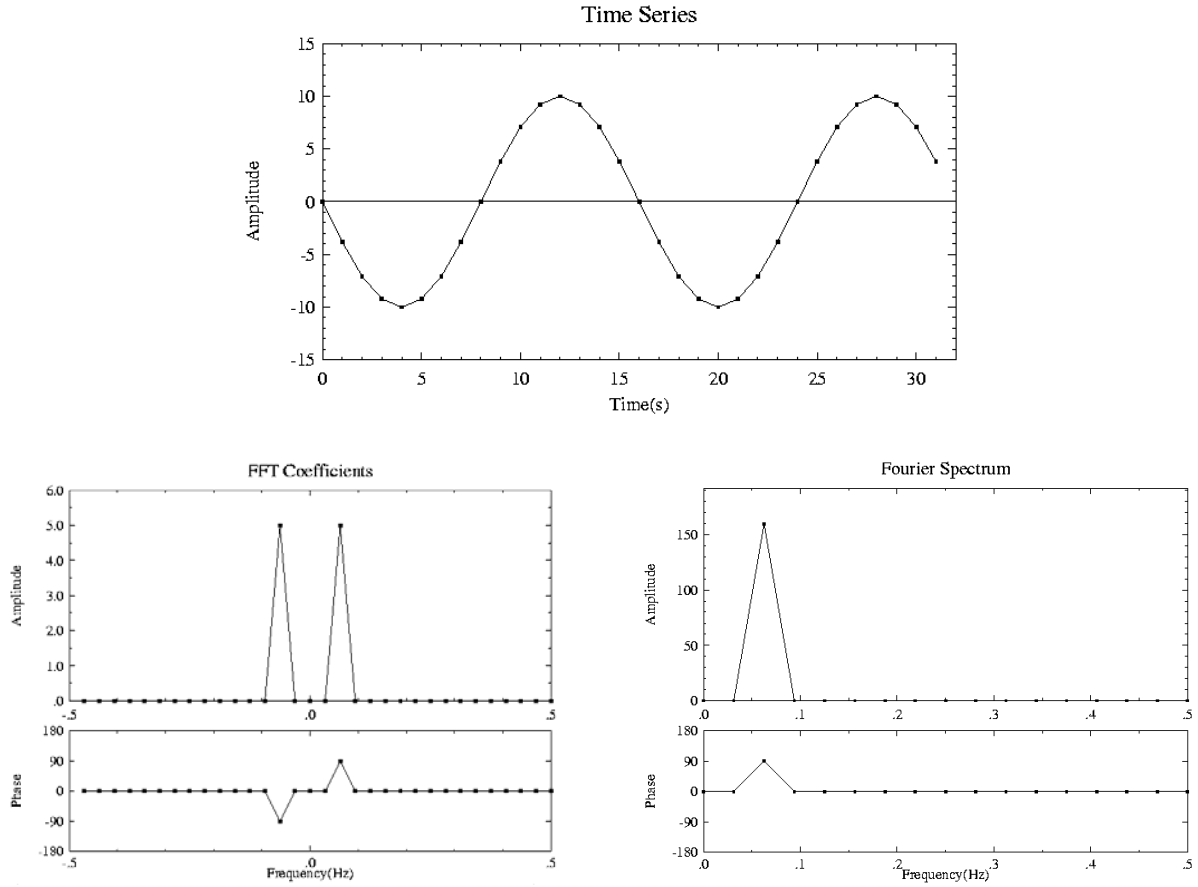


Fig. 16 Time series (*top*), coefficients of FFT (*left bottom*), discrete Fourier spectra (*right bottom*) of the given time series, i. e., a sine function.

Exercise: Cosine wave at the Nyquist frequency

Calculate the FFT of a cosine wave by using phase = 0.0 with Δt = 1.0 s, N = 32, amplitude = 10.0, and damping = 0.0, and with the period corresponding to the Nyquist frequency $f_{Nyquist} = 1/2\Delta t = 0.5$ *Hz*.

Draw the time series, FFT coefficients, and Fourier spectra.

Note that the coefficient for $f_{Nyquist}$ = 10.0, i. e., it does not share the amplitude with the coefficient for $-f_{Nyquist}$.

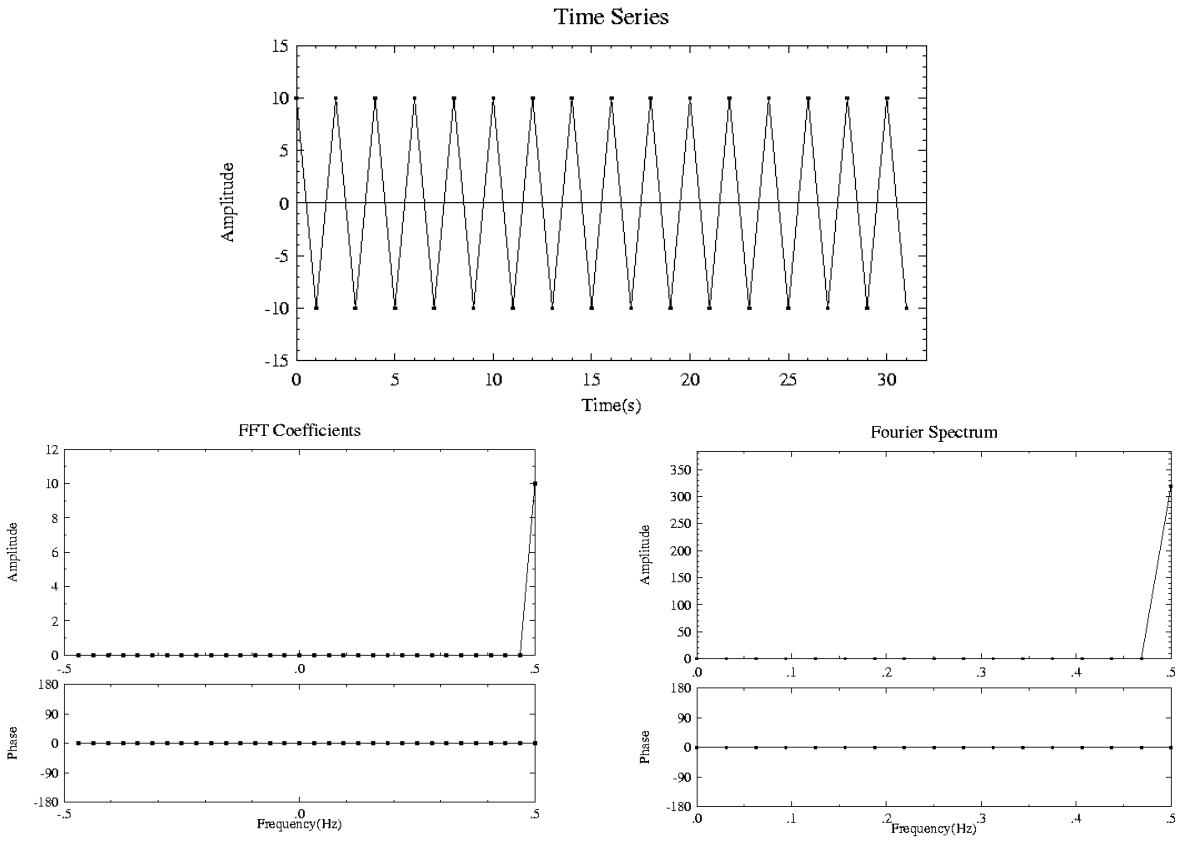


Fig. 17 Time series (*top*), coefficients of FFT (*left bottom*), discrete Fourier spectra (*right bottom*) of the given time series, i. e., a cosine function at the Nyquist frequency.

Exercise: Summation

Calculate the FFT of two cosine waves with (T = 16.0 s, A = 10.0) and (T = 2.0 s, A = 10). Other parameters are common, i. e., Δt = 1.0 s, N = 32, amplitude = 10, phase = 0, and damping = 0.0.

Draw the time series, FFT coefficients, and Fourier spectra for each cosine wave and for the summed one. Additivity is one of the basic characteristics of Fourier transforms.

$$u(t)+v(t)=\int_{-\infty}^{\infty}\left[U(f)+V(f)\right]e^{-2\pi ift}\,df\,. \tag{11.3}$$

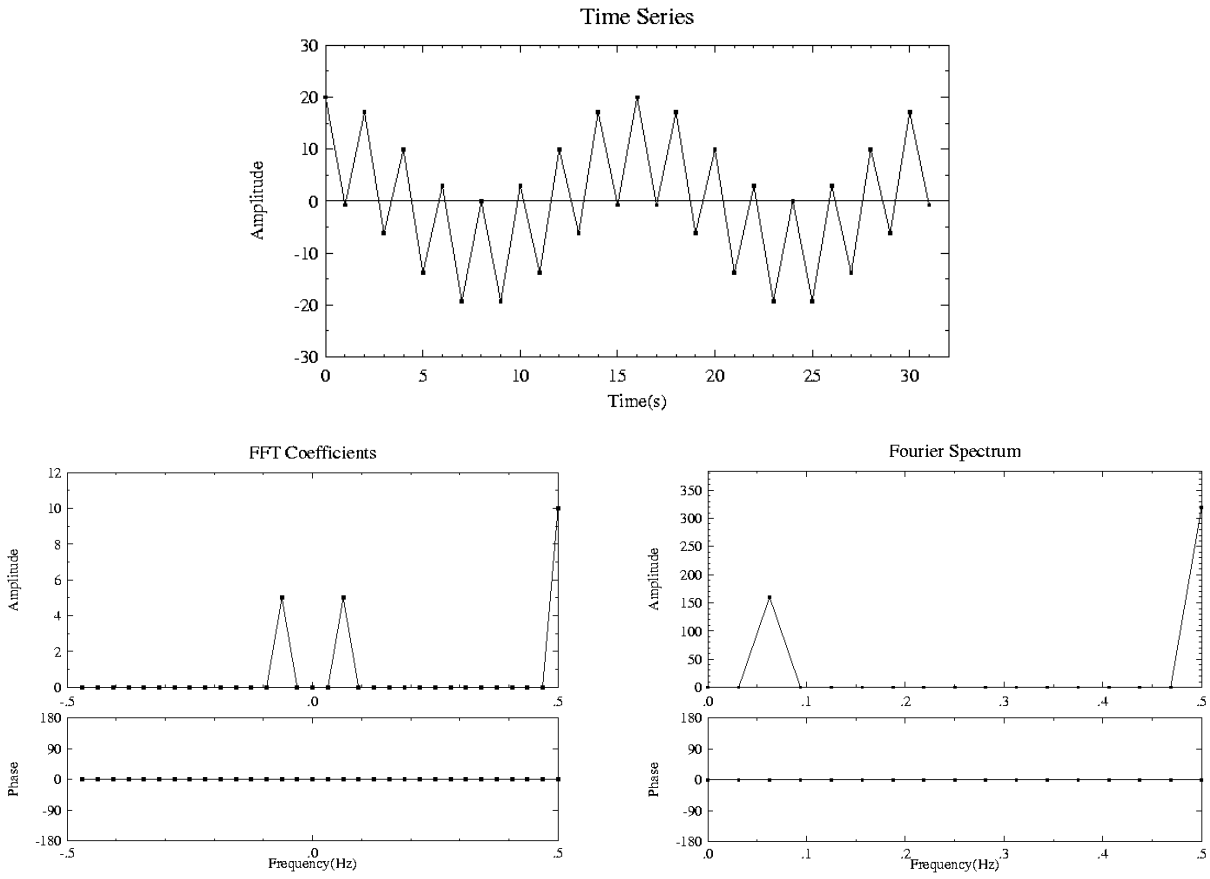


Fig. 18 Time series (*top*), coefficients of FFT (*left bottom*), discrete Fourier spectra (*right bottom*) of the given time series, i .e., two superposed cosine functions with different frequencies.

2.5.2. Constant

Exercise: Constant

Calculate the FFT for a constant with the constant = 10.0 and Δt = 1.0 s, N = 32.

Draw the time series, FFT coefficients, and Fourier spectra.

Note that the amplitude of the Fourier spectra at zero frequency is

$$TC_k = N\Delta t C_k = 32 \times 1.0 \times 10.0 = 320.0,$$

whereas the FFT coefficient at zero frequency is C_k = 10.0.

The FFT coefficient and Fourier spectra at the zero frequency correspond to the DC component of the time series.

Repeat the same procedure but with $N = 64$ and check the amplitude of the Fourier spectra and FFT coefficients.

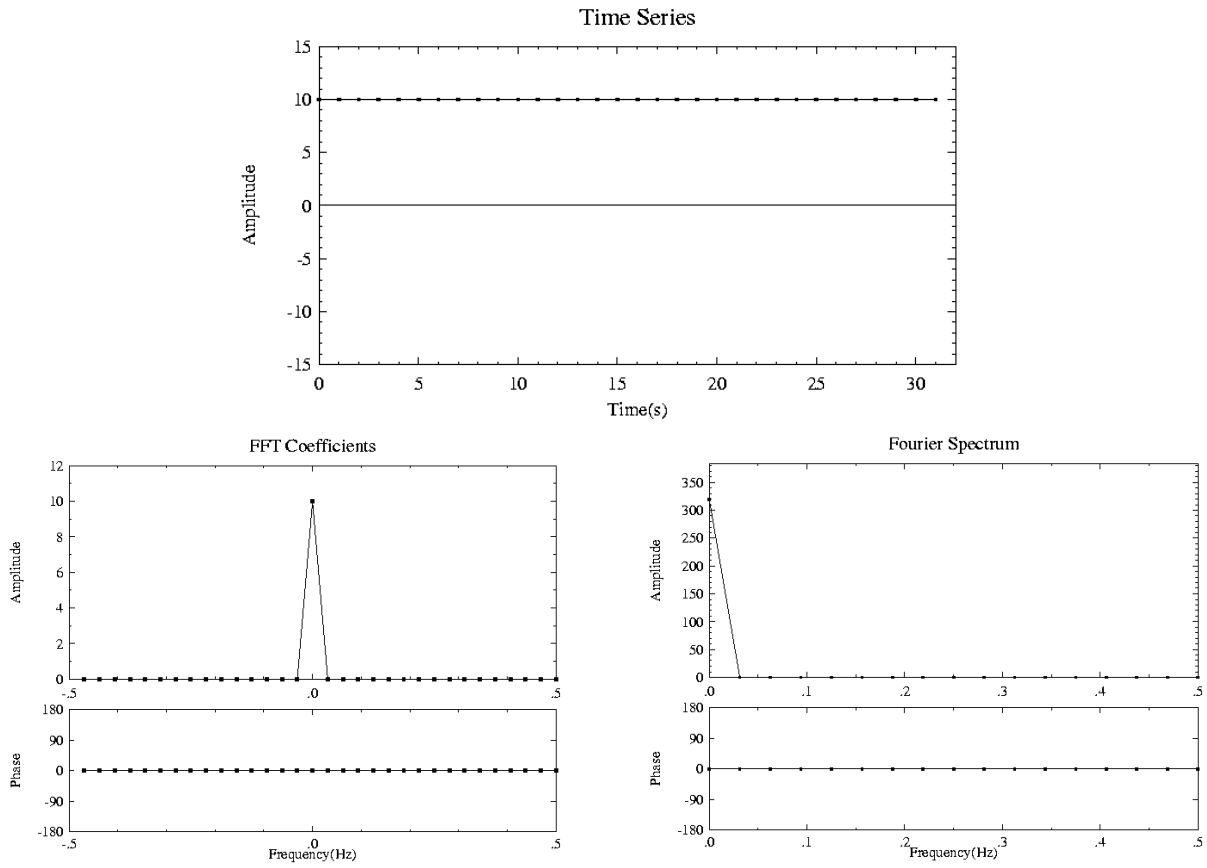


Fig. 19 Time series (*top*), the coefficients of FFT (*left bottom*), the discrete Fourier spectra (*right bottom*) of the given time series, i. e., a constant function.

2.5.3. Impulse

Exercise: Calculate the FFT for an impulse at t = 0.

Calculate the FFT for an impulse of amplitude = 10.0 with Δt = 1.0 s, N = 32, amplitude = 10.0, and location = 0.0.

Draw the time series, FFT coefficients, and Fourier spectra.

For a "continuous-infinite" case, the Fourier transform of a Dirac delta function

$$\delta(t) = \begin{cases} +\infty & t = 0 \\ 0 & t \neq 0 \end{cases}, \quad \int_{-\infty}^{\infty} \delta(t)dt = 1 \tag{11.4}$$

is known to be unity.

$$\int_{-\infty}^{\infty} \delta(t) e^{-i2\pi ft} dt = 1 .$$

Therefore, the mathematical Fourier transform of an impulse, i. e., the delta function at $t = 0$ is a constant and has zero phase.

Note that the amplitude of the Fourier spectra, which is constant for all frequencies, coincides with that of the impulse = 10.0, whereas the FFT coefficients have the value of $A_{impulse}/N$ = 10.0/32 = 0.3125.

Repeat the same procedure but with $N = 64$ and check the amplitude of the Fourier spectra and FFT coefficients.

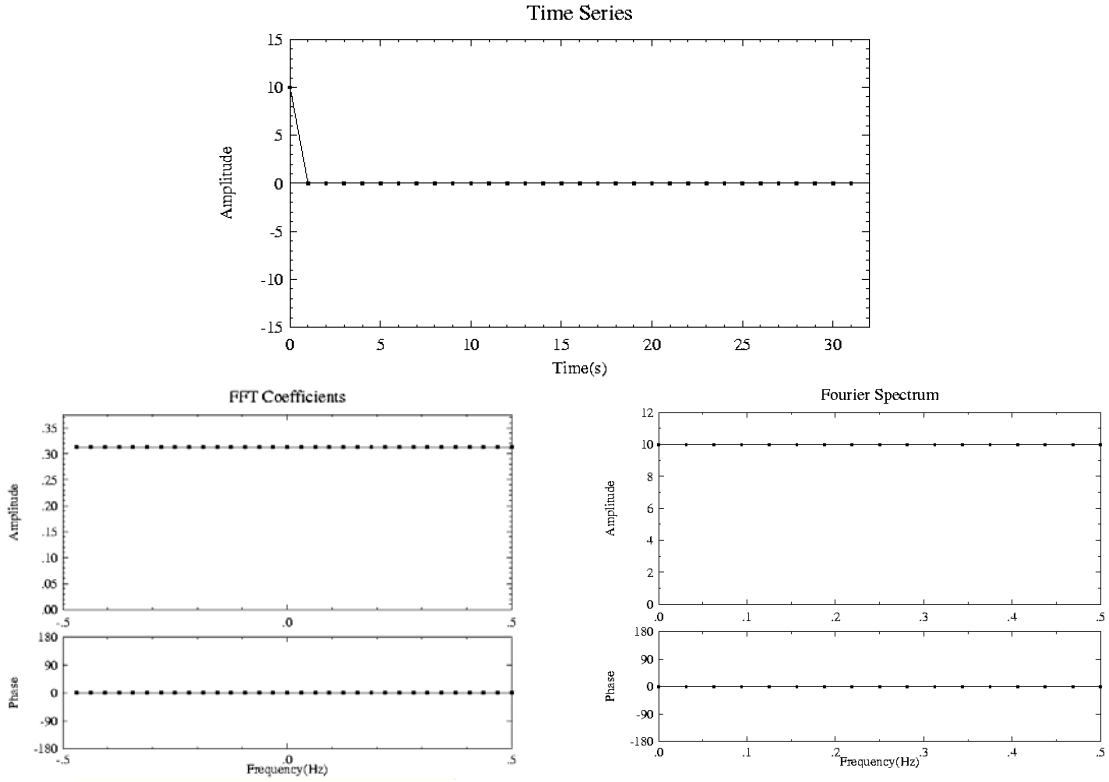


Fig. 20 Time series (*top*), coefficients of FFT (*left bottom*), discrete Fourier spectra (*right bottom*) of the given time series, i. e., an impulse at t = 0.0.

2.5.4. Time Shift

Exercise: Compute FFT of an impulse at t = 2.0, 4.0, 6.0, ...

Calculate the FFT of an impulse of amplitude = 10.0 with Δt = 1.0 s, N = 32, amplitude = 10.0, and location = 2.0, 4.0, 6.0…

Draw the time series, FFT coefficients, and Fourier spectra.

The Fourier transform of a time-shifted signal $u(t-\tau)$ is known to be as follows:

$$\int_{-\infty}^{\infty} u(t-\tau)e^{-i2\pi ft}dt = e^{-i2\pi f\tau}U(f),$$

(11.5)

which introduces $2\pi f\tau$ phase shifts to $U(f)$.

The Fourier transform of a time-shifted signal is $u(t-\tau)$ is known to be as follows:

$$\int_{-\infty}^{\infty} u(t-\tau)e^{-i2\pi ft}dt = e^{-i2\pi f\tau}U(f), \qquad (11.6)$$

which introduces $2\pi f\tau$ phase shifts to $U(f)$.

Note that there is no difference with respect to the amplitude Fourier spectra and FFT coefficients among the shifted impulses, and that the slope of the phase spectra increases with the time shift given to the impulse in the time domain.

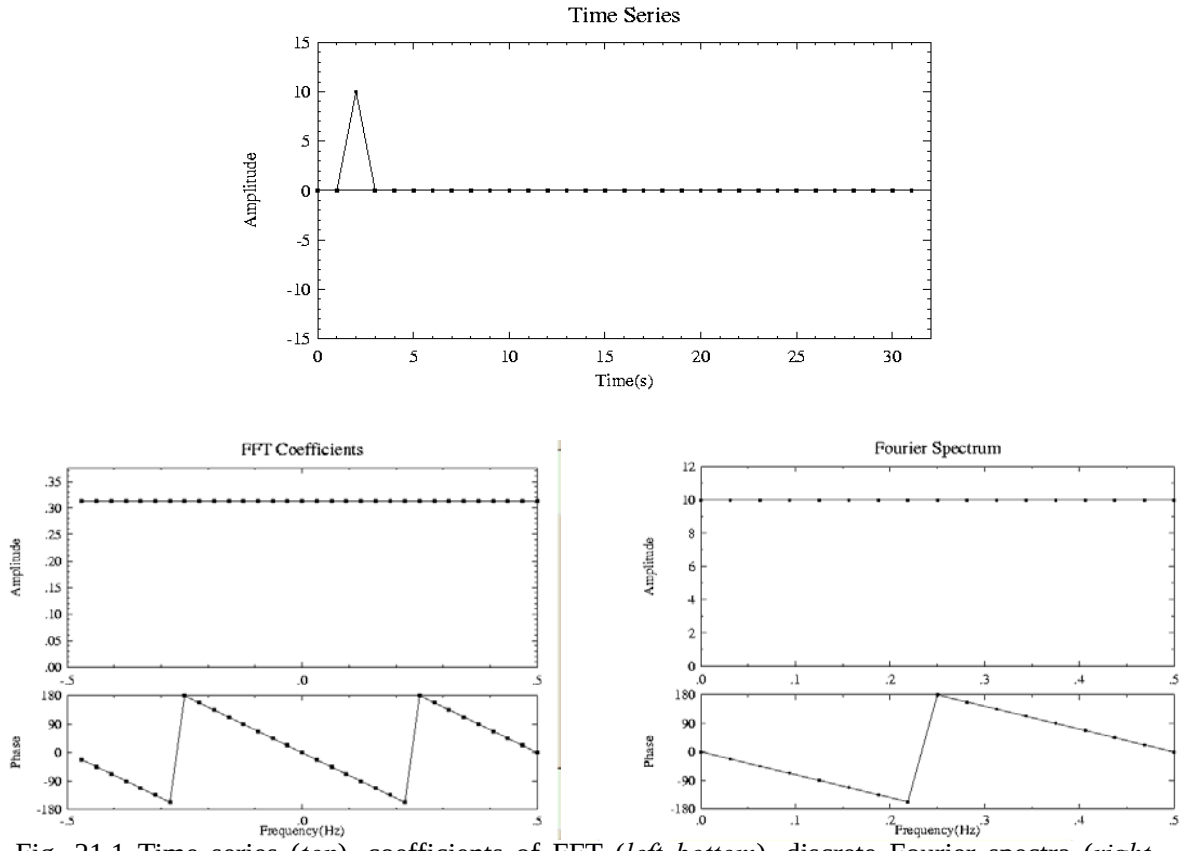


Fig. 21.1 Time series (*top*), coefficients of FFT (*left bottom*), discrete Fourier spectra (*right bottom*) of the given time series, i. e., an impulse at t = 2.0.

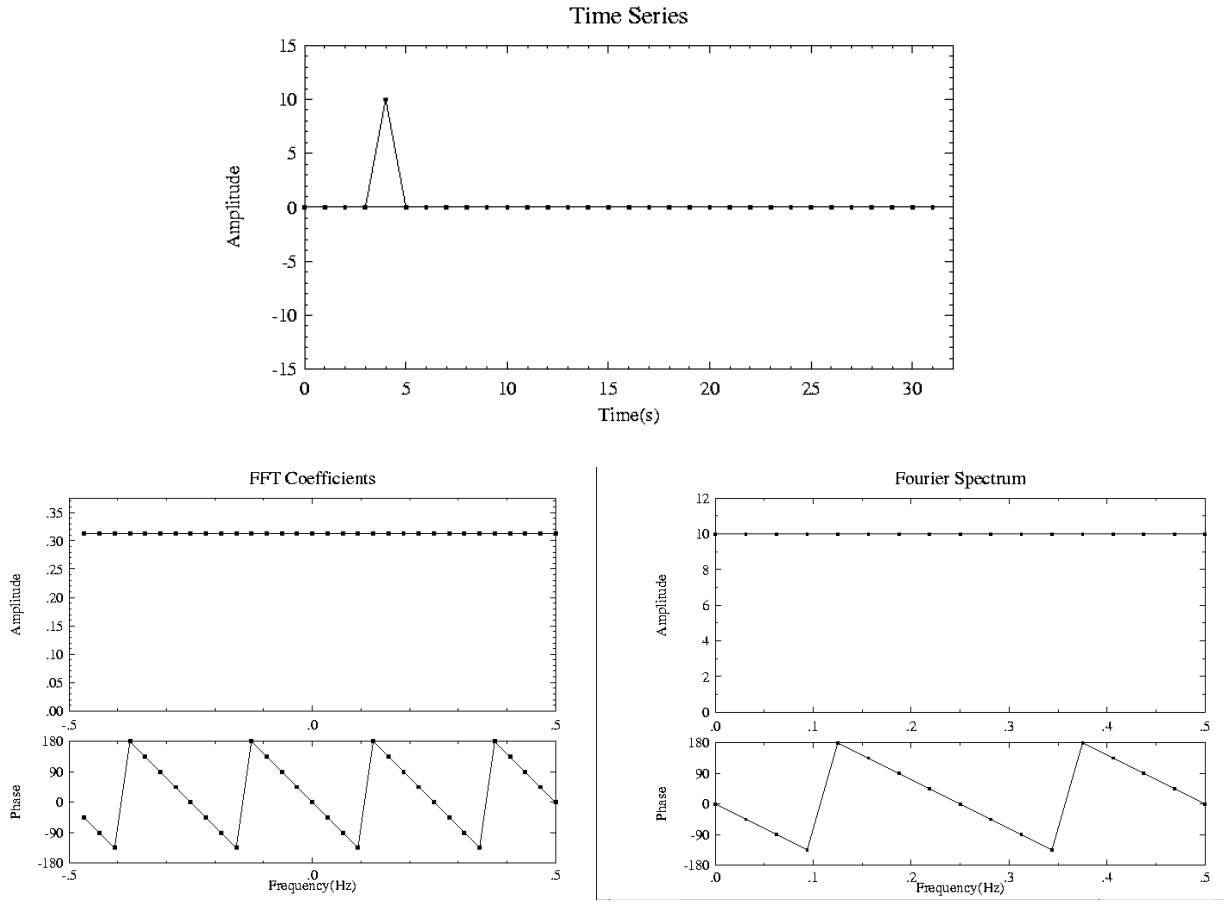


Fig. 21.2 Time series (*top*), Coefficients of FFT (*left bottom*), discrete Fourier spectra (*right bottom*) of the given time series, i. e., an impulse at $t = 4.0$.

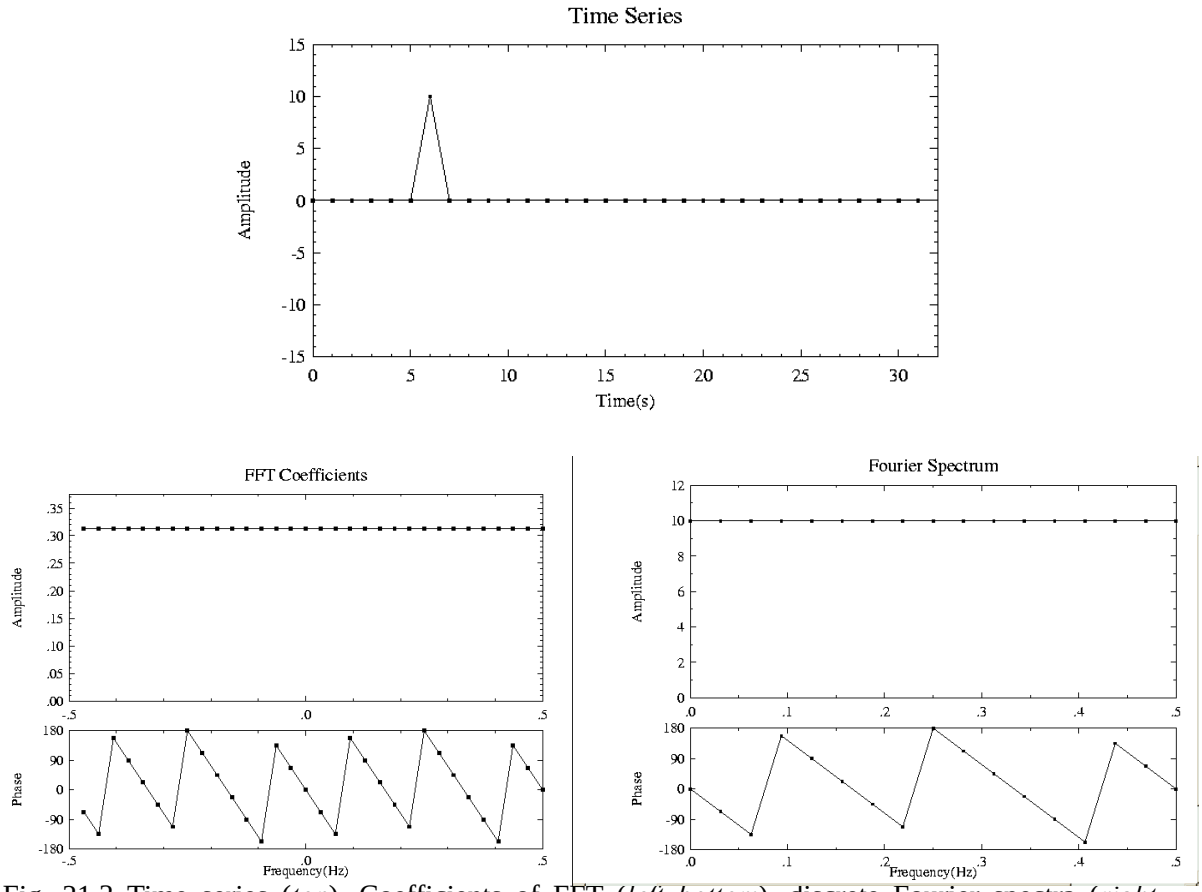


Fig. 21.3 Time series (*top*), Coefficients of FFT (*left bottom*), discrete Fourier spectra (*right bottom*) of the given time series, i. e., an impulse at $t = 6.0$.

2.5.5. Aliasing

Exercise: Simulate aliasing effect.

Calculate the cosine wave having a period of 0.8 s (1.25 Hz) with Δt = 1.0 s, N = 32, amplitude = 10.0, damping = 0.0, and phase = 0.0.

The frequency of 1.25 Hz is higher than the Nyquist frequency 0.5 Hz. The time series panel below does not appear to be a 1.25-Hz wave. A false peak due to aliasing is observed in the spectrum panels.

Aliasing occurs due to the $f = \pm(f_0 \pm 2f_{Nyquist})$ ambiguity of the frequency. In this example, the peak at 1.25 Hz is folded into 1.25 – 2*0.5 Hz = 0.25 Hz.

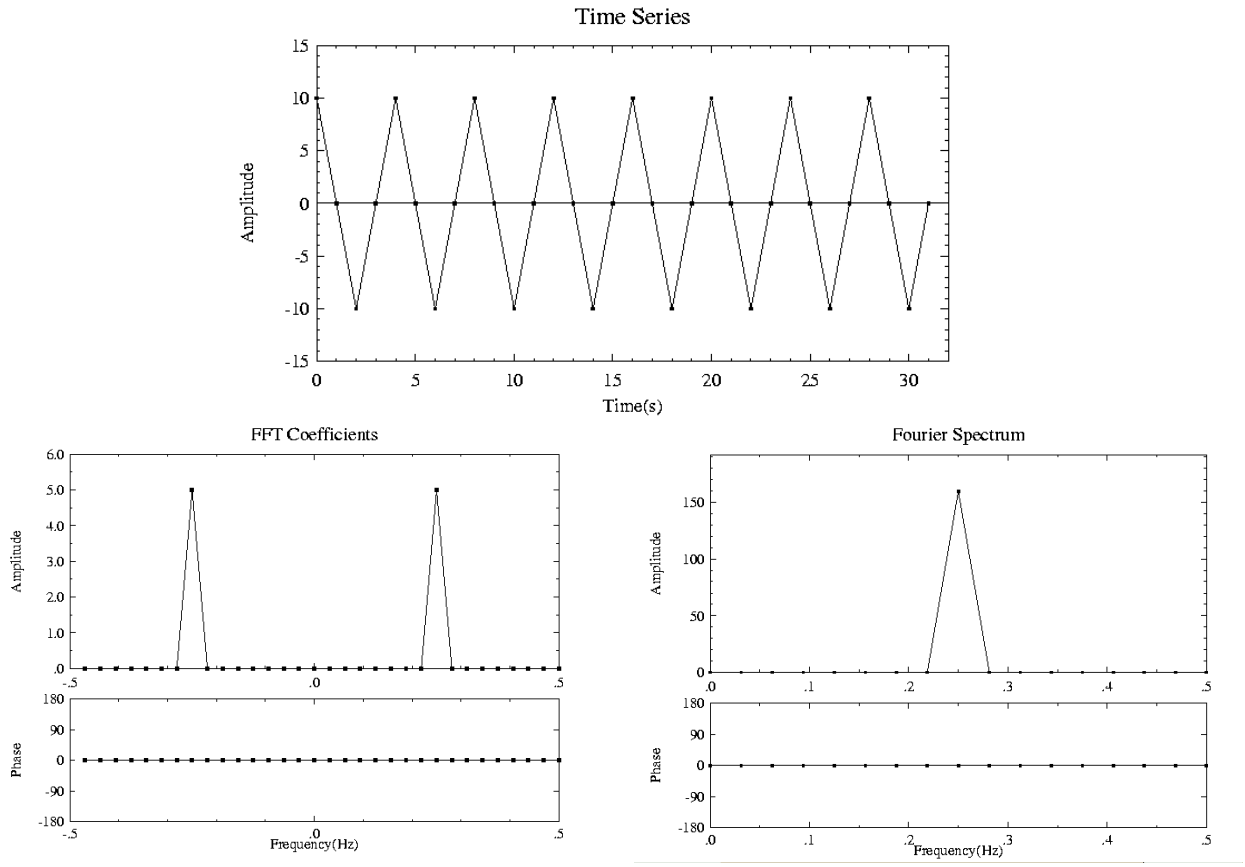


Fig. 22 Time series (*top*), coefficients of FFT (*left bottom*), discrete Fourier spectra (*right bottom*) of the given time series, i. e., a cosine function at the frequency higher than the Nyquist frequency.

2.5.6. Assumption of Periodicity

Exercise:

Calculate a cosine wave period = 7.0 s (0.14 Hz) Δt = 1.0 s, N = 32, amplitude = 10.0, damping = 0.0, phase = 0.0, and consider why a line spectrum at f = 0.14 Hz could not be obtained.

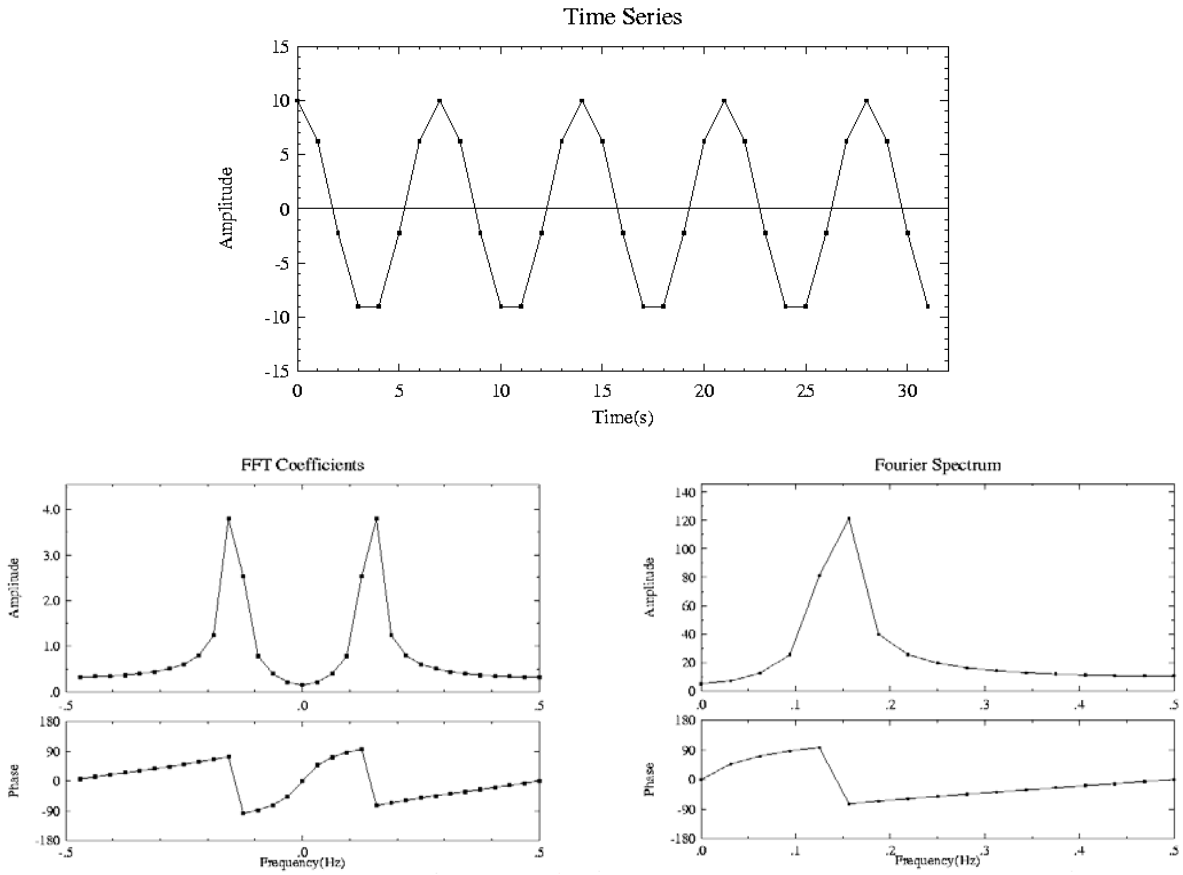


Fig. 23 Time series (*top*), coefficients of FFT (*left bottom*), discrete Fourier spectra (*right bottom*) of the given time series, i. e., a cosine function at the frequency mentioned above.

3. Filtering Techniques

Recorded signals are often contaminated by AC noise or high-frequency ground noise from nearby stations. Therefore, various filtering techniques are essential for digital data processing.

3.1. Weighted Moving Average

The simplest method to suppress high-frequency components included in a given time series $x_m = x(t_m)$ may be moving averages such as

$$y_m = \frac{x_{m-1} + x_m + x_{m+1}}{3}.$$

This equation shows a three-point moving average. The output y_m is defined by the one-step previous term of the input x_{m-1}, the present term x_m, and the future term x_{m+1}.

The five-point moving average is given by

$$y_m = \frac{x_{m-2} + x_{m-1} + x_m + x_{m+1} + x_{m+2}}{5}.$$

The performance of the moving average can be estimated by applying it to an impulse of the unit amplitude located at $t = 0.0$, because the moving average belongs to linear systems. Fig. 24 shows the Fourier spectra of the output corresponding to the impulse input for the two moving averages mentioned above. As shown here, the moving average can certainly be used to eliminate high-frequency components. However, it is difficult to control the performance and value of parameters such as the cut-off frequency, the slope of the cut-off, etc.

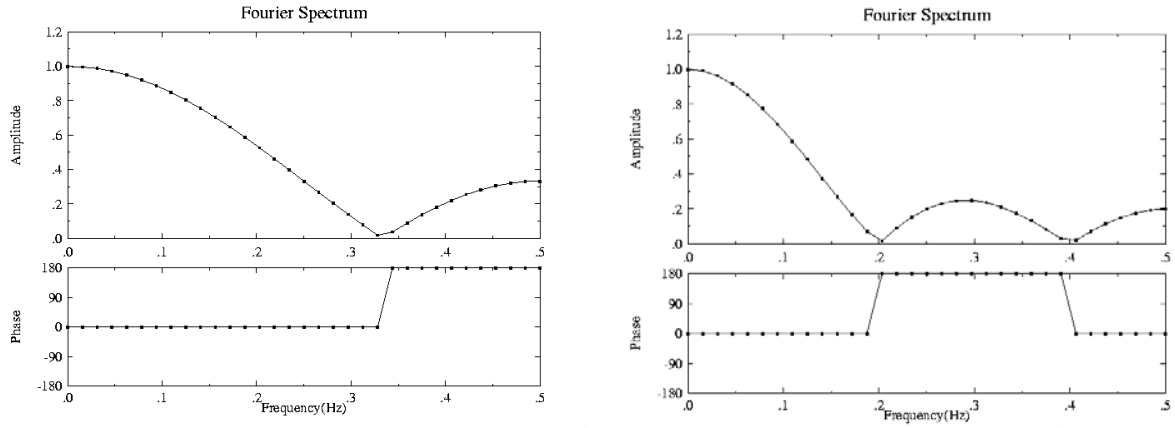


Fig. 24 Fourier spectra of the output from the three-point moving average corresponding to the impulse input (*left*) and that from the five-point moving average (*right*). These examples are calculated with Δt = 1.0 s and N = 64.

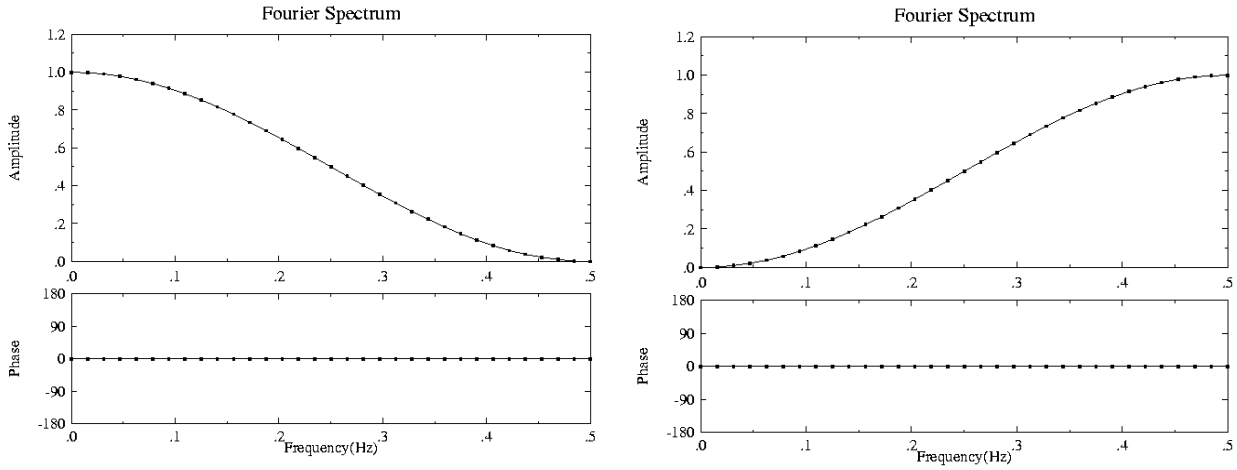


Fig. 25 Fourier spectra of the output from the weighted moving average corresponding to the impulse input. *Left*: for the weight coefficients (0.25, 0.5, and 0.25). *Right*: for the weight coefficients (–0.25, 0.5, –0.25). These examples are calculated with Δt = 1.0 s and N = 64

The weighted moving average is a similar procedure but with different weight coefficients. This gives a better performance than that given by the simple moving average.

For example, the three-point average

$$y_m = 0.25x_{m-1} + 0.5x_m + 0.25x_{m+1},$$

can eliminate high-frequency components, as shown in Fig. 25 (*left*). Note that this maintains the phase lag at zero for all frequencies.

The other example,

$$y_m = -0.25x_{m-1} + 0.5x_m - 0.25x_{m+1},$$

can eliminate low-frequency components as shown in Fig. 25 (*right*). Note that the phase lag is maintained at zero for all frequencies.

The differentiation of a continuous function *x(t)*,

$$y(t) = \frac{d}{dt}x(t),$$

can be approximated by the finite difference,

$$y_m = \frac{x_m - x_{m-1}}{\Delta t} = \left(-\frac{1}{\Delta t}\right)x_{m-1} + \left(\frac{1}{\Delta t}\right)x_m.$$

This also belongs to the weighted moving average.

These examples show that the weighted moving average can have a good performance. In other words, we can arrange its characteristics by selecting the weight coefficients. By using the idea of the impulse response, we can check the characteristics of its performance. However, we must design the weighted moving average by selecting the weight coefficients in such a way that the characteristics of the performance are obtained as desired.

3.2. Convolution—Filtering in the Time Domain

In order to understand the methods that are used to design weight coefficients, in this chapter, the procedure for "convolution" in the time domain is examined.

3.2.1. Convolution

Suppose that the Fourier transform of the time dependent functions *f(t)* and *g(t)* are *F(ω)* and *G(ω)*, respectively. The inverse Fourier transform of the product of *F(ω)* with *G(ω)* is given by

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)G(\omega)e^{i\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega t}d\omega\left(\int_{-\infty}^{\infty} g(\tau)e^{-i\omega\tau}d\tau\right)$$

$$= \int_{-\infty}^{\infty} g(\tau)d\tau\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega(t-\tau)}d\omega$$

$$= \int_{-\infty}^{\infty} g(\tau)f(t-\tau)d\tau.$$

This integration is referred to as the **convolution** of two functions *f(t)* and *g(t)* in the range $(-\infty, \infty)$. Convolution is usually expressed by an asterisk between two functions.

$$f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$$

If *f(t)* has a non-zero value only in the range $t_1 < t < t_2$,

$$f(t) * g(t) = \int_{t_1}^{t_2} f(\tau)g(t-\tau)d\tau = \int_{t-t_2}^{t-t_1} f(t-\tau)g(\tau)d\tau$$

Mathematically, convolution in the time domain corresponds to the product of the Fourier transforms in the frequency domain. The time domain operation may have advantages when one of the two time series has a short duration. In such a case, the short time series *f(t)* is considered as a filter for modifying the input signal *g(t)*. The effect of filtering must be controlled by the spectrum of the filter *f(t)*. Note that the amplitude spectrum of the output is the product of the amplitude spectra of the two original signals and the phase spectrum of the output is the sum of their phase spectra.

3.2.2. Filtering in the Time Domain by Convolution

Let us examine the calculation procedures in a computer for the convolution of two time series. Assume that *f(t)* has a short duration with non-zero values only within $[0, t_M]$.

$$h(t) = f(t) * g(t) = \int_{t-t_M}^{t} f(t-\tau)g(\tau)d\tau.$$

In a discretized form,

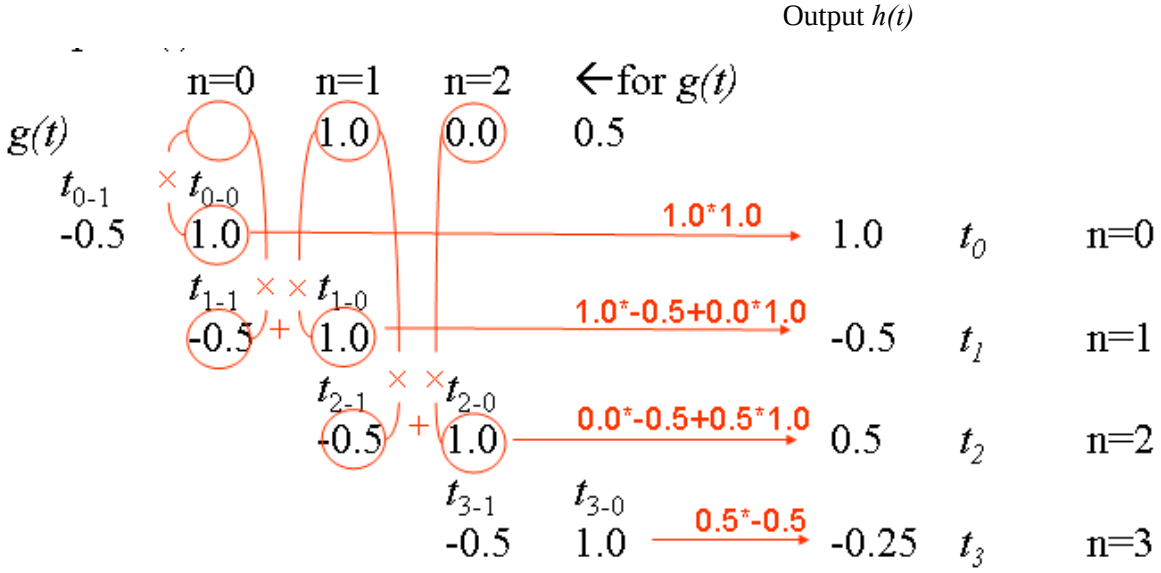
$$h(t_n) = \Delta\tau \sum_{m=n-M}^{n} f(t_{n-m})g(t_m)$$
$$= \Delta\tau f(t_M)g(t_{n-M}) + \Delta\tau f(t_{M-1})g(t_{n-M+1}) + \cdots + \Delta\tau f(t_0)g(t_n).$$

This means that the filter time series is reversed and used as the weight coefficients for the weighted moving average, as explained in the previous chapter.

Example: Time domain operation

Yilmaz(1994) show a graphical explabnation of the convolution as follows. The convolution of a filter *f(t)* = (1.0, –0.5) with a signal *g(t)* = (1.0, 0.0, 0.5). Assume that the sampling interval is equal to 1.0 s. Further, note that *M = 1*.

Reversing the filter *f(t)*: (1, –0.5) changes it into (–0.5, 1).



Add the product $f(t_{n-m})g(t_m)$ for $m = 0$ to M. The sum gives the value $h(t_n)$.

Shift the moving array one sample to the right and repeat the procedure for adding products. Try to examine whether the same result is given if *f(t)* and *g(t)* exchange their roles. (After Yilmaz (1994)).

3.3. Feature of Filter Wavelets

When a signal is composed of only a few cycles in the time domain, it is called a "wavelet." A wavelet is usually considered a transient signal. Undoubtedly, convolution with a wavelet can be considered equivalent to filtering. The characteristics of this filter are directly defined by the frequency spectrum of the filter wavelet. Every weighted moving average belongs to this category. The discussion in the previous chapter gave some typical examples.

Again, it must be noted that the amplitude spectrum of the output is the product of the amplitude spectra of the filter and the input signal, while the phase spectrum of the output is the sum of their phase spectra.

3.3.1. Phase

The time series are composed of limited and discrete numbers of sinusoidal functions with a constant interval of frequency Δf, that is, the reciprocal of the duration T. The wavelet that shows a symmetry around $t = 0$ and has a positive peak amplitude is the *zero phase* wavelet. Fig. 26 shows the decomposition of a wavelet into various sinusoids. Note that all the component sinusoids have *zero* time shifts. The phase lag is defined by $2\pi f t_{shift}$, where t_{shift} is the time shift. If the time shift is zero for all the frequencies, the wavelet is called "zero phase wavelet." If the time shift is a constant for all frequency components, it is equivalent to a *linear phase shift*, an example of which is given in Fig. 27. The tangent of

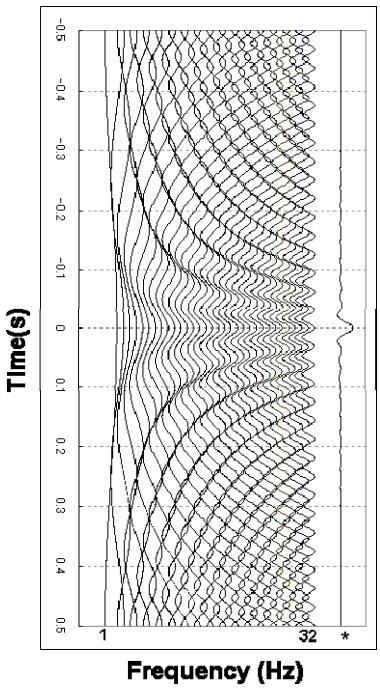


Fig. 26 Decomposition of a band limited symmetric (zero-phase) wavelet (denoted by an asterisk) into a discrete number of sinusoids with no phase lag, but with the same peak amplitude. Produced newly based on the concept of Yilmaz(1994).

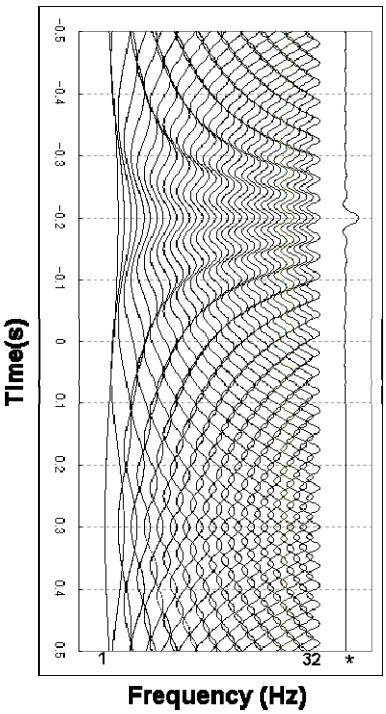


Fig.27 Constant time delay -0.2 sec given to the sinusoidal components same as those in Fig. 26 results in a wavelet of the same shape as that in Fig.26 (denoted by an asterisk), except that it is shifted in time by -0.2 sec. Produced newly based on the concept of Yilmaz(1994).

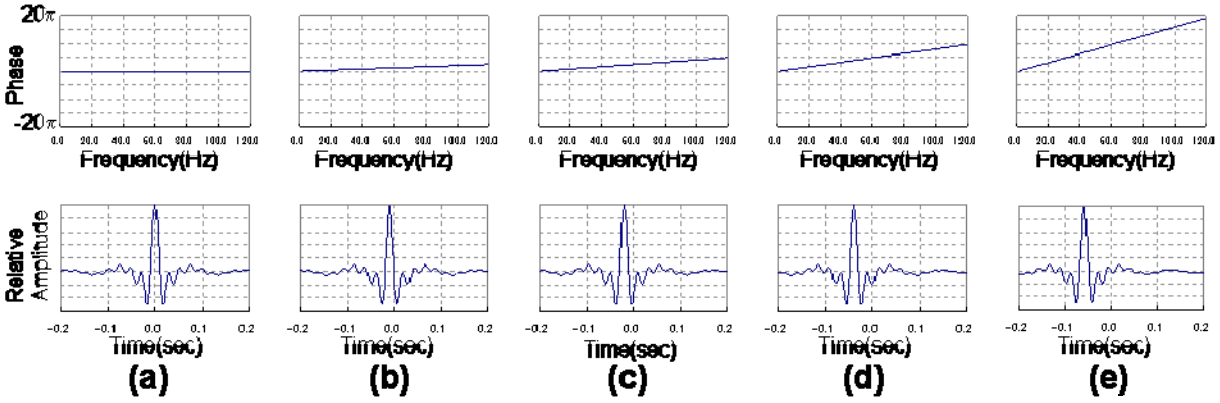


Fig.28 Wavelets in the time domain are shifted by th elinear phase shift starting with a zero-phase wavelet (a). The slope of the linear phase function is related to the time shift. Produced newly based on the concept of Yilmaz(1994).

the line for the phase spectra is proportional to the time shift for the linear phase shift, as shown in Fig. 28. Note that the linear phase shift keeps the waveform constant.

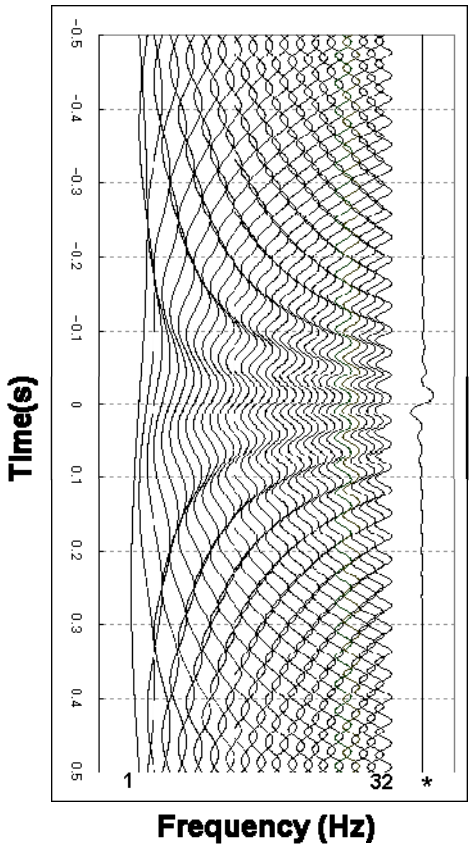


Fig. 29 Constant 90-degree phase shift given to the sinusoidal components same as those in Fig. 26 results in asymmetric wavelet but the zero crossing at t = 0 (indicated by an asterisk). Produced newly based on the concept of Yilmaz(1994).

In contrast to the linear phase shift, a constant phase shift changes the waveform. The wavelet shown in Fig. 29 has the same amplitude spectrum as that in Fig. 26. The difference is their phase spectrum. Note that zero crosses are aligned in Fig. 29, whereas the peaks are aligned in Fig. 26. Fig. 30 shows the way in which the waveform is changed by a constant phase shift. Note that a constant 180-degree phase shift changes the sign of the wavelet. Note the relation of the panels (a) and (c) and panels (b) and (d). The constant phase shift of 180 degree implies a reversal of sign.

The linear and constant phase shifts are two basic examples of phase change. The combined operation is defined as $a + b \cdot frequency$, where a is the constant phase shift and b is the tangent of the linear phase shift, gives a time shift with a waveform change. The result is a combination of both effects, as shown in Fig. 31.

Note that the shape of the wavelet can be changed by modifying the phase spectrum even while keeping the amplitude spectrum constant. Several examples for this combination are shown

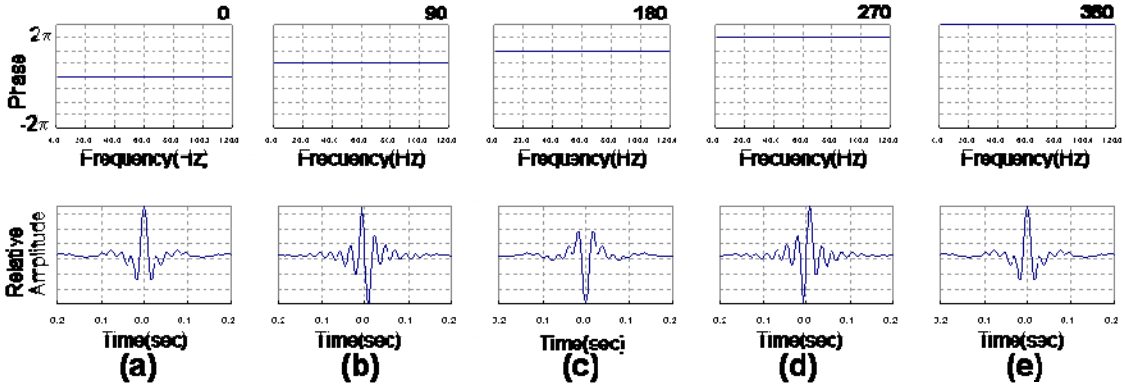


Fig. 30 Series of waveform change caused by a constant phase shift starting with the zero-phase wavelet (a). A 90-degree phase shift converts the zero-phase wavelet to an antisymmetric wavelet (b),while a 180-degree phase shift reverses its polarity (c). A 270-degree phase shift reverses the polarity, while making the wavelet antisymmetric (d). Finally, a 360-degree phase shift does not influence the wavelet (e). Produced newly based on the concept of Yilmaz(1994).

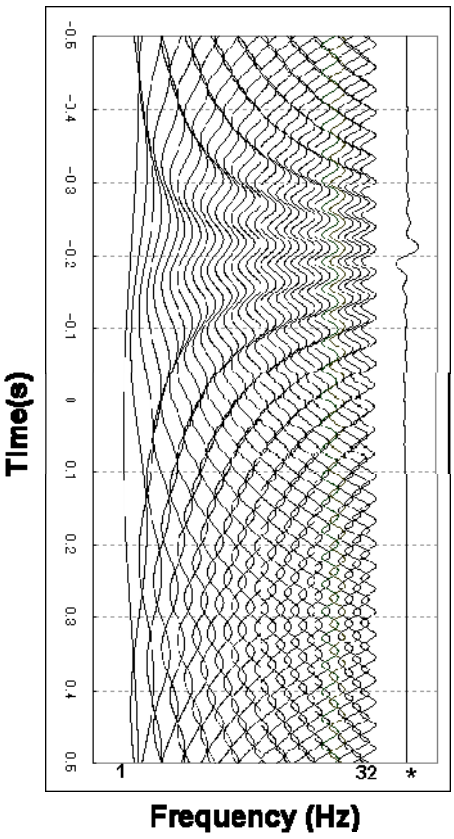


Fig. 31 Time-shifted antisymmetric wavelet (denoted by an asterisk) caused by a linear phase shift combined with a constant phase shift for the component sinusoids same as those in Fig.26. Produced newly based on the concept of Yilmaz(1994).

in Figs. 32 (a), (b), and (c). An arbitrary change in the phase spectra, however, can break the wavelet.

The tangent of the phase shift is called "delay."

$$delay = -\frac{d\phi}{d\omega}$$

The linear phase shift is an example of constant delay for all frequencies. In general, "delay" can be dependent on the frequency.

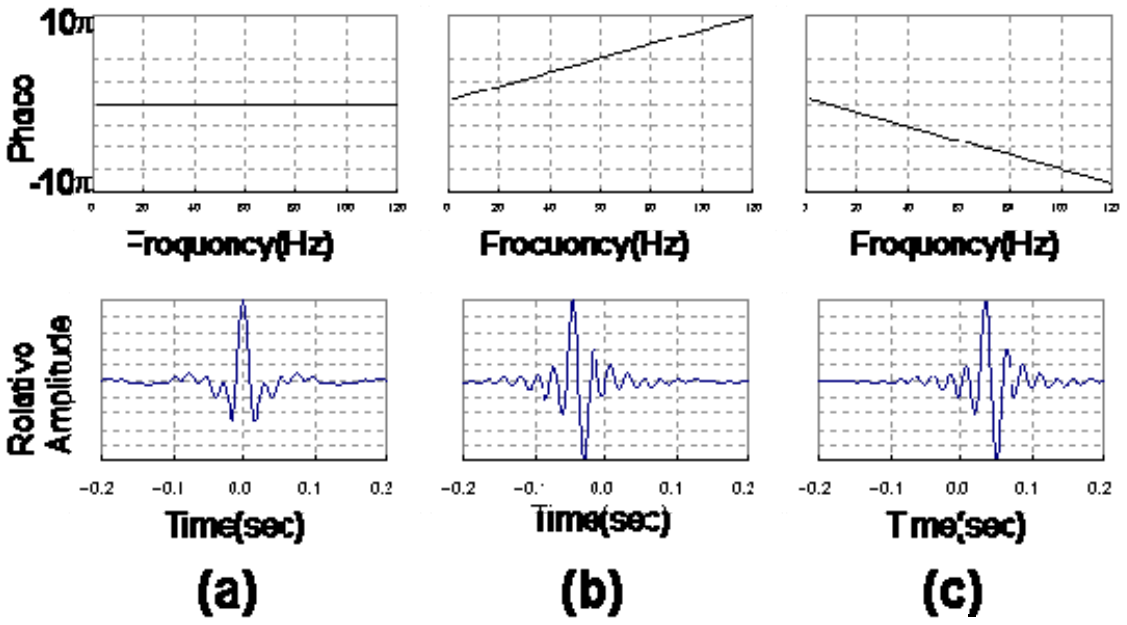


Fig. 32 a non-zero-phase spectrum of any form in (b) and (c) modifies the shape of a zero-phase wavelet (a). Produced newly based on the concept of Yilmaz(1994).

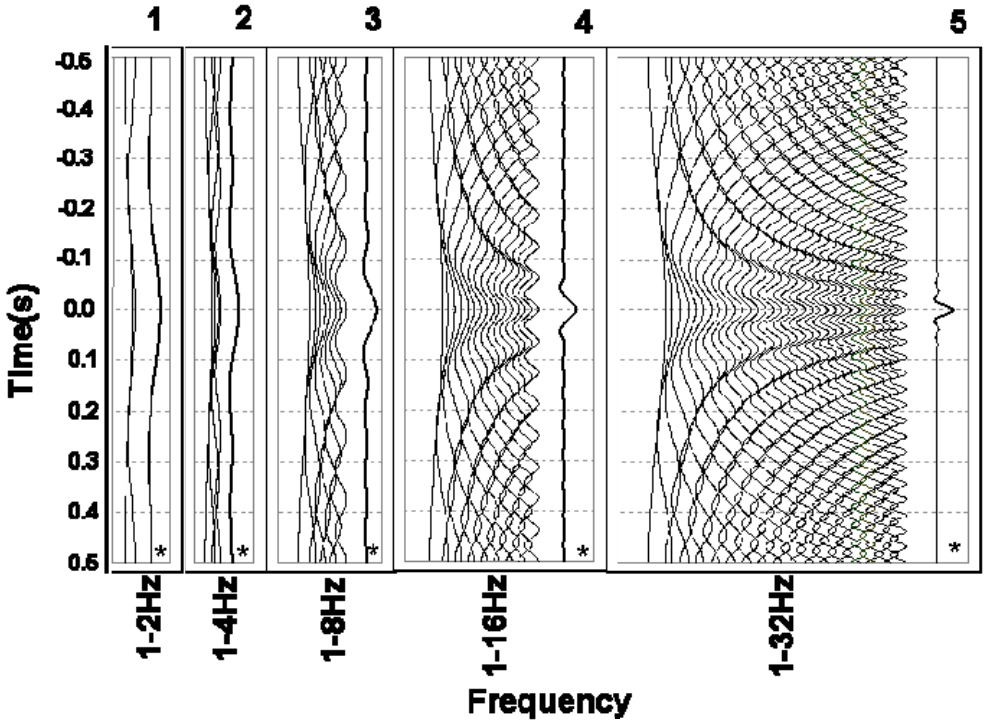


Fig. 33 The summation of zero-phase sinusoids with an identical peak amplitude shows that the incresing frequency bandwidth results in the synthesized zero-phase wavelet increasingly compressed. Produced newly based on the concept of Yilmaz(1994).

3.3.2. Frequency Components

In the previous chapter, wavelets with varying phase spectra and fixed amplitude spectra are observed. By changing the amplitude spectrum or selecting the frequency contents, the wavelet changes its shape even when its phase spectrum is maintained constant. Here, zero-phase wavelets are used for simplicity.

Fig. 33 shows a clear example of the changes in zero-phase wavelets by the selection of frequency contents. As more frequency components are summed, the synthesized zero-phase wavelet is increasingly compressed. If they are summed till the Nyquist frequency, a spike is formed (Fig. 34).

The broader the bandwidth, the more compressed the wavelet; in other words, a shorter wavelet is obtained. This property also follows from the fundamental concept that the effective time span of a time series is inversely proportional to its effective spectral bandwidth (Fig. 35).

The shape of the frequency spectrum also influences the wavelet shape. Fig. 36 shows a typical case. A short wavelet requires a tapered amplitude spectrum, although the width of the passband for all cases is identical.

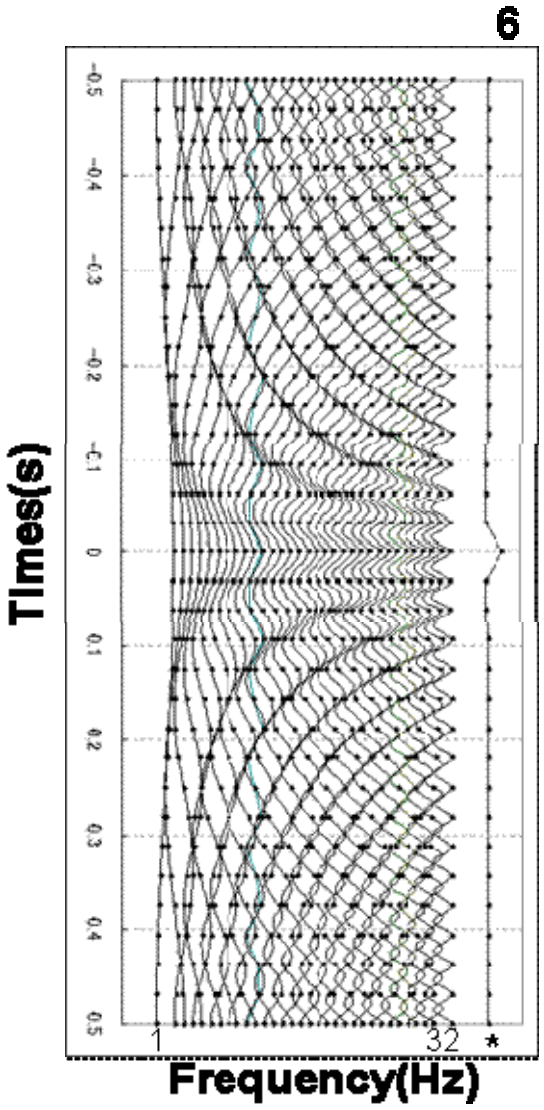


Fig. 34 The output wavelet becomes a spike when the summation includes sinusoids at all frequencies up to the Nyquist frequency. Small dots denote the sampling points at 64Hz. Produced newly based on the concept of Yilmaz(1994).

Filtering in the frequency domain can be performed by the inverse Fourier transform of the product of the Fourier transforms of the filter and input time series. This is equivalent to the filtering in the time domain that is performed by the weighted moving average of the input time series, the weight coefficients of which are the reversed filter time series. This, in general, can be written as

$$y_i = \cdots + a_2 x_{i-2} + a_1 x_{i-1} + a_0 x_i + a_{-1} x_{i+1} + a_{-2} x_{i+2} + \cdots,$$

where $(\cdots, a_{-2}, a_{-1}, a_0, a_1, a_2, \cdots)$ is the filter time series.

If we consider causality, i. e., the idea that the results cannot proceed to the cause, we cannot use the terms of the future x_{i+1}, x_{i+2},… to obtain the present output y_i. Thus,

$$y_i = \cdots + a_2 x_{i-2} + a_1 x_{i-1} + a_0 x_i .$$

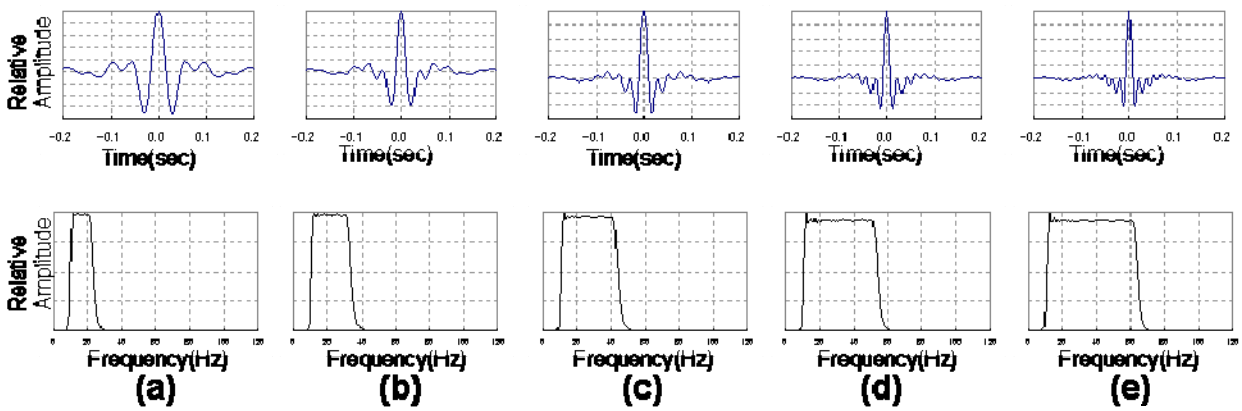


Fig. 35 Incresing bandwidth in the frequency domain (*bottom panels*) corresponds to more compressed wavelet in the time domain(*top panels*). Produced newly based on the concept of Yilmaz(1994).

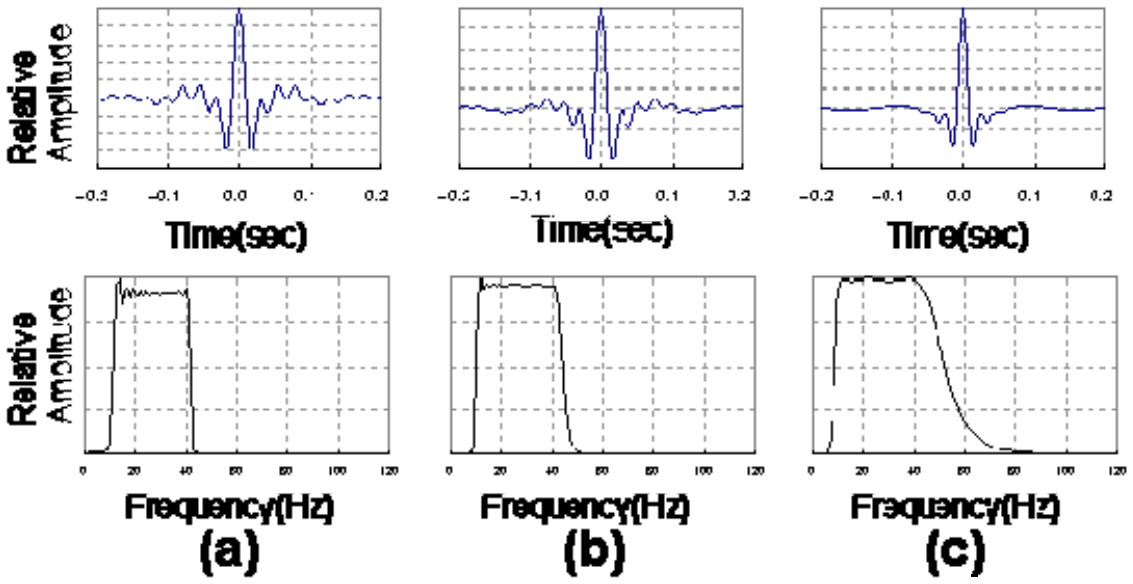


Fig. 36 More gentle slope in the frequency domain(*bottom panels*) corresponds to smoother wavelet in the time domain (*top panels*). (a) The steep slopes of the passband cause ripples in the wavelet and the actual amplitude spectrum. (b) A moderate and (c) gentle slope help eliminate the ripples. Produced newly based on the concept of Yilmaz(1994).

3.3.3. Causality or Non-Causal Filtering

A phenomenon that is the result of another phenomenon (the cause) never occurs before the cause itself does. This is called a “causal relation” or “causality” and it is strictly maintained in the real world. However, in a computer, this relation can be broken. Such a breakage often affects the seismological analyses. The following shows us examples.

Exercise: Filtering in the time domain and in the frequency domain: an example

The topics in this chapter can be learned much better by practicing with the distributed software. Here, the following programs are prepared for practice.

FFILT.EXE creates a set of coefficients of Fourier expansion from given bandpass characteristics.

FPRDCT.EXE calculates the product of two given sets of the coefficients of Fourier expansion, i. e., filtering in the frequency domain.

FWVLET.EXE creates a filter wavelet from the given time series that may be obtained by the inverse Fourier transform of a given set of the coefficients of Fourier expansion.

FCONV.EXE calculates convolution, i. e., filtering in the time domain for a given filter wavelet and input signal.

The programs *TESTSIG.EXE*, *PTIME.EXE*, *FFT.EXE*, *PCFFT.EXE*, *PSPEC.EXE*, and *IFFT.EXE* are also used.

(0) Prepare the test input signal *UT1*, that is a unit impulse located at $t = 8.0$ s of the time series with N = 64, Δt = 1.0, and its Fourier transform *UF1* by using *TESTSIG.EXE* and *FFT.EXE* (Fig. 36a).

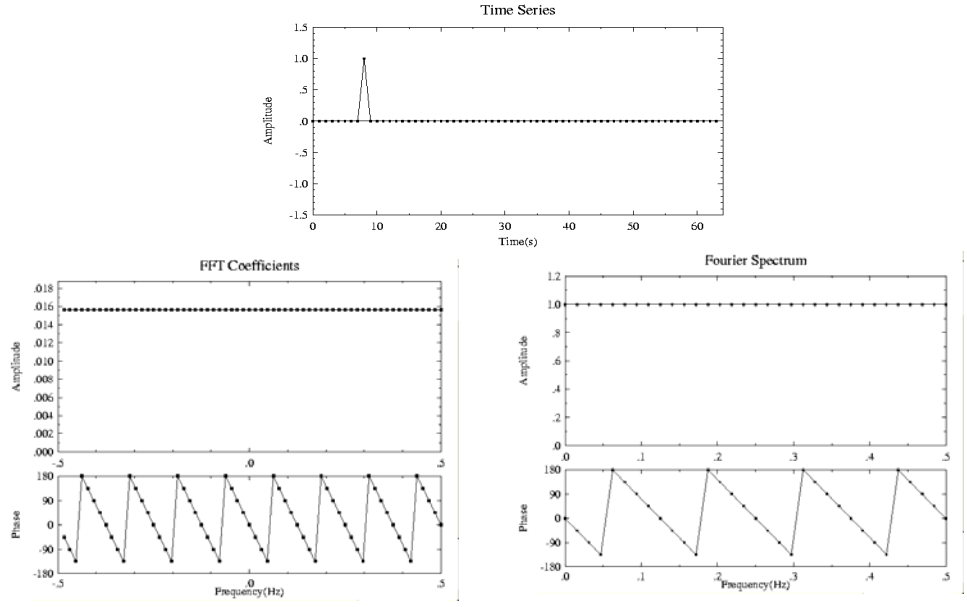


Fig. 36a Test input signal for the exercise *UT1* and its spectra. Time dependence (*upper panel*), the coefficients of Fourier expansion (*lower left panel*), and Fourier spectra (*lower right panel*). The impulse is located at $t = 8.0$ s of the time series with N = 64, Δt = 1.0. The linear phase shift due to the shifted location of the impulse is shown.

Filtering in the frequency domain:

(1) Design a band pass filter in the frequency domain by using *FFILT*. The number of data for the corresponding time series N, its sampling interval Δt, and four frequencies f_1, f_2, f_3, and f_4 must be given when we run *FFILT*. Let N = 64, Δt = 1.0, f_1 = 0.1 Hz, f_2 = 0.2 Hz, f_3 = 0.3 Hz, and f_4 = 0.4 Hz and the output file name be *UF2*. Draw the coefficients of the Fourier expansion by using *PCFFT* and Fourier spectra by using *PSPEC* (Fig. 37.1). The figure is stored in the PostScript file *G.PS*.

The following steps are schematically shown in Fig. 37.2.

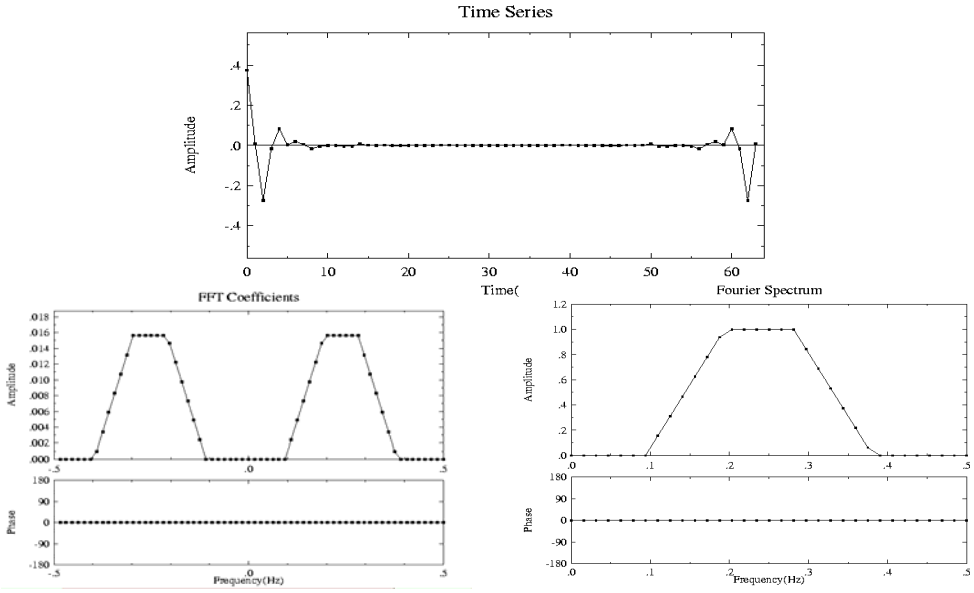


Fig. 37.1 Filter given in the frequency domain *UF2* (*lower panels*) and the corresponding time series *UT1* (*upper panel*). Since t is assumed that phase lag is zero at all frequencies, the time series is symmetrical till the point $t = 0.0$ s. Remember the implicitly assumed periodicity. The next step of $t = 63.0$ s is $t = 0.0$ s.

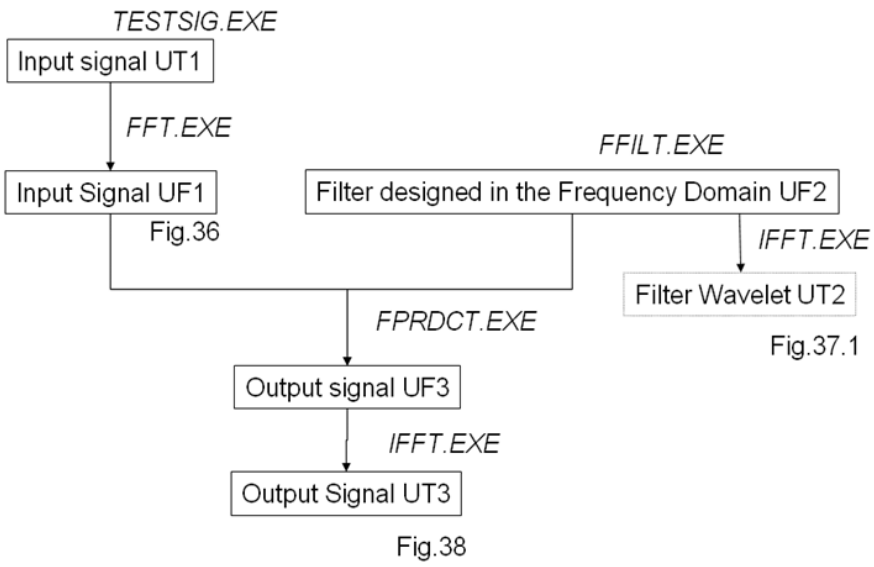


Fig. 37.2 Procedure for designing a filter in the frequency domain.

(2) Apply the filter in the frequency domain by using *FPRDCT*. Use the stored data in *UF2* for the filter in the frequency domain. Give the output the file name *UF3*.
(3) Apply an inverse Fourier transform by using *IFFT* and store the results in the output file *UT3*. Draw this time series and its Fourier components by using *PTIME*, *PCFFT*, and *PSPEC* (Fig. 38).
(4) Obtain the time series corresponding to the band pass filter in the frequency domain designed by the procedure in (1) by using *IFFT*. Give the input file the name *UF2* and the output file the name *UT2*. Draw this time series by using *PTIME* (Fig. 37.1). The figure is stored in the PostScript file *GPS*.

Remember that the input impulse is located at $t = 8.0$ and notice that there are signals in Fig. 38 before $t = 8.0$. This means that the causal relation is broken because the result (output) is occurring before the cause (input) does. As shown in Fig. 37, the phase of the filter wavelet is assumed to be zero for all frequencies. Since there has been breakage of causal relation, this assumption may not be a valid one. It may be necessary to arrange the phase of the filter wavelet in order to maintain the causal relation. However, this is difficult to achieve this in the design of the filter wavelet in the frequency domain.

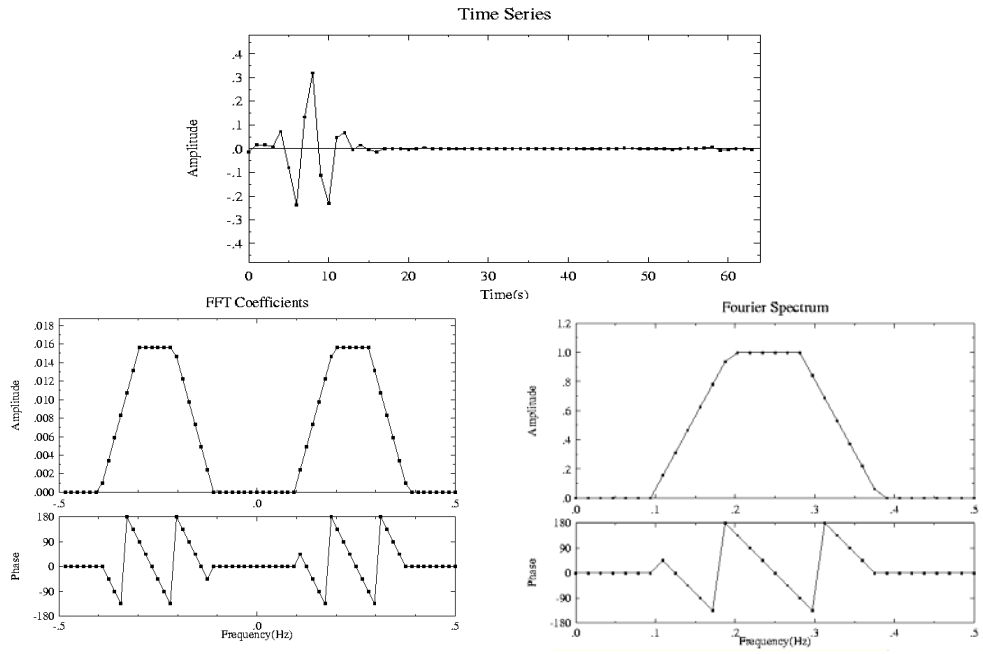


Fig. 38 The spectra of filtered signal *UF3*, which is obtained by the product of the Fourier transform of the filter shown in Fig. 37 and that of the input time series shown in Fig. 36 (*lower panels*). The upper panel shows the time series obtained by their inverse Fourier transform, *UT3*.

Filtering in the time domain:

(5) Extract the filter wavelet from the time series stored in the file *UT2* by using *FWVLET*. Give the output the file name *FWV1*. We have to select either a causal filter or a zero-phase filter. Here, we select a zero-phase filter with 13 coefficients.

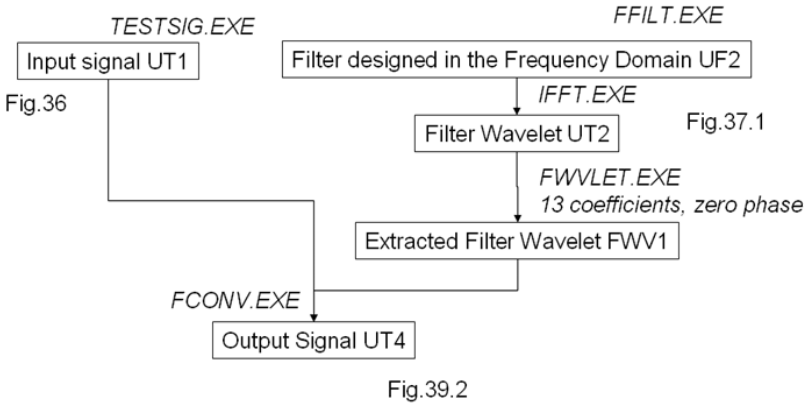


Fig. 39.1 Procedure for designing a filter in the time domain

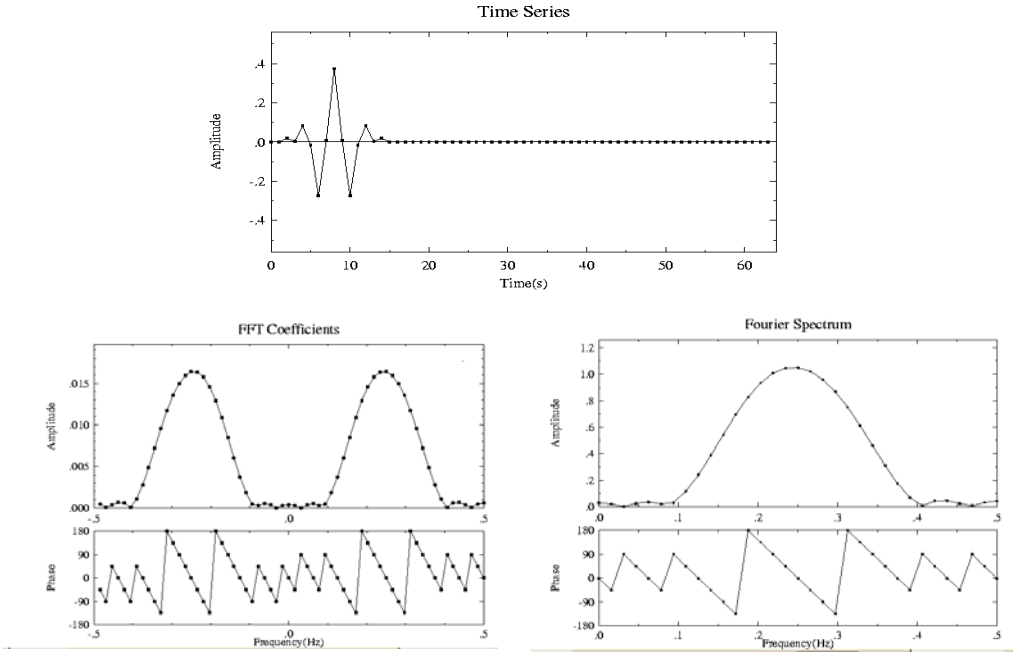


Fig. 39.2 Output time series from the filtering in the time domain with the truncated filter wavelet designed for zero phase filtering, *UT4* (*upper panel*), and its spectra, *UF4* (*lower panels*). Note the stability of the output time series and the negligible phase change in the pass band in comparison with Fig. 38. The change in the amplitude spectral shape is the effect of the truncation of the wavelet.

(6) Apply the filter in the time domain obtained in (5) by using *FCONV*. The input filter file name is FWV, the file name of the input time series is *UT1*, and the output file name is *UT4*. Draw the time series stored in *UT4* by using *PTIME* and compare it with the figures for *UT3* obtained in (3). Due to the truncation of the filter time series, the time series stored in *UT4* is slightly different from that in *UT3*. Check the performance of the filtering by using *FFT*, *PCFFT*, and PSPEC with *UT4* (Fig. 39.2).
(7) Extract the filter wavelet from the time series stored in the file *UT2* by using *FWVLET*. Give the output file the name *FWV2*. We must select either a causal filter or a zero-phase filter. In this case, we select a causal filter with 6 coefficients.
(8) Apply the filter in the time domain obtained in (5) by using *FCONV*. The input filter file name is FWV, the file name of the input time series is *UT1*, and the output file name is *UT5*. Draw the time series stored in *UT4* by using *PTIME* and compare it with the figure for *UT3* obtained in (3). The output in this case is clearly different from the results obtained in the time domain in (6) and from those in (3). Note that the causality is satisfied in the time domain. Check the performance of the filtering by using *FFT*, *PCFFT*, and *PSPEC* with *UT5* (Fig.40).
(9) Repeat the procedures explained above after changing the number of weight coefficients for *FWV* and compare them.

This example shows the problems with designing a filter wavelet with desirable characteristics both in the time frequency domains. In general, however, the filtering in the frequency domain works well for zero-phase filtering.

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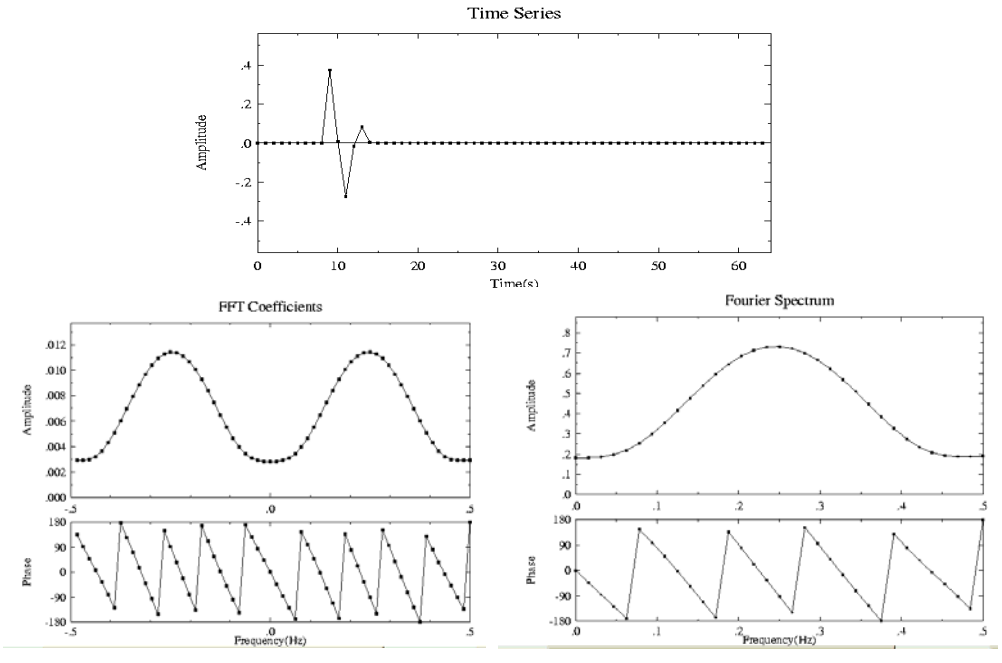


Fig. 40 Output time series from the filtering in the time domain with the truncated filter wavelet designed for zero phase filtering *UT5* (*upper panel*) and its spectra *UF5* (*lower panels*). Note that the causality with Fig.36 is maintained. The change in the amplitude and phase spectral shape is the effect of the truncation of wavelet. The truncation of a former half of the filtering wavelet results in the phase shift by filtering and reduction in the amplitude spectra even in the pass band.

3.4. Recursive Filter

In the previous chapter, we have checked the features of the filter wavelet that can be replaced by using the weighted moving average. The general formula of this filter is given by the following equation after taking causality into account.

$$y_i = \cdots + a_2 x_{i-2} + a_1 x_{i-1} + a_0 x_i = a_0 x_i + a_1 x_{i-1} + a_2 x_{i-2} \cdots .$$

We have checked that the differentiation can be expressed by the weighted moving average that belongs to this category, i. e.,

$$y_m = \frac{x_m - x_{m-1}}{\Delta t} = \left(-\frac{1}{\Delta t} \right) x_{m-1} + \left(\frac{1}{\Delta t} \right) x_m .$$

Let us consider the integration given by

$$y(t) = \int_0^t x(\tau) d\tau .$$

The discretization gives

$$y_m = \Delta t \cdot \sum_{n=0}^{m} x_n .$$

This implies the relation,

$$y_m = y_{m-1} + \Delta t \cdot x_m .$$

Note that the term in the output that corresponds to the single step after y_{m-1} is used to construct the output for the current y_m. The filter that has had such a recursive usage of the output in the past is called a "recursive filter" (Fig. 40). The general formula for the recursive filter after taking the causality into account is given by

$$b_0 y_i = a_0 x_i + a_1 x_{i-1} + a_2 x_{i-2} \cdots - (b_1 y_{i-1} + b_2 y_{i-2} + \cdots).$$

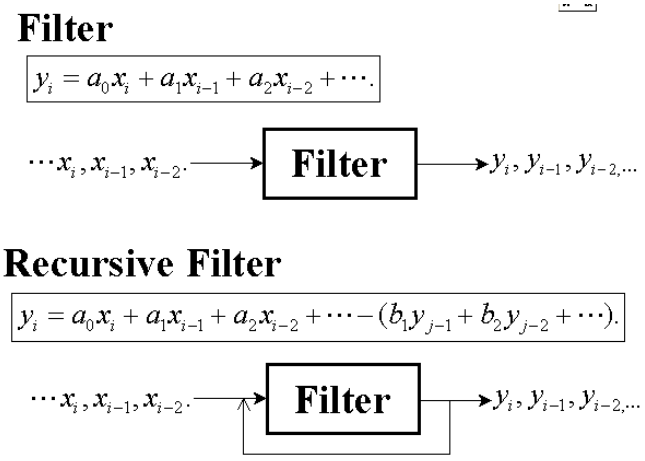


Fig. 41 Block diagram for the filter that can be expressed by the weighted moving average (upper) and the recursive filter (lower).

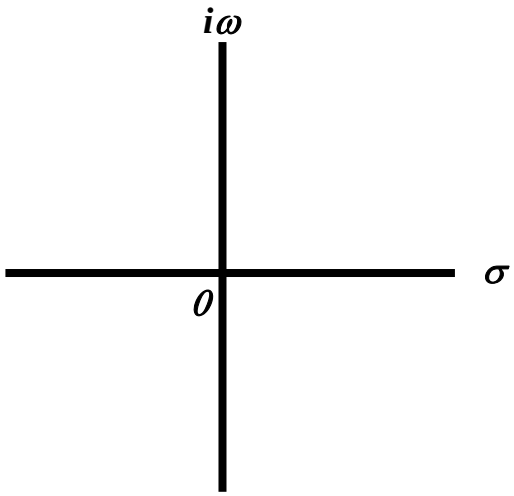


Fig.42 Complex s-plain

3.4.1. Laplace Transform

The Fourier transform of a continuous function has been defined previously. The meaning of Fourier transform is basically an expansion of the function on the basis of the sinusoidal function *exp(iωt)*. The sinusoidal function with an exponential decay or amplitude *exp((σ+ iω)t)* can also be used as the basis for expansion. Such an integral transform is called *Laplace Transform*.

The Laplace transform and its inverse transform are defined by the following (here $s = \sigma + i\omega$).

$$F(s) = \int_0^\infty f(t) e^{-st} dt ,$$

$$f(t) = \frac{1}{2\pi i} \int_{\gamma - i\omega}^{\gamma + i\omega} F(s) e^{st} ds.$$

The Laplace transform *F(s)* is defined in the complex *s*-domain (Fig. 42).

3.4.2. Filter Operation in the *s*-domain

The instrument characteristics of seismometers, seismographs, and every electronic circuit can be described by an appropriate transfer function. The analog transfer function may be given by using the variable for the Laplace transform as follows:

$$T(s) = \frac{A_L s^L + A_{L-1} s^{L-1} + \cdots + A_2 s^2 + A_1 s + A_0}{B_M s^M + B_{M-1} s^{M-1} + \cdots + B_2 s^2 + B_1 s + B_0} \quad , \; s = \sigma + i\omega. \tag{12}$$

The stability of this analog filter is obtained simply when all the solutions of the equation,

$$B_M s^M + B_{M-1} s^{M-1} + \cdots + B_2 s^2 + B_1 s + B_0 = 0,$$

s_n has to satisfy the following condition for the stability of the system.

$Re(s_n) = \sigma < 0$.

Otherwise, the circuit becomes a noise generator.

Example: **Simple Moving Coil Type Seismometer (Transfer function in the s-domain)**

The equation of motion for a pendulum's displacement in a seismometer relative to the ground *x(t)* induced by the ground motion *y(t)* is given by

$$\frac{d^2x}{dt^2} + 2h\omega_0 \frac{dx}{dt} + \omega_0{}^2 x = -\frac{d^2y}{dt^2}, \tag{13}$$

where ω_0 denotes the natural frequency of the pendulum and h denotes the damping factor. Applying the Fourier transform to both sides yields

$$-\omega^2 x_m + 2ih\omega_0\omega x_m + \omega_0{}^2 x_m = \omega^2 y_m \quad .$$

Thus, the response in the frequency domain is given by

$$-x_m \Big/ y_m = \frac{1}{1 - 2ih(\omega_0/\omega) - (\omega_0/\omega)^2}.$$

This response belongs to a high pass filter, and therefore, the seismometer has an equivalent digital filter. Define the transfer function in the frequency domain,

$$T(i\omega) = -x_m \Big/ y_m = \frac{1}{1 - 2ih(\omega_0/\omega) - (\omega_0/\omega)^2} = \frac{(i\omega)^2}{(i\omega)^2 + 2h\omega_0(i\omega) + \omega_0^2}. \tag{14}$$

The Laplace transform of Eq. (14) gives the transfer function in the *s*-domain. The substitution of $i\omega$ with s in Eq. (15) gives the result:

$$T(s) = -\frac{X(s)}{Y(s)} = \frac{s^2}{s^2 + 2h\omega_0 s + \omega_0^2}. \tag{15}$$

The solutions obtained when the denominator = 0 are called “poles.” In contrast, the solutions of the numerator = 0 are called “zeros,” because these cause the transfer function to be equal to zero. Eq. (12) can be factorized by using these poles and zeros as follows.

$$T(s) = \frac{A_L}{B_M} \cdot \frac{(s - s_L^0)\cdots(s - s_2^0)(s - s_1^0)}{(s - s_M)\cdots(s - s_2)(s - s_1)} = G_0 \cdot \frac{(s - s_L^0)\cdots(s - s_2^0)(s - s_1^0)}{(s - s_M)\cdots(s - s_2)(s - s_1)} \quad . \tag{16}$$

The suffix *0* denotes the “zero” point. As shown, “zeros” and “poles” determine the transfer function with a constant G_0. If s coincides with one of the poles, *T(s)* becomes infinite. If s coincides with one of the zeros, *T(s)* becomes zero. Actually, $s = \sigma + i\omega$ moves only along the imaginary axis in the complex *s*-plane, and poles must locate at $\sigma < 0$ in stable systems. Then, s cannot coincide exactly with any of the poles. Poles located near the imaginary axis can induce resonance. If one of the zeros lies on the imaginary axis, *T(s)* becomes zero sharply at the corresponding frequency. This feature is important for the design of a notch filter. The pole-zero representation of an analog transfer function provides a method for the design of circuits. Readers are recommended to study books on electronics, especially on *active filters* for more information. Several examples for simple transfer functions will be given in the following description.

Today, many seismic observation organizations release their data to the public via the Internet so that any researcher can use them. Some of these organizations provide information on instrumental characteristics using the pole-zero representation. Hence, it may be useful to show the method of reconstructing the transfer function in the frequency domain from a given value of poles and zeros.

At an angular frequency ω, the variable of the Laplace transform s is located at $(0, i\omega)$. $(s - s_m)$ in the denominator of Eq. (13) implies the distance between $s = (0, i\omega)$ and the pole s_m taking phase into consideration as well. Namely,

$$(s - s_m) = \left|(s - s_m)\right| \exp(\arg(s - s_m)).$$

When all poles and zeros are similarly considered, the following relations are obtained:

$$T(s) = G_0 \cdot \frac{(s - s_L^0)\cdots(s - s_2^0)(s - s_1^0)}{(s - s_M)\cdots(s - s_2)(s - s_1)}$$
$$= G_0 \frac{|s - s_L^0|\cdots|s - s_2^0||s - s_1^0|}{|s - s_M|\cdots|s - s_2||s - s_1|} \cdot \exp\{i\arg(s - s_L^0) + \cdots + i\arg(s - s_1^0) - i\arg(s - s_M) - \cdots - i\arg(s - s_1)\}.$$

Suppose that $X_L \ldots X_2$, X_1 denote the absolute values of $(s - s_m)$ and $\Theta_L \ldots \Theta_2$, Θ_1 denote the absolute values of their phases. Similarly, $x_M \ldots x_2$, x_1 and $\theta_M \ldots \theta_2$, θ_1 for poles. Then,

$$T(s) = G_0 \frac{X_L \cdots X_2 X_1}{x_M \cdots x_2 x_1} \cdot \exp\{i\Theta_L + \cdots + i\Theta_1 - i\theta_M - \cdots - i\theta_1\}.$$

This shows that the transfer function can be reconstructed from the given values of poles and zeros by a direct graphical measurement on the complex s-plane without any special software. This simple feature is one of the advantages of introducing Laplace transforms in the analysis of transfer functions.

Example: **Simple Moving Coil Type Seismometer (Poles and Zeros)**

Eq. (15) is factorized in the following manner.

$$T(s) = -\frac{X(s)}{Y(s)} = \frac{s^2}{s^2 + 2h\omega_0 s + \omega_0^2} = \frac{(s - s_2^0)(s - s_1^0)}{(s - s_2)(s - s_1)}.$$

The solutions of the equation, achieved by equating the denominator to zero are $s = \omega_0\left(-h \pm \sqrt{h^2 - 1}\right)$.

This gives the pole position at

$\left(-\omega_0 h,\ \omega_0\sqrt{1 - h^2}\right),\ \left(-\omega_0 h,\ -\omega_0\sqrt{1 - h^2}\right)$ for $h < 1.0$, under-damped case,

$(-\omega_0,\ 0.)$ doubled for $h = 1.0$, critically damped case,

$\left(-\omega_0\left(h - \sqrt{h^2 - 1}\right), 0.\right),\ \left(-\omega_0\left(h + \sqrt{h^2 - 1}\right), 0.\right)$ for $h > 1.0$, over-damped case.

For all these three cases, the poles are located in the left half of the s-plane. This guarantees the stability of the system. The doubled zeros are located at (0,0).

3.4.3. *Z*-transform

Remember the discrete Fourier transform:

$$X_k = TC_k = \Delta t \sum_{m=1}^{N} x_m e^{-im\omega_k \Delta t},$$

$$x_m = \frac{1}{N\Delta t} \sum_{k=1}^{N} X_k e^{im\omega_k \Delta t}, \qquad (17)$$

$$where \ \ \omega_k = \frac{2\pi k}{N\Delta t}.$$

X_k has a certain physical meaning. Let us change Eq. (17) slightly in the following way.

$$\tilde{X}_k = \sum_{m=1}^{N} x_m e^{-im\omega_k \Delta t},$$

$$x_m = \frac{1}{N} \sum_{k=1}^{N} \tilde{X}_k e^{im\omega_k \Delta t}.$$

This gives an abstract quantity in the transformed domain. A new variable is introduced as

$$z = e^{i\omega_k \Delta t}. \qquad (18)$$

Thus,

$$Z(x_m) = \tilde{X}_k = \sum_{m=1}^{N} x_m z^{-m},$$

$$x_m = \frac{1}{N} \sum_{k=1}^{N} \tilde{X}_k z^k. \qquad (19)$$

This new integral transform for discrete systems is called *z-transform*. Eq. (18) can be extended to relate the discrete z-transform with a continuous Laplace transform with $s = \sigma + i\omega$.

$$z = e^{s\Delta t}. \qquad (20)$$

The product with z implies a time shift of Δt toward the future, whereas that with z^{-1} implies one toward the

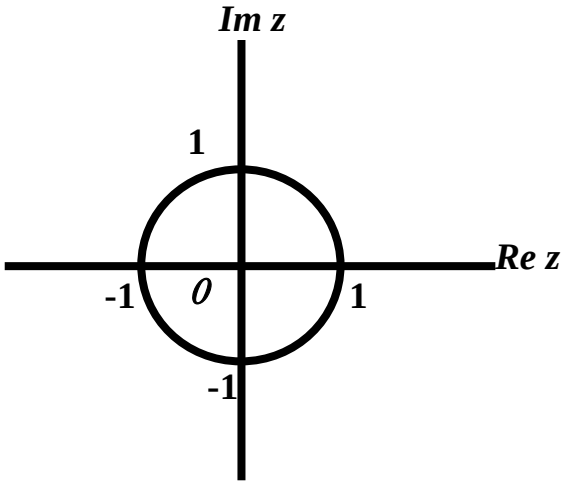


Fig. 43 Complex *z*-plain. The left half of the complex s-plane is mapped into a unit circle centered at the origin by $z = exp(st)$. The points (0, $i\omega$/2.), (0, -$i\omega$/2) on the *s*-plane are mapped to (-1, 0). In other words, the positive and negative parts of the imaginary axis on the *s*-plane are mapped to the upper and lower halves of the unit circle on the *z*-plain, respectively. The origin of the *s*-plane is mapped to (1, 0) on the *z*-plane.

past.

3.4.4. Filter Operator in the *Z*-domain

Suppose *x(t)* denotes the input time series; *X(ω)*, its Fourier spectrum; *y(t)*, the filtered output; *Y(ω)*, its spectrum; and *F(ω)*, the spectrum of the applied filter. Then,

$$Y(\omega) = F(\omega)X(\omega). \tag{21}$$

Suppose the filter spectra can be written, *e. g.*, in the following form in order to facilitate ease of discussion.

$$F(z(\omega)) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{b_0 + b_1 z^{-1} + b_2 z^{-2}}, \tag{22}$$

Eq. (21) gives the relation

$$\left[b_0 + b_1 z^{-1} + b_2 z^{-2}\right] Y(\omega) = \left[a_0 + a_1 z^{-1} + a_2 z^{-2}\right] X(\omega).$$

The inverse Fourier transform of both sides gives

$$b_0 y(t) + b_1 y(t - \Delta t) + b_2 y(t - 2\Delta t) = a_0 x(t) + a_1 x(t - \Delta t) + a_2 x(t - 2\Delta t),$$

because of the relation

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} z^{-n} Y(\omega) e^{i\omega t} d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega) e^{i\omega(t - n\Delta t)} d\omega = y(t - n\Delta t) \quad .$$

Then, the filtered output can be calculated rapidly with a defined value of coefficients, a few preceding data of the input time series, and a few preceding data of the output. For Eq. (22),

$$\left.\begin{aligned} y_0 &= \frac{a_0}{b_0} x_0 \\ y_1 &= \frac{1}{b_0}\left(a_0 x_1 + a_1 x_0 - b_1 y_0\right) \\ y_j &= \frac{1}{b_0}\left(a_0 x_j + a_1 x_{j-1} + a_2 x_{j-2} - b_1 y_{j-1} - b_2 y_{j-2}\right),\ j \geq 2 \end{aligned}\right\} . \tag{23}$$

This shows an example for a recursive filter operating in the time domain. The filter *F(ω)* might give a phase lag. In order to compensate for the phase lag, namely, in order to apply a zero-phase filter, inverse the time axis and apply the same filter in such way that

$$\left.\begin{aligned} y_N &= \frac{a_0}{b_0} x_{N-1} \\ y_{N-2} &= \frac{1}{b_0}\left(a_0 x_{N-2} + a_1 x_{N-1} - b_1 y_{N-1}\right) \\ y_{N-j} &= \frac{1}{b_0}\left(a_0 x_{N-j} + a_1 x_{N-j+1} + a_2 x_{N-j+2} - b_1 y_{N-j+1} - b_2 y_{N-j+2}\right),\ j \geq 3 \end{aligned}\right\} . \tag{24}$$

Of course, we can employ more coefficients a_l and b_m if necessary. The general form of Eq. (22) may be

$$F(z)=\frac{a_L z^{-L}+a_{L-1}z^{-(L-1)}+\cdots+a_2 z^{-2}+a_1 z^{-1}+a_0}{b_M z^{-M}+b_{M-1}z^{-(M-1)}+\cdots+b_2 z^{-2}+b_1 z^{-1}+b_0}, z=e^{i\omega\Delta t}. \tag{25}$$

The corresponding recursive filter may be

$$\begin{aligned} y_j=\frac{1}{b_0}\{\ & a_0 x_j+a_1 x_{j-1}+a_2 x_{j-2}+\cdots+a_{L-1}x_{j-L+1}+a_L x_{j-L} \\ & -(b_1 y_{j-1}+b_2 y_{j-2}+\cdots+b_{M-1}y_{j-M+1}+b_M y_{j-M})\ \}. \end{aligned} \tag{26}$$

This filter given by Eq. (25) is sometimes called a *transfer function* by analogy with the filters of electronic circuits, which are analog filters. Since the denominator of Eq. (25) controls the feedback part of Eqs. (23) and (24), b_m must be selected carefully in order to avoid any instability in the filtering. The transform $z = exp(s\Delta t)$ maps the left half of the complex s-plane to the unit circle centered at the origin on the complex z-plane. Therefore, a stable and causal filtering requires that all the solutions of the equation

$$b_M z^{-M}+b_{M-1}z^{-(M-1)}+\cdots+b_2 z^{-2}+b_1 z^{-1}+b_0=0$$

z_n must satisfy the condition

$$|z_n|<1. \tag{27}$$

Additionally, if there are no zeros outside the unit circle on the complex z-plane, it is called minimum phase condition.

Focus on the direct coincidence of the coefficients of a filter wavelet in the time domains shown in Eq. (23) and Eq. (24) with the coefficients used in the transfer function in the Z-domain that is shown in Eq. (22). This shows that the analysis of the transfer function in the Z-domain gives the value of the coefficients for recursive filtering in the time domain.

The transfer function given in the frequency domain and that given in the s-domain are analogous functions, whereas that represented by a recursive filter is applied to a discrete time series in the computer. Z-transform behaves like an interpreter at the border between two worlds that are different each other—one a continuous world and the other a digital one.

The relation between the transfer function in the Z-domain and that in the s-domain is given approximately by the so called ***bilinear transform***:

$$s=\frac{2}{\Delta t}\cdot\frac{1-z^{-1}}{1+z^{-1}},\ z=\frac{1+\frac{\Delta t\cdot s}{2}}{1-\frac{\Delta t\cdot s}{2}}. \tag{28}$$

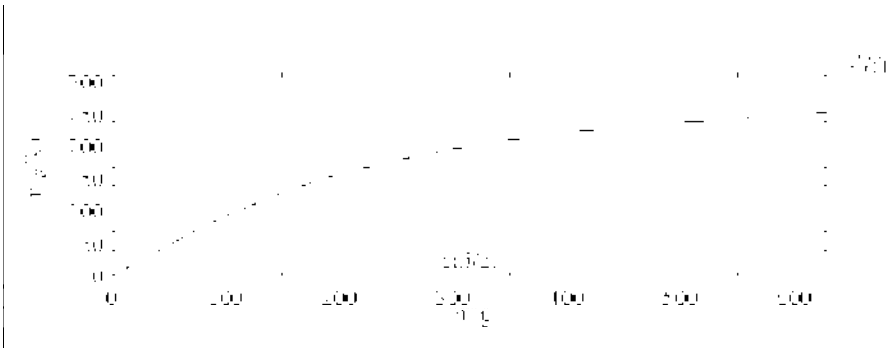


Fig. 44.1 Mapping of the continuous angular frequency ω on to the discrete frequency ω_k. This is an example of $\Delta t=0.01$, $N=1024$. The Nyquist angular frequency is $\pi/\Delta t$.

This is an approximation of Eq. (18) that is equivalent to the following:

$$s = \frac{1}{\Delta t}\ln(z).$$

The discrete angular frequency ω_k is also transformed. For $z = exp(i\omega_k\Delta t)$, the bilinear transform gives

$s = \frac{2}{\Delta t}\cdot\frac{1-e^{-i\omega_k\Delta t}}{1+e^{-i\omega_k\Delta t}} = \frac{2i}{\Delta t}\cdot\tan(\omega_k\Delta t/2)$. Since $s = i\omega$ for $\sigma = 0$,

$$\omega = \frac{2}{\Delta t}\cdot\tan(\omega_k\Delta t/2) \; or \; \omega_k = \frac{2}{\Delta t}\cdot\tan^{-1}(\omega\Delta t/2) \tag{29}$$

This shows the distortion of the continuous angular frequency ω by the bilinear transform to the discrete one ω_k (Fig. 44.1). To compensate this distortion, a warped angular frequency

$$\omega'_c = \frac{2}{\Delta t}\cdot\tan(\omega_c\Delta t/2) \tag{30}$$

is introduced (Sherbaum(1996)). The critical angular frequencies ω_c of the continuous transfer function are first converted to the corresponding warped angular frequencies ω'_c, and then the bilinear transform using the warped ones is applied to obtain the equivalent discrete transfer function (Fig. 44.2). Eq. (30) shows that ω'_c tends to ω_c for a small value of

$$\frac{\omega_c\Delta t}{2} = \frac{\pi f_c}{f_{Sampling}} = 2\pi\cdot\frac{f_c}{f_{Nyquist}}$$

This means that considering the warped frequency is not necessary for frequencies that are considerably smaller than the Nyquist frequency. The natural frequency of a seismometer is usually much smaller than the Nyquist frequency, whereas the anti-aliasing filter has a cut-off frequency that is comparable with the Nyquist one.

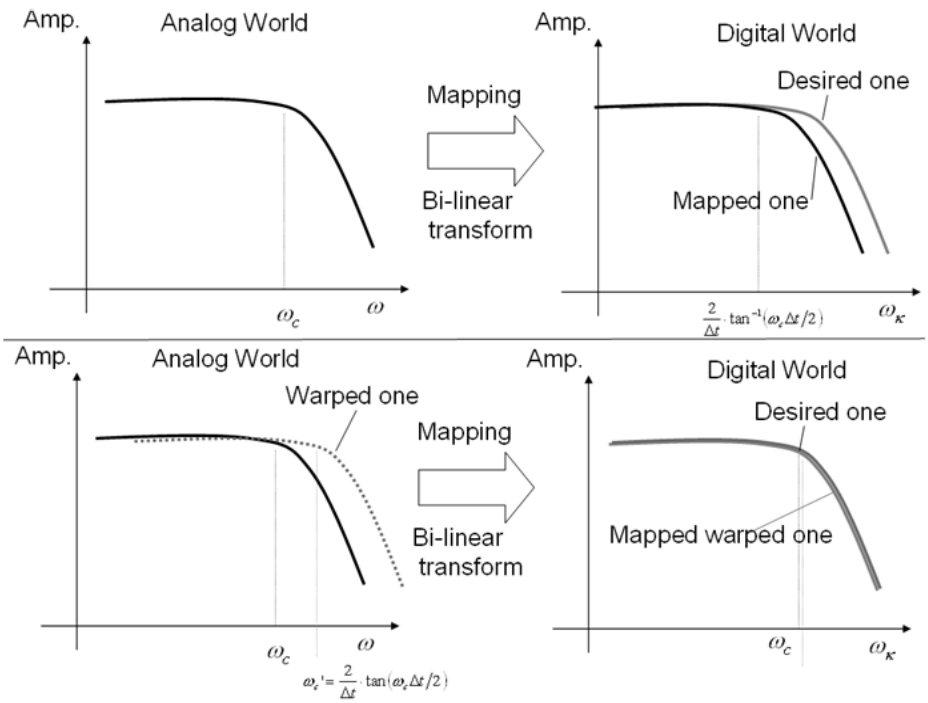


Fig. 44.2 Schematic drawing that shows how warped frequency works.

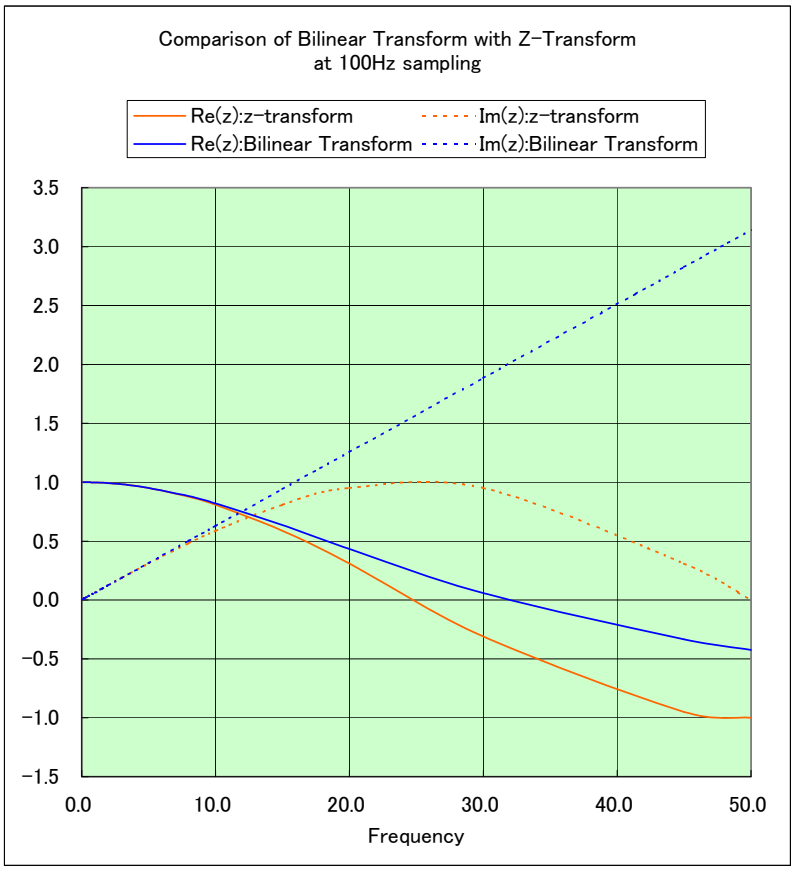


Fig. 44a Example of a comparison of bi-linear transform with z-transform for Δt = 0.01 s. The difference is negligible at frequencies less than 10 Hz.

Example: **Simple Moving Coil Type Seismometer (z-transform and equivalent recursive filter)**

Eq. (15) is factorized in the following manner:

$$T(s) = -\frac{X(s)}{Y(s)} = \frac{s^2}{s^2 + 2h\omega_0 s + \omega_0^2} = \frac{(s - s_2^0)(s - s_1^0)}{(s - s_2)(s - s_1)}.$$

This reduces to the following with frequency warping as given by Eq. (30).

$$T(s) = \frac{s^2}{s^2 + 2h\omega'_0 s + \omega'^2_0}. \tag{31}$$

Applying the bilinear transformation to Eq. (31) gives a simulated transfer function in the *z*-domain,

$$\begin{aligned} T(z) &= \frac{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2}{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 2h\tan\left(\frac{\omega_0\Delta t}{2}\right)\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + \tan^2\left(\frac{\omega_0\Delta t}{2}\right)} \\ &= \frac{(1-z^{-1})^2}{(1-z^{-1})^2 + 2h\tan\left(\frac{\omega_0\Delta t}{2}\right)(1-z^{-1})(1+z^{-1}) + \tan^2\left(\frac{\omega_0\Delta t}{2}\right)(1+z^{-1})^2} \\ &= \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{b_0 + b_1 z^{-1} + b_2 z^{-2}}, \end{aligned} \tag{32}$$

where

$$\begin{aligned} a_0 &= 1.0, a_1 = -2.0, a_2 = 1.0, \\ b_0 &= 1 + 2h\tan(\omega_0\Delta t/2) + \tan^2(\omega_0\Delta t/2), \\ b_1 &= -2 + 2\tan^2(\omega_0\Delta t/2), \\ b_2 &= 1 - 2h\tan(\omega_0\Delta t/2) + \tan^2(\omega_0\Delta t/2). \end{aligned}$$

These coefficients of the recursive filter give an approximately equivalent discrete transfer function.

Exercise: **Filter Equivalent to a Simple Moving Coil Type Seismometer—1.**

The program *DSEISM.EXE* calculates the coefficients of the recursive filter equivalent to the relative motion of the pendulum mass of a seismometer and applies the recursive filter to an input time series.

(1) Prepare an input time series by using *TESTSIG.EXE*. An impulse of a unit amplitude at $t = 0.0$ s will give you the response characteristics of the filter. For example, use $\Delta t = 0.05$ s and $N = 128$.

(2) Run *DSEISM.EXE* with a natural period $T_0 = 0.5$ s, damping factor $h = 0.71$, $\Delta t = 0.05$, and gain $G_0 = 1.0$.

(3) Draw the filtered time series by using *PTIME.EXE* and its Fourier spectra by using *FFT.EXE* and *PSPEC.EXE*. An example is shown in Fig. 45.1.

(4) Repeat the above procedure with different values of the natural period and damping factor.

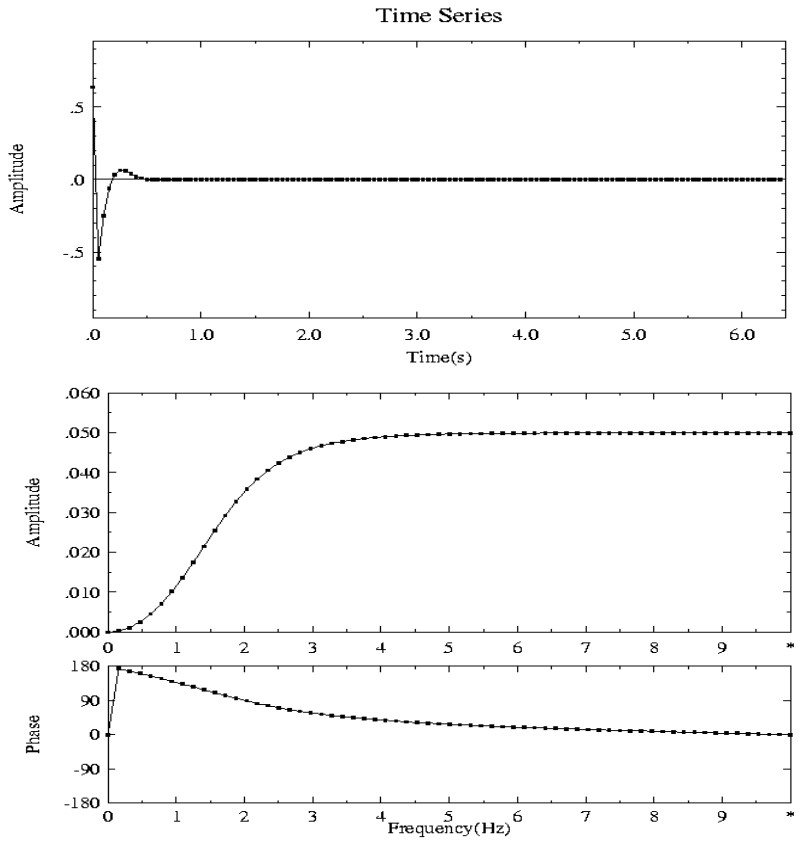


Fig. 45.1 Filtered time series and its Fourier spectra obtained by *DSEISM.EXE*. These are equivalent to the impulse response of the relative motion of a pendulum mass of a seismometer with the natural period $T_0 = 0.5$ s, the damping factor $h = 0.71$, $\Delta t = 0.05$, and gain $G_0 = 1.0$. The phase at zero frequency should converge to 180 degrees. However, to avoid division by zero, it is forced to be zero.

Example: Filter equivalent to a Simple Moving Coil Type Seismometer— 2

In the previous example, the recursive filter that gives the relative displacement of a pendulum $-x_m$ for ground displacement y_m is given. Usually, the data obtained by a digital recorder are given numerically and a constant is given for conversion into volts. The potential difference, which is the output from seismometer, is given as follows:

$$e_m = \frac{G_0 R_s}{R_0 + R_s}(i\omega)x_m,$$

where $(i\omega)$ shows the effect of differentiation due to a moving coil type transducer; R_0, the coil resistance; R_s, the shunt resistance; and G_0, the product of the sensitivity of the seismometer with the conversion constant of a digital recorder. Therefore, the system response is

$$e_m / y_m = \frac{G_0 R_s}{R_0 + R_s} \cdot \frac{(i\omega)}{1 - 2ih(\omega_0/\omega) - (\omega_0/\omega)^2}.$$

Fig. 45.2 shows its frequency dependency.

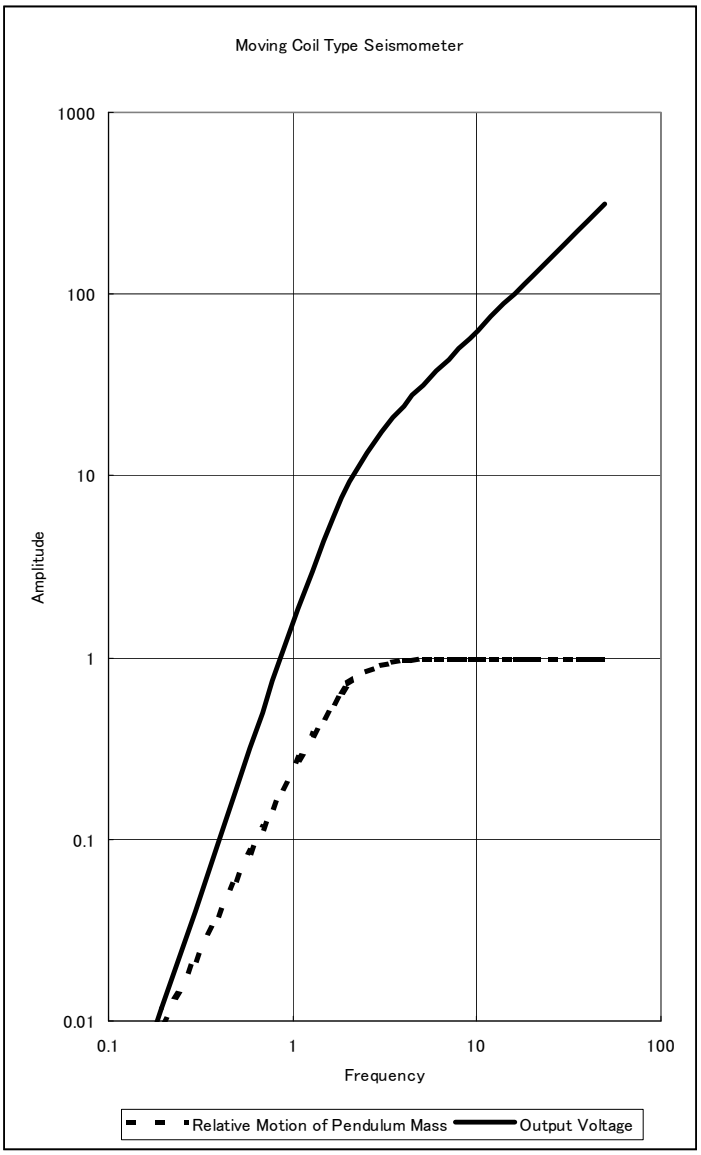


Fig. 45.2 Response of the recursive filters calculated by the formula written above in this page. *Dashed line*: Filter equivalent to the relative motion of pendulum mass of a simple moving coil type seismometer, against ground displacement. *Solid line*: Filter equivalent to the voltage change between two output terminals of a simple moving coil type seismometer, against ground displacement. The parameters used are T_0=0.5, h=0.71.

This has an equivalent digital filter. The corresponding transfer function in the *s*-domain is

$$T(s) = G \cdot \frac{s^3}{s^2 + 2h\omega_0 s + \omega_0^2},\ G = \frac{G_0 R_s}{R_0 + R_s}. \tag{33.1}$$

The solutions of the equation, i. e., the denominator is equal to zero, are

$$s = \omega_0\left(-h \pm \sqrt{h^2 - 1}\right).$$

This gives the pole position at

$$\left(-\omega_0 h,\ \omega_0\sqrt{1-h^2}\right),\ \left(-\omega_0 h,\ -\omega_0\sqrt{1-h^2}\right) \quad \text{for } h < 1.0, \text{ under-damped case,}$$

$$(-\omega_0,\ 0) \quad \text{doubled for } h = 1.0, \text{ critically damped case,}$$

$$\left(-\omega_0\left(h-\sqrt{h^2-1}\right), 0\right),\ \left(-\omega_0\left(h+\sqrt{h^2-1}\right), 0\right) \quad \text{for } h > 1.0, \text{ over-damped case.}$$

However, the numerator gives a tripled zero at (0, 0).

By using the warping frequency, the transfer function is given approximately as follows:

$$T(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}, \tag{33.2}$$

where

$$a_0 = G, a_1 = -3G, a_2 = 3G, a_3 = -G,.$$

$$b_0 = (\Delta t/2)\cdot\left\{1 + 2h\tan(\omega_0\Delta t/2) + \tan^2(\omega_0\Delta t/2)\right\},$$
$$b_1 = (\Delta t/2)\cdot\left\{-1 + 2h\tan(\omega_0\Delta t/2) + 3\tan^2(\omega_0\Delta t/2)\right\},$$
$$b_2 = (\Delta t/2)\cdot\left\{-1 - 2h\tan(\omega_0\Delta t/2) + 3\tan^2(\omega_0\Delta t/2)\right\},$$
$$b_3 = (\Delta t/2)\cdot\left\{1 - 2h\tan(\omega_0\Delta t/2) + \tan^2(\omega_0\Delta t/2)\right\}.$$

Example: Reconstruction of Transfer Function from Given Poles and Zeros

Suppose that the following data are given for an observation system. In fact, these data are for a STS—2 feedback type seismometer.

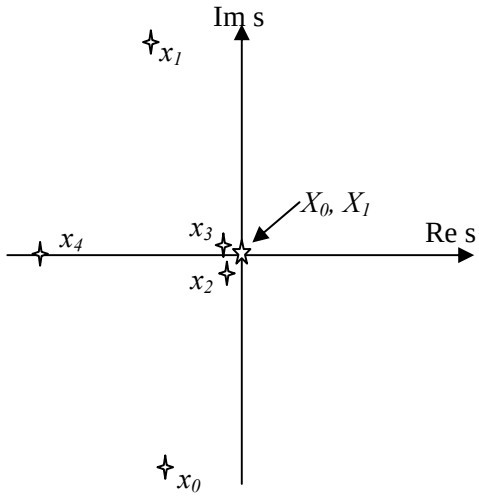


Fig. 45.3 Configuration of poles and zeros, for the example.

Normalization factor: A_0 = 5.42787E+07
Normalization frequency: 0.02 (Hz)

Complex **zeros**:

i	real part	imaginary part	Index
0	0.000000E+00	0.000000E+00	X_0
1	0.000000E+00	0.000000E+00	X_1

Complex **poles**:

i	real part	imaginary part	Index
0	–1.247510E+02	–4.171480E+02	x_0
1	–1.247510E+02	4.171480E+02	x_1
2	–4.873870E–02	–1.552120E–02	x_2
3	–4.873870E–02	1.552120E–02	x_3
4	–2.513300E+02	0.000000E+00	x_4

Sensitivity: G_0 = 6.291456E+08 (digit/(M/s))

Frequency of sensitivity: 0.02 (Hz)

The normalized amplitude spectra of the transfer function are given as follows:

$$\left|\tilde{T}(\omega)\right| = \frac{\left|s - X_0\right| \cdot \left|s - X_1\right|}{\prod_{m=0}^{4} \left|s - x_m\right|} = \frac{\omega^2}{\prod_{m=0}^{4} \sqrt{\left(0 - \mathrm{Re}(x_m)\right)^2 + \left(\omega - \mathrm{Im}(x_m)\right)^2}}$$

The normalizing factor A_0 is the reciprocal of this value at f = 0.02 (Hz). $A_0\left|\tilde{T}(\omega)\right| = 1.0$ at f = 0.02 Hz.

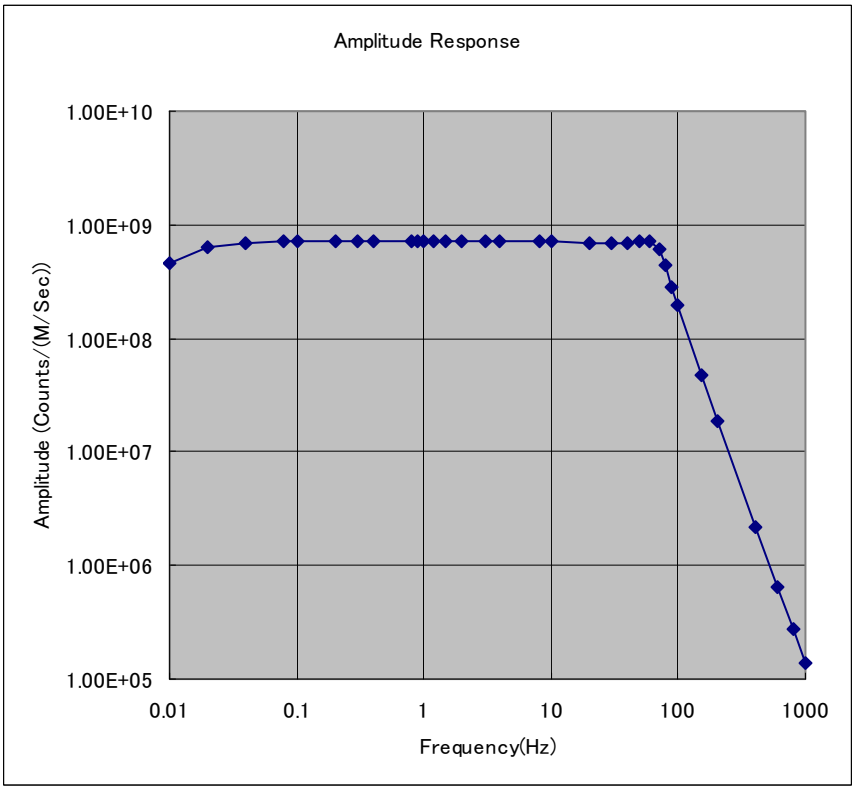
The reconstructed transfer function is given as follows:

$$\left|T(\omega)\right| = G_0 A_0 \left|\tilde{T}(s)\right| = \frac{G_0 A_0 \omega^2}{\prod_{m=0}^{4} \sqrt{\left(\mathrm{Re}(x_m)\right)^2 + \left(\omega - \mathrm{Im}(x_m)\right)^2}}$$

For phase spectra,

$$Arg(T(\omega)) = \sum_{l=0}^{1} \frac{\pi}{2} - \sum_{m=0}^{4} \left\{ \tan^{-1}\left(\frac{\omega - \mathrm{Im}(x_m)}{0 - \mathrm{Re}(x_m)} \right) \right\}$$

The first term corresponds to two zeros at the origin. These spectra can be calculated even with a handy calculator.



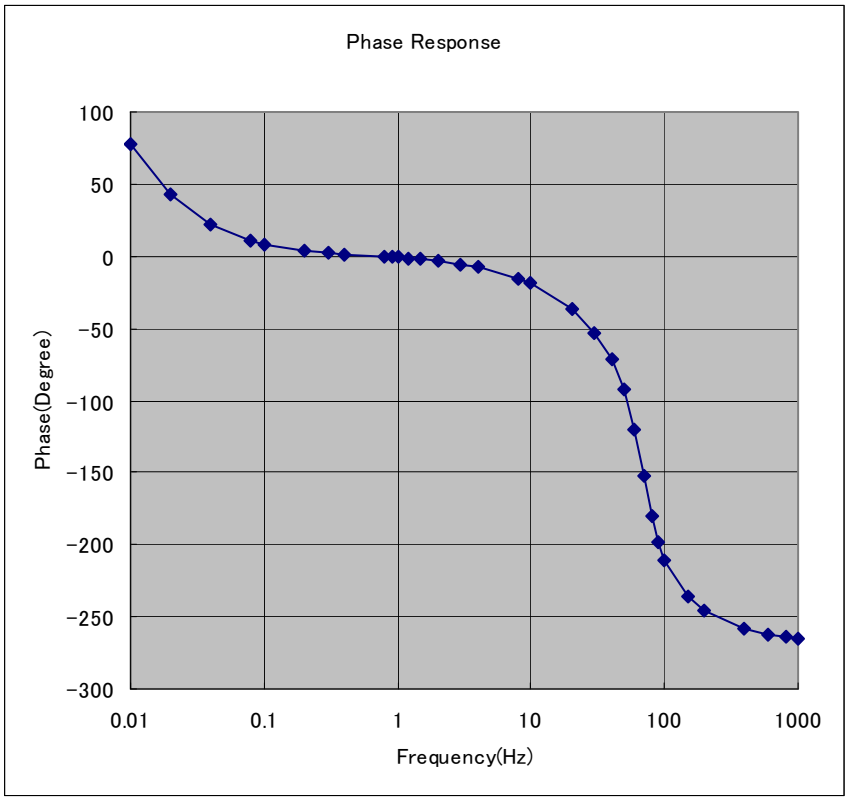


Fig. 45.4 Amplitude (top) and phase (bottom) response of an STS-2 feed back type seismometer. These are calculated by the formula using poles and zeros data given in the previous page. The calculations were performed easily in Microsoft Excel.

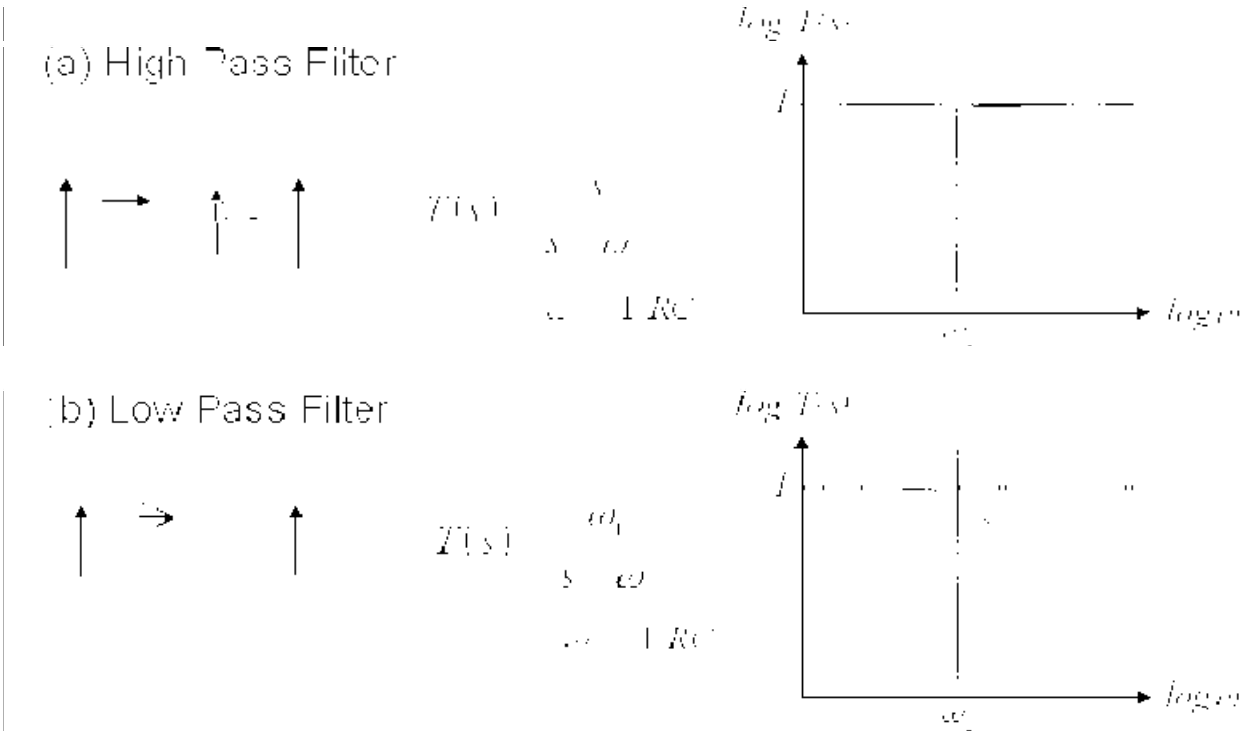


Fig. 46 (*Top)* RC high pass filter. Its circuit, transfer function, and response characteristics. (*Bottom)* RC low pass filter.

3.4.5. Analog Filters and their Transfer Functions

Several electronic analog filters are commonly used in seismometry. Their transfer functions are introduced here. Naturally, each circuit has its equivalent digital filter.

(1) **RC filter**: One of the simplest electronic circuits is an *RC* filter that is composed of a resistor and a capacitor, as shown in Fig. 46.

High Pass RC Filter

If the resistor is connected parallel to the output (Fig. 46 *top*), it is a high pass filter. Suppose *x(t)* and *y(t)* denotes the input and output voltage imbalance, respectively. The equation for a high pass filter is as follows:

$$\begin{aligned} y(t) + V_c(t) &= x(t), \\ I(t) &= C\frac{d}{dt}V_c(t), \\ RI(t) &= y(t), \end{aligned} \tag{34.1}$$

where *R* and *C* denotes the resistance and the capacitance integrated in the circuit (Fig. 46 *top left*); *I(t)*, the current that passes through *R* and *C*; and V_c, the voltage across the capacitor. This formula gives

$$\frac{d}{dt}y(t) + \frac{1}{CR}y(t) = \frac{d}{dt}x(t). \tag{34.2}$$

The transfer function is given by the Laplace transform of Eq. (34.2).

$$T(s) = \frac{s}{s + \omega_0}, \text{ where } \omega_0 = 1/RC. \tag{34.3}$$

The amplitude spectra are drawn schematically in Fig. 46 *top right*.

Low Pass RC Filter

If the capacitor is connected parallel to the output, it works as a low pass filter (Fig. 46 *bottom*). The low pass filter shown in Fig. 46 *bottom left* corresponds to the following equation.

$$\begin{aligned} RI(t) + y(t) &= x(t), \\ I(t) &= C\frac{d}{dt}y(t), \end{aligned} \tag{35.1}$$

This can be combined into

$$RC\frac{d}{dt}y(t) + y(t) = x(t). \tag{35.2}$$

The transfer function is given by the Laplace transform of (35.2).

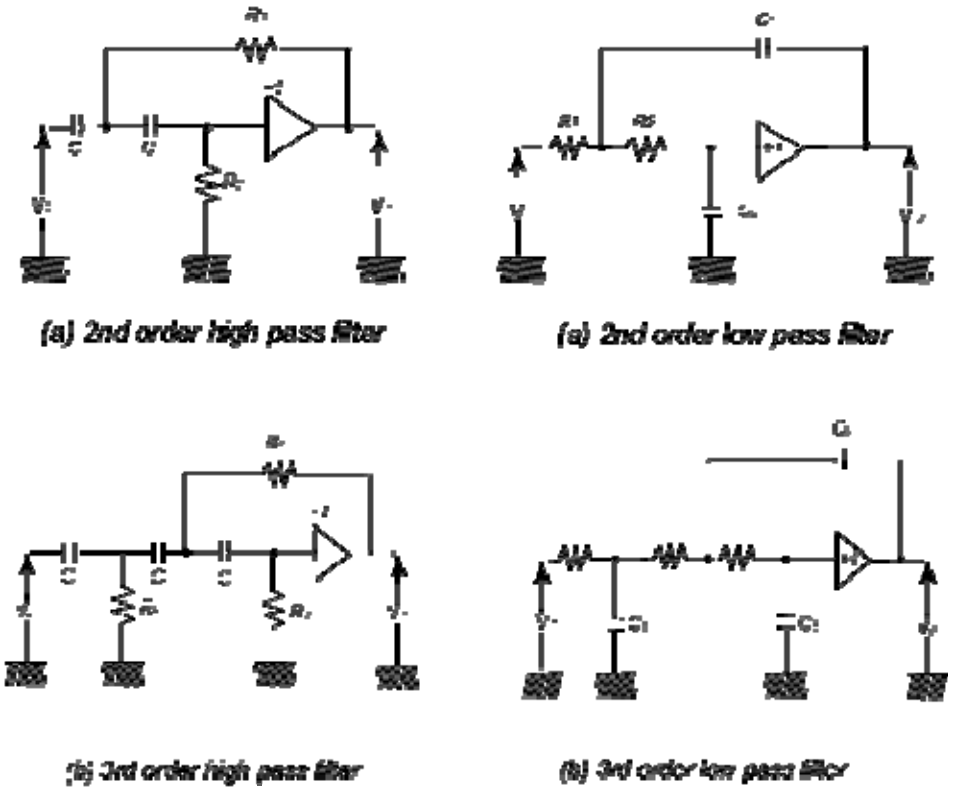


Fig. 47 Schematic circuit of an active filter composed of OpAmps. Reproduced based on Yanagisawa and Kanematsu (1980)

$$T(s) = \frac{\omega_c}{s + \omega_0}, where\ \omega_0 = 1/RC. \tag{35.3}$$

The amplitude spectra are drawn schematically in Fig. 46 *bottom right*.

(2) **Active filter**.

Today, many electronic circuits with known characteristics are widely used for filtering. Some of those characteristics have their proper names. The requirements for the filters may be flat amplitude characteristics in the pass band, sharp cutoff, and flat delay characteristics in the pass band. The last implies linear phase characteristics because delay is defined as phase differentiated by the angular frequency.

A ***Butterworth filter*** has its amplitude characteristics as

$$|T(s)| = \frac{1}{\sqrt{1 + (\omega/\omega_c)^{2n}}},$$

where ω_c is the cutoff angular frequency. n denotes the order of the filter and the slope of amplitude characteristics in the stop frequency band. In other words, an n-order filter has a decay of $20\ n$ db/oct. The transfer function itself is given for a low pass filter

$$T(s) = \Pi \frac{\Omega_0^2}{(s/\omega_c)^2 + (\Omega_0/Q)(s/\omega_c) + \Omega_0^2}, for\ even\ n,$$

$$T(s) = \frac{\Omega_a}{(s/\omega_c) + \Omega_a} \Pi \frac{\Omega_0^2}{(s/\omega_c)^2 + (\Omega_0/Q)(s/\omega_c) + \Omega_0^2}, for\ odd\ n,$$

and for a high pass filter

$$T(s) = \Pi \frac{\Omega_0^2}{(\omega_c/s)^2 + (\Omega_0/Q)(\omega_c/s) + \Omega_0^2}, for\ even\ n,$$

$$T(s) = \frac{\Omega_a}{(\omega_c/s) + \Omega_a} \Pi \frac{\Omega_0^2}{(\omega_c/s)^2 + (\Omega_0/Q)(\omega_c/s) + \Omega_0^2}, for\ odd\ n,$$

where ω_c is the cut-off angular frequency; other coefficients are given in the following table.

Table 10 Coefficients for *Butterworth* filter

n	Q	Ω_0	Ω_a	C_1	C_2	C_3
2	0.707107	1.000000	---	1.4142	0.7071	---
3	1.000000	1.000000	1.000000	1.3926	3.5468	0.2025
4	0.541196	1.000000	---	1.0824	0.9239	---
	1.306536	1.000000	---	2.6131	0.3827	---
5	0.618034	1.000000	1.000000	1.3541	1.7529	0.4213
	1.618034	1.000000	---	3.2361	0.3090	---

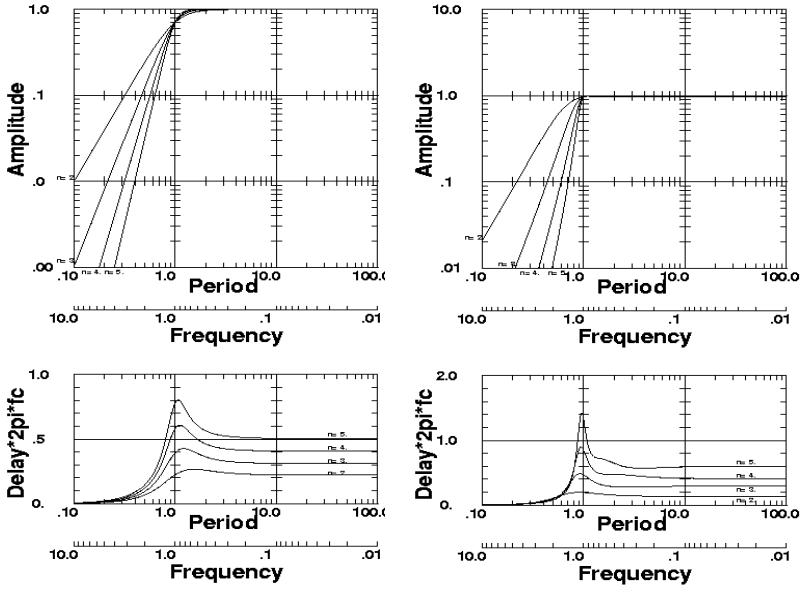


Fig. 48 Amplitude and delay of (a) Butterworth filter and (b) Chevyshev filter in the frequency domain. Both are plotted for the case of a low pass filter

The value of C_n ($n = 1, 2, 3$) in Table 10 corresponds to the actual value of capacitor C_a ($a = 1, 2, 3$) in Fig. 47 (*right*) but normalized one, i.e., $C_a = C_n/\omega_c R$, where R is the resistance selected in advance. For high pass filter (Fig. 47 *left*), R_n ($n = 1, 2, 3$) are calculated by $1/C_n$ ($n = 1, 2, 3$) in Table 10. The value of the actual resistor R_a ($a = 1, 2, 3$) in Fig. 47 (*left*) is given by $R_a = R_n/\omega_c C$, where C is the value of the capacitance selected in advance.

Butterworth filter has plane amplitude characteristics in the pass frequency band and is used for shaping of the spectra. The delay characteristics of this filter, however, are not flat even in the pass band. These have a peak of delay around the cut off frequency. This means that the shape of the signal around the cut off frequency in the time domain is considerably distorted by filtering operation.

Chevyshev filter : If we allow the ripple in the pass band, we can perform a sharp cut off. *Chevyshev* filter belongs to this category. The transfer function is given for low pass filter

$$T(s) = \frac{1}{\sqrt{10^{Rp/10}}} \prod \frac{\Omega_0^2}{(s/\omega_c)^2 + (\Omega_0/Q)(s/\omega_c) + \Omega_0^2}, \; for\ even\ n,$$

$$T(s) = \frac{\Omega_a}{(s/\omega_c) + \Omega_a} \prod \frac{\Omega_0^2}{(s/\omega_c)^2 + (\Omega_0/Q)(s/\omega_c) + \Omega_0^2}, \; for\ odd\ n,$$

and for a high pass filter

$$T(s) = \frac{1}{\sqrt{10^{Rp/10}}} \prod \frac{\Omega_0^2}{(\omega_c/s)^2 + (\Omega_0/Q)(\omega_c/s) + \Omega_0^2}, \; for\ even\ n,$$

$$T(s) = \frac{\Omega_a}{(\omega_c/s) + \Omega_a} \prod \frac{\Omega_0^2}{(\omega_c/s)^2 + (\Omega_0/Q)(\omega_c/s) + \Omega_0^2}, \; for\ odd\ n,$$

where R_p is the ripple amplitude in a pass band measured in (db).

The coefficients are given in the following table.

Table 11 Coefficients for Chevyshev filter with ripple of 0.25 db in the pass band

n	Q	Ω_0	Ω_a	C_1	C_2	C_3
2	0.809254	1.453972	---	1.1132	0.4249	---
3	1.508026	1.156992	0.767223	1.6110	6.8272	0.0885
4	0.657249	0.674422	---	1.9491	1.1280	---
	2.536110	1.077939	---	4.7055	0.1829	---
5	1.035932	0.732405	0.436951	2.6625	5.0919	0.3147
	3.875683	1.046630	---	7.4060	0.1233	---

The delay characteristics of this filter are not flat in the pass band. In such a case, the filtering operation causes considerable distortion of waveform in the time domain.

Bessel filter: It can produce flat delay characteristics in the pass band. However, the cutoff is not sharp. The amplitude characteristics are also flat in the pass band. The transfer function is given by the same formulas as those of a **Butterworth** filter. The coefficients are given in the following table.

Table 12 Coefficients for Bessel filter

n	Q	Ω_0	Ω_a	C_1	C_2	C_3
2	0.577350	1.732051	---	0.6667	0.5000	---
3	0.691047	2.541547	2.322165	0.5647	0.8136	0.1451
4	0.521935	3.023265	---	0.3453	0.3169	---
	0.805538	3.389366	---	0.4753	0.1831	---
5	0.563536	3.777894	3.646739	0.3601	0.4171	0.1280
	0.916479	4.261031	---	0.4302	0.1280	---

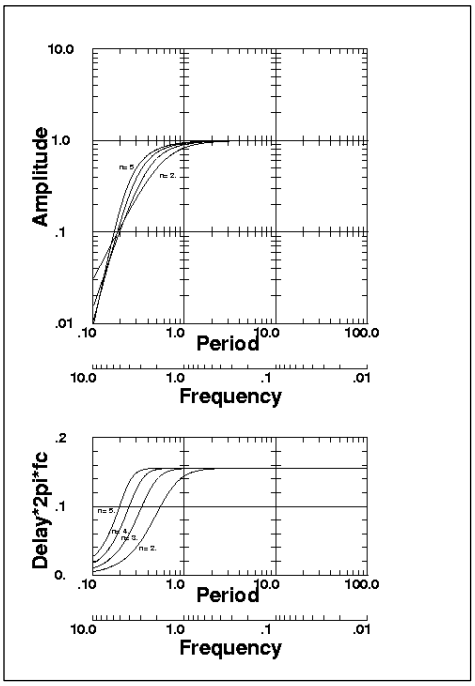


Fig. 48 (continued) (c) Amplitude and delay of the Bessel filter in the frequency domain plotted for the case of a low pass filter.

Example: Recursive filter equivalent to analog filter circuit

Low Pass Filter: The transfer function of the second-order analog low pass Butterworth filter circuit is

$$T_{2L}(s)=\frac{\Omega_0^2}{(s/\omega_c)^2+(\Omega_0/Q)(s/\omega_c)+\Omega_0^2}.$$

The warped cutoff angular frequency and bi-linear transform are applied. Then,

$$s/\omega_c{}'=\frac{1-z^{-1}}{\alpha(1+z^{-1})},\ \alpha=\tan\left(\frac{\omega_c\Delta t}{2}\right).$$

Therefore,

$$T_{2L}(z)=\frac{a_0+a_1z^{-1}+a_2z^{-2}}{b_0+b_1z^{-1}+b_2z^{-2}},\qquad \begin{matrix} a_0=1, a_1=2, a_2=1, b_0=1+(1/\Omega_0\alpha)/Q+(1/\Omega_0\alpha)^2, \\ b_1=2\{1-(1/\Omega_0\alpha)^2\},\ b_2=1-(1/\Omega_0\alpha)/Q+(1/\Omega_0\alpha)^2. \end{matrix}$$

The third-order filters can be written as

$$T_{3L}(s)=T_{1L}(s)\cdot T_{2L}(s),\qquad T_{1L}(s)=\frac{\Omega_a}{(s/\omega_c)+\Omega_a}$$

The warped cutoff angular frequency and bi-linear transform give

$$T_{1L}(z)=\frac{c_0+c_1z^{-1}}{d_0+d_1z^{-1}},\ c_0=1,\ c_1=1,\ d_0=1+1/(\alpha\Omega_a), d_1=1-1/(\alpha\Omega_a).$$

Third-order filtering can be achieved by applying $T_{1L}(z)$ and $T_{2L}(z)$ sequentially.
It is also possible to apply it immediately by using the following.

$$T_{3L}(z)=T_{1L}(s)\cdot T_{2L}(s)=\frac{a_0c_0+(a_1c_0+a_0c_1)z^{-1}+(a_2c_0+a_1c_1)z^{-2}+a_2c_1z^{-3}}{b_0d_0+(b_1d_0+b_0d_1)z^{-1}+(b_2d_0+b_1d_1)z^{-2}+b_2d_1z^{-3}}.$$

High Pass Filter: The transfer function of an analog second-order high pass Butterworth filter is

$$T_{2H}(s)=\frac{\Omega_0^2}{(\omega_c/s)^2+(\Omega_0/Q)(\omega_c/s)+\Omega_0^2}.$$

The equivalent recursive filter is

$$T_{2H}(z)=\frac{a_0+a_1z^{-1}+a_2z^{-2}}{b_0+b_1z^{-1}+b_2z^{-2}},\qquad \begin{matrix} a_0=1, a_1=-2, a_2=1, b_0=1+(\alpha/\Omega_0)/Q+(\alpha/\Omega_0)^2, \\ b_1=2\{(\alpha/\Omega_0)^2-1\},\ b_2=1-(\alpha/\Omega_0)/Q+(\alpha/\Omega_0)^2. \end{matrix}$$

The third-order filters can be written as

$$T_{3H}(s)=T_{1H}(s)\cdot T_{2H}(s),\quad T_{1H}(s)=\frac{\Omega_a}{(\omega_c/s)+\Omega_a}$$

The warped cutoff angular frequency and bi-linear transform give

$$T_{1H}(z)=\frac{c_0+c_1z^{-1}}{d_0+d_1z^{-1}},\ c_0=1,\ c_1=-1,\ d_0=1+\alpha/\Omega_a, d_1=-1+\alpha/\Omega_a.$$

These formulas are valid also for Chebyshev and Bessel filters because their transfer functions are composed of $T_{1L}(s), T_{2L}(s), T_{3L}(s)$ or $T_{1H}(s), T_{2H}(s), T_{3H}(s)$.

3.4.6. Exercise for Digital Filtering

The transfer function of a given recursive filter is easily obtained by using Eqs. (20) and (25). However, it is not easy to design the filter's coefficients; to achieve this, its amplitude and phase characteristics must be arranged in the desired manner. Saito (1978) has published a subroutine package written in Fortran, which gives the coefficients of a recursive filter whose frequency characteristics coincide with one of the four filters popularly used in electronics, i.e., Butterworth, Chebyshev-I (constant ripple in pass band), Chebyshev-II (constant ripple in stop band), and Elliptic filters. Each of these four can be arranged as high pass, low pass, band pass, and band stop filters, respectively. These programs have been provided as a free software.

However, it is requested that users of the software acknowledge this by stating that **"the subroutines for digital filtering published by Saito (1978) are used for processing"** with the reference **" Saito, M. (1978):An automatic Design Algorithm for Band Selective Recursive Digital Filters, BUTURI-TANSA, Vol. 31, No. 4, pp112-135 (in Japanese)."** The distributed programs have been compiled and linked and provided as the library for *g77* on *Cygwin*. The filtering operation can be performed by just calling the subroutine *BANDP1* in the main routine as follows:

CALL BANDP1 (X,N,DT,FL,FH,FS,AP,AS, ntype, nchara,n causal)

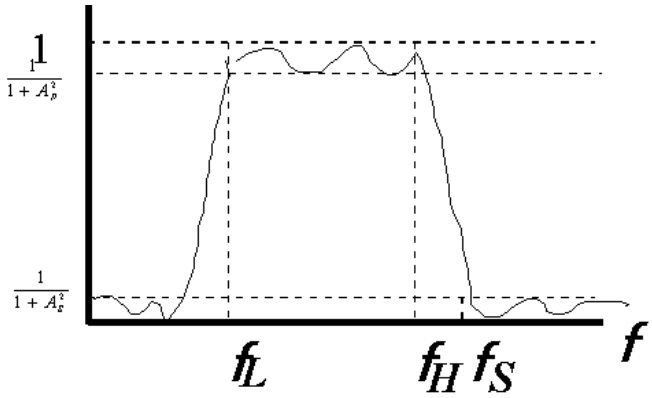
where *X* : Input time series / Filtered time series,
N : Number of data included in *X*,
DT : Sampling interval,
FL : Lower limit frequency of the pass-band,
FH : Upper limit frequency of the pass-band,
FS : Limit frequency of the stop-band,
AP : $1/(1 + A_P^2)$ denotes the ripple in the pass-band,
AS : $1/(1 + A_S^2)$ denoted the ripple in the stop-band.

A_p and A_s are schematically indicated in Fig. 49.

NTYPE : Flag that indicates the type of filter
1 Butterworth filter
2 Chevyshev-I
3 Chevyshev-II
4 Elliptic

NCHARA: Flag that indicates the frequency characteristics of filter
1 Low cut (high pass)
2 High cut (low pass)
3 Band pass
4 Band stop

NCAUSAL: Flag that indicates the causality of filter
1 Causal
2 Zero phase



Zero-phase filtering always breaks the causality. For example, a small ringing caused by the filtering, appears before the initial break of the P-wave and makes it difficult to judge the arrival time. Causal filtering can be performed by these identical subroutines. The flag *NCAUSAL* controls the subroutine *GNF* for both causal and non-causal filtering.

Exercise:

(1) Construct a time series that has an impulse of unit amplitude at $t = 1.28$ s with $\Delta t = 0.01$, $N = 256$ and store the data in the file *UT* by using *TESTSIG*.

(2) Draw the time series *UT* and check it by using *GSTOOLS*.

(3) Apply FFT to the time series *UT* by using *FFT*.

(4) Draw the Fourier spectra from *UF* by using *PSPEC*.

(5) Apply a digital filter by using *TRFILT*. The output file name is *UT1;* A Butterworth type low cut causal filter with FS = 15.0 Hz, FL = 30.0 Hz, AP = 10.0, As = 0.1 is selected for the following example.

```
a:\> TRFILT UT   UT1
 ??? Filter Type ???
       Butterworth   type              ==> 1
       Chebyshev-I   type               ==> 2
       Chebyshev-II type               ==> 3
       Elliptic        type             ==> 4
1
 ??? CHARACTERISTICS OF FILTER ???
       Low Cut (High Pass) Filter ==> 1
       High Cut (Low Pass) Filter ==> 2
       Band Pass                  Filter ==> 3
       Band Stop                  Filter ==> 4
1
              FL
              /------------
            /
          /
        /
------/
       FS
 ??? FS,FL,AP,AS ???
       AP,AS: Parameter defining the ripple
              in pass band and stop band.
              Use AP = 0.1, AS = 10.0, if you do
              not like to think.
15. 30. 0.1 10.0
 ??? Causal or Zero-Phase ???
       Causal        FIlter ==> 1
       Zero-Phase Filter ==> 2
1
```

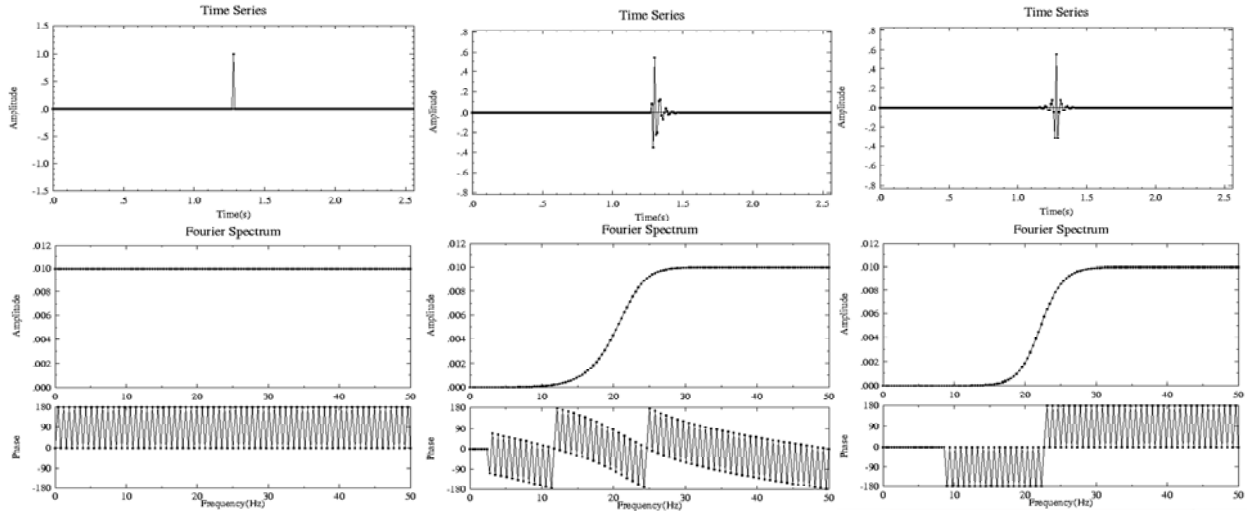


Fig. 50 Examples of the performance of the recursive filter designed by *TRFILT.EXE*. *Left*: the input signal that is an impulse of unit amplitude located at $t = 1.28$ s. *Center*: the output from the causal Butterworth low pass filter. *Right*: the output from the zero phase Butterworth low pass filter. The parameters used are $\Delta t = 0.01$, $N = 256$, $F_S = 15.0$ Hz, $F_L = 30.0$ Hz, $A_p = 0.1$, $A_s = 10.0$.

(6) Draw the filtered time series *UT1* by using *PTIME*.

(7) Apply *FFT* to the filtered time series *UT1* and store the result in the file *UF1*.

(8) Draw the Fourier spectra stored in *UF1* by using *PSPEC* and compare it with the drawn *UF*.

(9) Check the performance for filters of other types by repeating the above procedure with different values of control parameters.

3.5. Deconvolution or Inverse Filtering

Every observed seismic waveform data is an output from some filters. For example, the seismometer itself is a high pass filter. Amplifiers, sometimes, have frequency characteristics that are not uniform for all frequencies. Moreover, we use other high cut filter to suppress AC noise that has energy at 50 or 60 Hz since these distort seismic signals. Moreover, an anti-aliasing filter is used in digital data acquisition systems. Since seismometry is conducted not only for obtaining travel time data but also for obtaining waveform data, i. e., ground motion, we must compensate this distortion in order to obtain true ground motion. Remember that recorded signals are obtained by the convolution of the instrumental characteristics to the ground motion in the time domain and are given by their product in the frequency domain. The instrumental characteristics are identical to the response of instruments to an impulsive signal (Fig. 51 *top*).

$$\begin{aligned} r(t) &= f(t) * g(t), \\ R(\omega) &= F(\omega) G(\omega). \end{aligned} \tag{36}$$

where $r(t)$, $f(t)$, and $g(t)$ denote the recorded signals, the instrumental response, and the ground motion in the time domain, respectively; $R(\omega)$, $F(\omega)$, and $G(\omega)$, respectively, are the same parameters in the frequency domain.

Deconvolution in seismology is usually the process for eliminating the effect of the instrumental characteristics from the observed data and to recover the true ground motion. Mathematically, this is the reverse process of convolution. Deconvolution in the time domain corresponds to the quotient in the frequency domain. In comparison with the complexity of deconvolution in the time domain, the frequency domain operation is composed of only three steps. These are for applying the FFT to the instrument response and recorded signal, to divide the recorded signal spectra by the instrumental response spectra, and to apply the inverse FFT to the quotient (Fig. 51 *middle*).

$$G(\omega) = \{F(\omega)\}^{-1} \cdot R(\omega). \tag{37}$$

However, we must consider the following. The information once lost in the observation or processing can never be recovered by any technique, even by extremely sophisticated and efficient ones. This is because we cannot avoid noises that are recorded simultaneously with the signal or those that invade into the record during the processing. Once the signals weaken and become smaller than the noise level, any recovery process only amplifies such noises. Such amplified noises can be dominant in the recovered ground motion. The signals weaken a little either only during the recording or the processing can be strengthened or recovered by the deconvolution technique.

Typically, true ground motion has band limited feature at the far field and low pass feature at the near field, whereas the ground noise is present at every frequency. Let us handle only far field ground motion in order to have a simple demonstration. , $G(\omega)$ is band limited. The instrumental response $F(\omega)$ is also band limited, because a seismometer is a low cut filter and used with a high cut anti-alias filter. The recovering operator in the frequency domain $\{F(\omega)\}^{-1}$ has a large amplitude at a frequency outside this limited frequency band. The recorded signal $R(\omega)$ is almost band limited but it has little energy outside the frequency band of $G(\omega)$, i.e., the contribution of the noise. By applying the inverse filter, i.e., the recovering operator $\{F(\omega)\}^{-1}$, this small contribution of the noise will be considerably amplified and

contaminates the recovered ground motion transformed into the time domain by the inverse FFT (Fig. 51 *bottom*).

This shows that we have to select a frequency range such that the signal is sufficiently larger than the noise in order to prevent instability due to the application of the inverse filter. We cannot recover the ground motion outside this frequency band.

However, the recommended method is to handle the data only within the pass band of the instrumental response. Usually, this is given by the natural frequency of the seismometer and the cutoff frequency of the anti-alias filter. It is possible but not easy to use the information outside of this range. Even within this frequency range, the above mentioned instability problem can take place due to the smaller frequency band of $G(\omega)$. Thus, it is also recommended that the shape of $R(\omega)$ be observed and the useful frequency band be selected before beginning the data processing.

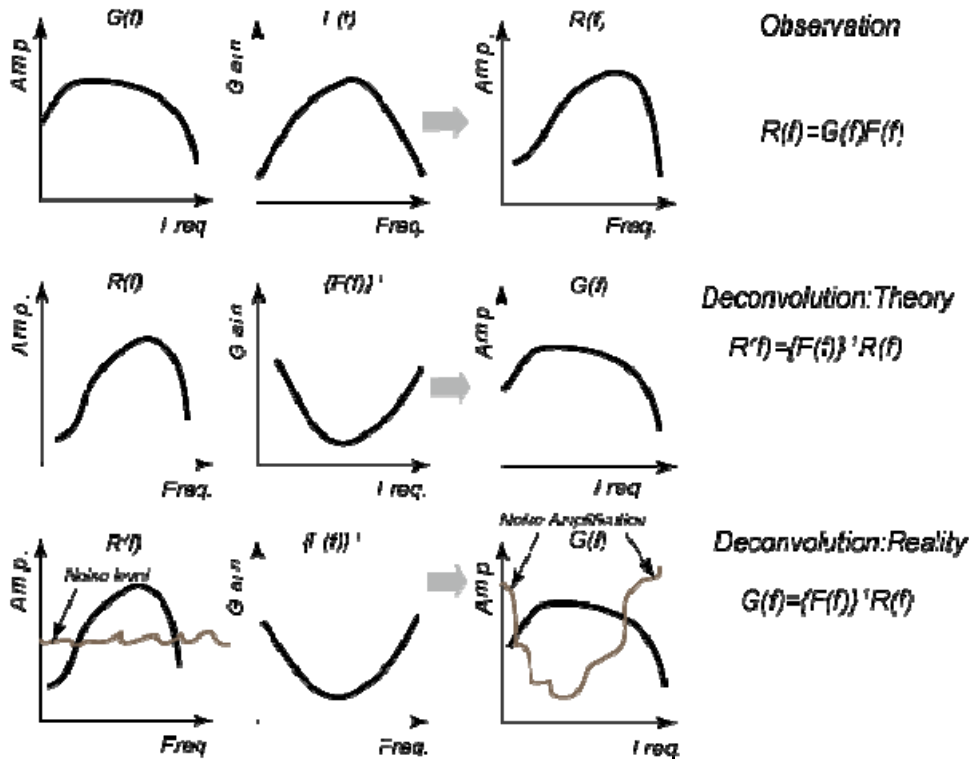


Fig. 51 Schematic illustration of Eq. (36) (*top*), theoretical inverse filtering process given by Eq. (37) (*middle*) and actual inverse filtering process with noise (*bottom*). Reproduced based on Sherbaum (1996).

Example: Inverse Filter for a Simple Moving Coil Type Seismometer

The transfer function that gives the inverse response to the previous examples is given as follows:

$$T(s) = G^{-1} \cdot \frac{s^2 + 2h\omega_0 s + \omega_0^2}{s^3},\ G = \frac{G_0 R_s}{R_0 + R_s}. \qquad (36)$$

Fig. 52 shows the amplitude response of this transfer function.

The solutions of the equation, i.e., the numerator is equal to zero, are $s = \omega_0\left(-h \pm \sqrt{h^2 - 1}\right)$. This gives the zero position at

$\left(-\omega_0 h,\ \omega_0\sqrt{1-h^2}\right),\ \left(-\omega_0 h,\ -\omega_0\sqrt{1-h^2}\right)$, for h < 1.0, the under-damped case,

$(-\omega_0,\ 0.)$ doubled for h = 1.0, critically damped case,

$\left(-\omega_0\left(h-\sqrt{h^2-1}\right), 0.\right),\ \left(-\omega_0\left(h+\sqrt{h^2-1}\right), 0.\right)$ for h > 1.0, over-damped case.

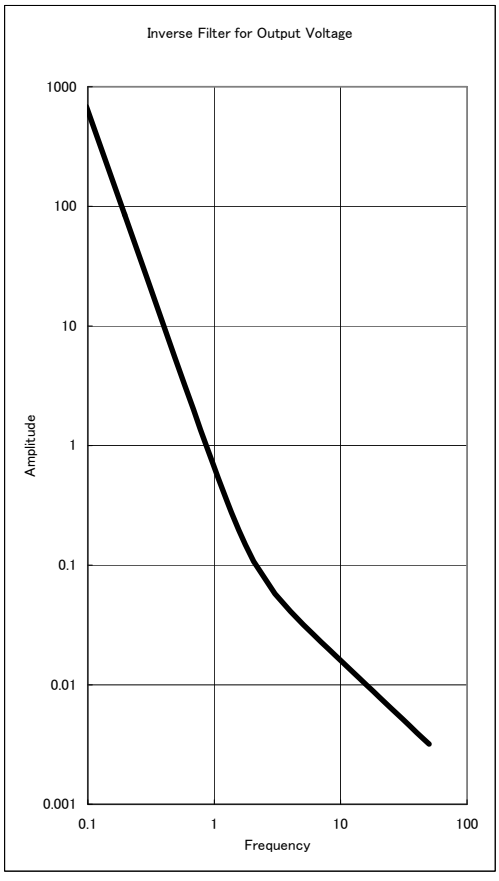
The denominator, however, gives a tripled pole at (0, 0).

By using the warped frequency, the transfer function is given approximately as follows.

$$T(s) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}},$$

where

$$a_0 = (\Delta t/2)\cdot\left\{1 + 2h\tan(\omega_0\Delta t/2) + \tan^2(\omega_0\Delta t/2)\right\},$$
$$a_1 = (\Delta t/2)\cdot\left\{-1 + 2h\tan(\omega_0\Delta t/2) + 3\tan^2(\omega_0\Delta t/2)\right\},$$
$$a_2 = (\Delta t/2)\cdot\left\{-1 - 2h\tan(\omega_0\Delta t/2) + 3\tan^2(\omega_0\Delta t/2)\right\},$$
$$a_3 = (\Delta t/2)\cdot\left\{1 - 2h\tan(\omega_0\Delta t/2) + \tan^2(\omega_0\Delta t/2)\right\}$$
$$b_0 = G, b_1 = -3G, b_2 = 3G, b_3 = -G.$$



Since a simple moving coil type seismometer is a low cut filter, its inverse filter makes the component grow in a frequency range that is lower than the natural frequency of the seismometer. This causes an instability in the output from the filter, because the signal to noise ratio is small at this frequency range.

Fig. 52 Amplitude response of the inverse filter for the voltage change between two output terminals of a simple moving coil type seismometer that is calculated by Eq. (36). The parameters used are T_0 = 0.5, h = 0.71. Theoretically, this filter can convert the output voltage variation with the ground displacement. However, this amplifies the low frequency components considerably as shown clearly in this figure.

Exercise: **Design of a recursive filter equivalent to a seismometer (simulated seismogram).**

The program *ISEISM.EXE* calculates the coefficients of the abovementioned inverse filter and applies the recursive filter to an input time series. Fig. 53.1 shows the following procedure.

(1) Prepare an input time series by using *TESTSIG.EXE*. For example, an impulse of the unit amplitude located at $t = 0.0$ s, with $\Delta t = 0.05$ s, $N = 128$ (Output: UT0). Then, apply a band pass filter by using *TRFILT.EXE* with this impulse. For example, for the Butterworth type, $F_L = 0.3$Hz, $F_H = 8.0$ Hz, $F_S = 9.5$ Hz, $A_P = 0.1$, $A_S = 10.0$ (Input: UT0, Output: UT1). This band pass impulse is used for artificial ground motion.

Calculate the Fourier transform (Input: UT1, Output: UF1)

(2) Run *VSEISM.EXE* with the natural period $T_0 = 0.25$ s, damping factor $h = 0.71$, $\Delta t = 0.05$, and gain $G_0 = 1.0$ (Input: UT1, Output: UT2). This output is the simulated seismogram for this exercise. Calculate the Fourier transform by using *FFT.EXE* (Input: UT2, Output: UF2).

(4) Run *ISEISM.EXE* with the natural period $T_0 = 0.25$ sec, the damping factor $h = 0.71$, $\Delta t = 0.05$, and gain $G_0 = 1.0$ (Input: UT2, Output: UT3). Calculate the Fourier transform by using *FFT.EXE* (Input: UT3, Output: UF3).

(5) Draw the time series UT1 and UT3 by using *PTIME.EXE* and its Fourier spectra UF1 and UF3 by using *FFT.EXE* and *PSPEC.EXE* and compare them. Consider the reason why the band pass filter is applied.

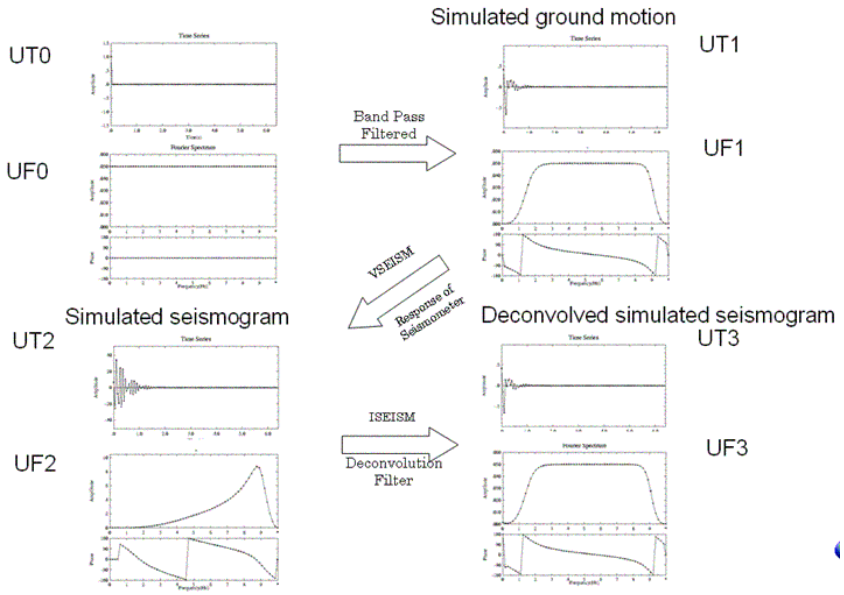


Fig. 53.1 Waveforms and its Fourier transform of each step of the example explained above. *Upper-Left*: Impulse. This is the beginning of the exercise, *Upper-Right*: Band pass filtered impulse, *Bottom Left*: Recursive filter equivalent to a seismometer is applied to the waveform shown in *Upper-Right*, *Bottom-Right*: Inverse filter is applied to the waveform shown in *Bottom-Left* panel. Compare *Upper-Right* and *Bottom-Right* panels to recognize the effect of inverse filter. Note that the band pass filtering at the second step is necessary to prevent the instability of output due to the big amplification at a high frequency by *VSEISM.EXE* and at a low frequency by *ISEISM.EXE*.

Exercise: Inverse Filter for a Simple Moving Coil Type Seismometer—II (Real Seismogram)

Ch1.dat is a seismogram (NS-component) observed by the L4C seismometer, the parameters of which are $\Delta t = 0.05$, $N = 8192$, $T_0 = 1.0$ s, $h = 0.64$.

(1) Draw wave forms of ch1.dat using *PTIME.EXE*.

(2) Run *ISEISM.EXE* with $T0 = 1.0$ s, $h = 0.64$, $\Delta t = 0.02$ and gain $G_0 = 2.7$ *(V/cm/s)* (Input: ch1.dat, Output: UT0).

(3) Apply a band pass filter by using *TRFILT.EXE* to this impulse. For example, the Butterworth type, $F_L = 0.1$ Hz, $F_H = 18.0$ Hz, $F_S = 20.0$ Hz, $A_P = 0.1$, $A_S = 10.0$ (Input: CH1.dat, Output: UT).

(4) Run *ISEISM.EXE* with $T_0 = 1.0$ s, $h = 0.64$, $\Delta t = 0.02$ and gain $G_0 = 2.7$ *(V/cm/s)* (Input: UT, Output: UT1).

(5) Apply a band pass filter by using *TRFILT.EXE* with the Butterworth type, $F_L = 0.1$ Hz, $F_H = 18.0$ Hz, $F_S = 20.0$ Hz, $A_P = 0.1$, $A_S = 10.0$ (Input: UT1, Output: UT2).

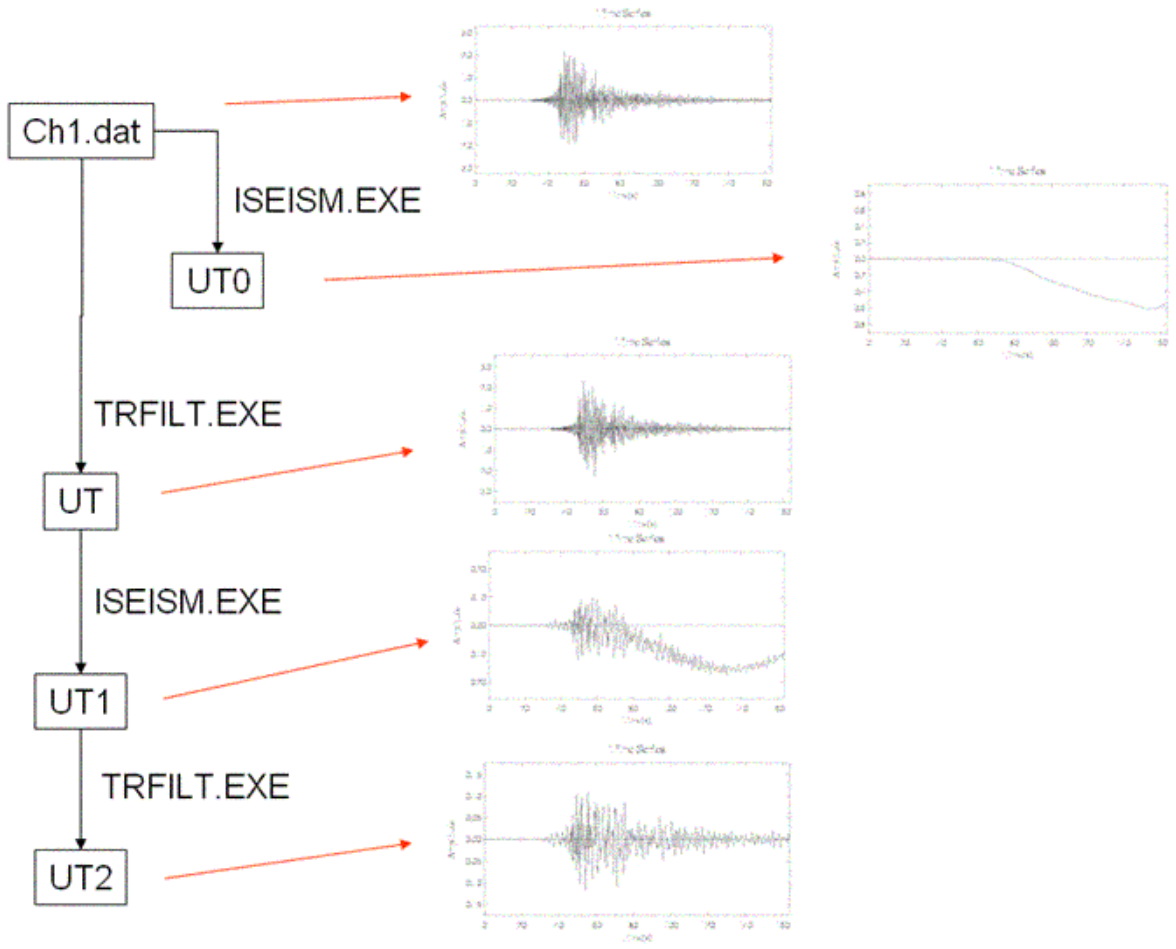


Fig. 53.2 Procedure and output of the example for a real seismogram.

Example: Filter for Conversion of Seismogram obtained by a Simple Moving Coil Type Seismometer to that of Longer Period Seismometer.

A simple way to avoid the abovementioned problem is to convert the system response with a longer-period seismometer.

Suppose that the damping constant and natural angular frequency of the seismometer used for the observations are h_0, ω_0, then its response is

$$e_m \Big/ y_m = G_0 \cdot \frac{(i\omega)}{1-2ih_0(\omega_0/\omega)-(\omega_0/\omega)^2}.$$

Let us convert this with the response from a seismometer, in which the damping constant and natural angular frequency are h_1, ω_1 .

$$e_m \Big/ y_m = G_1 \cdot \frac{(i\omega)}{1-2ih_1(\omega_1/\omega)-(\omega_1/\omega)^2}.$$

The inverse function in the frequency domain is

$$G^{-1} \cdot \frac{1-2ih_0(\omega_0/\omega)-(\omega_0/\omega)^2}{1-2ih_1(\omega_1/\omega)-(\omega_1/\omega)^2},\ G = G_0/G_1.$$

The transfer function in the s-domain is given as follows:

$$T(s) = G^{-1} \cdot \frac{s^2 + 2h_0\omega_0 s + \omega_0^2}{s^2 + 2h_1\omega_1 s + \omega_1^2}. \tag{37}$$

Then, using the warped frequency gives

$$T(s) = G^{-1} \cdot \frac{\left(1-z^{-1}\right)^2 + 2h_0 \tan\left(\frac{\omega_0\Delta t}{2}\right)\left(1-z^{-1}\right)\left(1+z^{-1}\right) + \tan^2\left(\frac{\omega_0\Delta t}{2}\right)\left(1+z^{-1}\right)^2}{\left(1-z^{-1}\right)^2 + 2h_1 \tan\left(\frac{\omega_1\Delta t}{2}\right)\left(1-z^{-1}\right)\left(1+z^{-1}\right) + \tan^2\left(\frac{\omega_1\Delta t}{2}\right)\left(1+z^{-1}\right)^2}$$

$$= \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{b_0 + b_1 z^{-1} + b_2 z^{-2}},$$

where

$$\begin{aligned}
a_0 &= 1 + 2h_0 \tan(\omega_0\Delta t/2) + \tan^2(\omega_0\Delta t/2),\\
a_1 &= -2 + 2\tan^2(\omega_0\Delta t/2),\\
a_2 &= 1 - 2h_0 \tan(\omega_0\Delta t/2) + \tan^2(\omega_0\Delta t/2),\\
b_0 &= G\{1 + 2h_1 \tan(\omega_1\Delta t/2) + \tan^2(\omega_1\Delta t/2)\},\\
b_1 &= G\{-2 + 2\tan^2(\omega_1\Delta t/2)\},\\
b_2 &= G(1 + 2h_1 \tan(\omega_1\Delta t/2) + \tan^2(\omega_1\Delta t/2)).
\end{aligned}$$

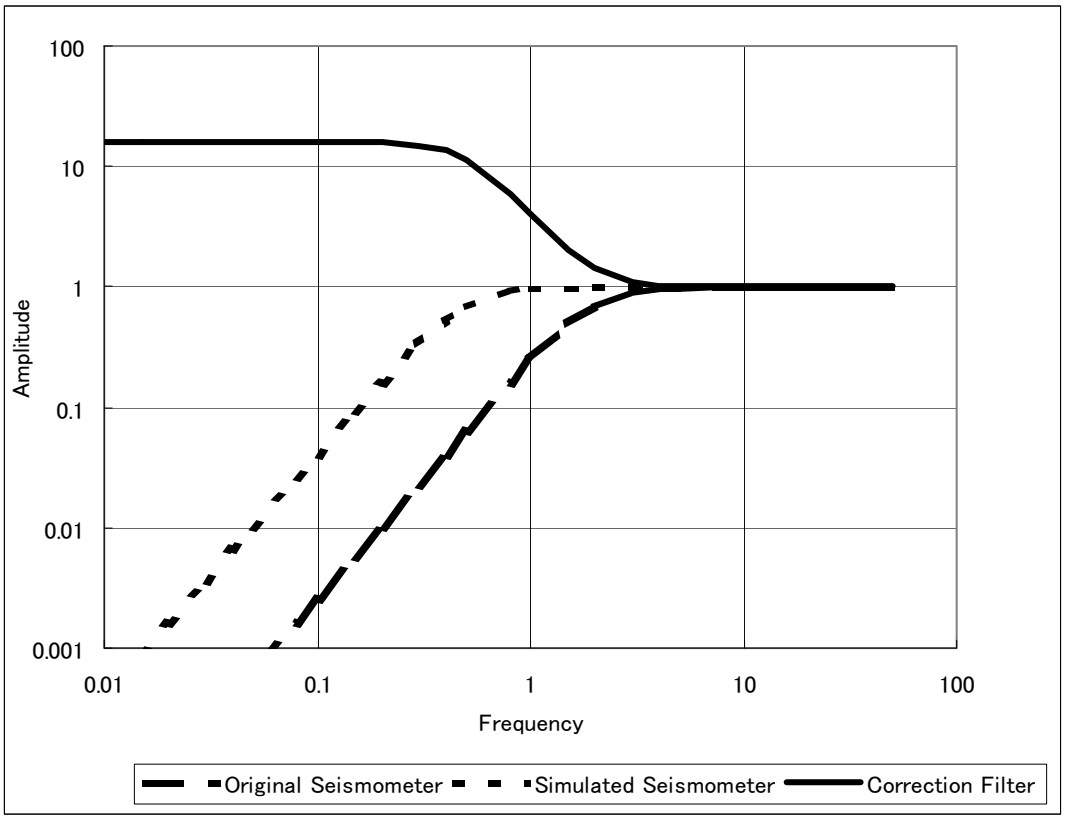


Fig. 54 Amplitude spectra of the correction filter explained above.

Exercise: Filter for Conversion of Seismogram obtained by a Simple Moving Coil Type Seismometer to that of Longer Period Seismometer （An Impulse）.

(1) The program *CSEISM.EXE* calculates the coefficients of the recursive filter that can convert the characteristics of the simple moving coil type seismometer used originally for (1) observing those of the simulated seismometer with a longer period.

(2) Prepare an input time series by using *TESTSIG.EXE*. For example, an impulse of unit amplitude located at $t = 0.0$ s, with $\Delta t = 0.05$ s, $N = 128$ (Output: UT0)

(3) Calculate Fourier transform by using *FFT.EXE* (Input: UT0, Output: UF0).

(4) Run *DSEISM.EXE* with the natural period $T_0 = 0.25$ s, the damping factor $h_0 = 0.71$, $\Delta t = 0.05$ and the gain $G_0 = 1.0$ (Input: UT0, Output: UT1).
Calculate Fourier transform by using *FFT.EXE* (Input: UT1, Output: UF1).

(5) Run *CSEISM.EXE* with $T_0 = 0.25$ s, $h_0 = 0.71$, $T_1 = 1.0$ s, $h_1 = 0.71$, $\Delta t = 0.05$, and gain $G_0 = 1.0$ (Input: UT1, Output: UT2).
Calculate Fourier transform by using *FFT.EXE* (Input: UT2, Output: UF2).

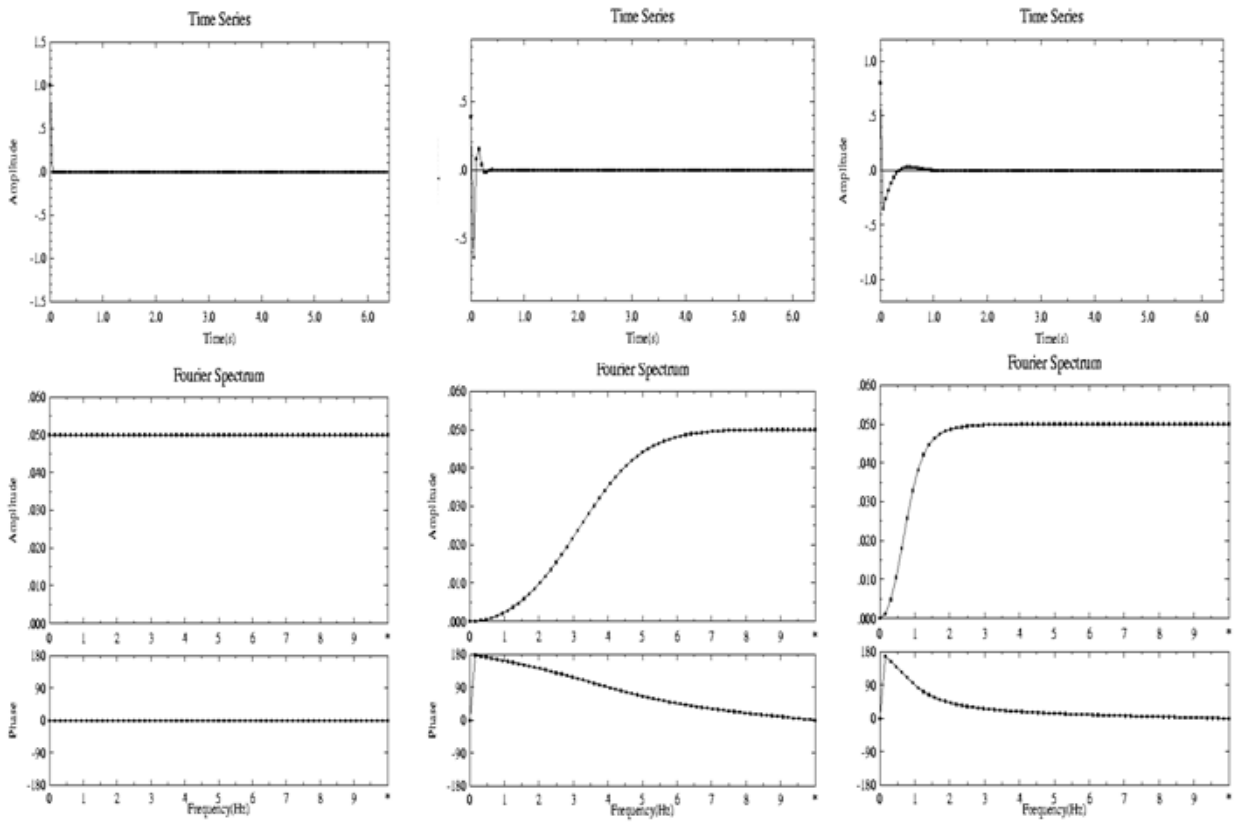


Fig. 55.1 Waveforms and spectra of the example mentioned above. *Left*: Impulse used as the input. *Center*: Simulated output for a seismometer T_0 = 0.25 s, $h_0 = 0.71$ by *DSEISM.EXE*. *Right*: Converted output from T_0 = 0.25, $h_0 = 0.71$ s to T_0 = 1.0, $h_0 = 0.71$ s by *CSEISMO.EXE*.

Exercise: **Filter for Conversion of Seismogram obtained by a Simple Moving Coil Type Seismometer to that of Longer Period Seismometer （Real Seismogram）**

Ch1.dat is a seismogram (NS-component) observed by L4C seismometer, the parameters of which are $\Delta t = 0.05$, $N = 8192$, $T_0 = 1.0$ s, $h = 0.64$.

(1) Draw wave forms of ch1.dat using *PTIME.EXE*.

(2) Run *CSEISM.EXE* with

Original Seismometer

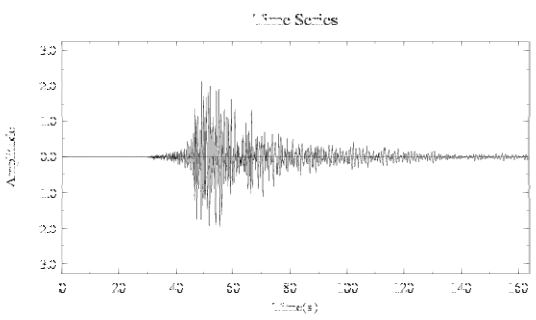
T_0, h, dt, g_0: 1.0, 0.64, 0.02, and 2.7 (V/cm/s)

Simulated Seismometer

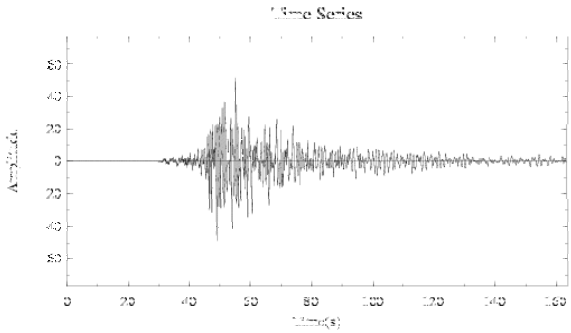
T_0, h, dt, g_0: 5.0, 0.71, 0.02, and 50.0 (V/cm/s)

(Input: ch1.dat, Output: output.dat).

Calculate Fourier transform by using *FFT.EXE* (Input: output.dat, Output:output.spc).



Input.dat



Output.dat

Fig. 55.2 Input signal (*left*): Ch1.dat and output signal (*right*) of *CSEISMO.EXE*

3.6. Integration of accelerogram or Base Line Correction

3.6.1 Origin of Low-Frequency Errors

Chiu (1997) explains the origins of low-frequency errors in the following manner. We are interested mainly in low-frequency noises because the effects of high-frequency noises are considerably reduced by double integration since their frequency band is lower than Nyquist frequency.

(1) Deviations of the instrument response from flat amplitude response and from linear phase shift. These are large in a high-frequency range. Careful calibration by the manufacturer is essential.
(2) Limited resolution of analog-to-digital Converter. This causes the approximations and the drift.
(3) Insufficient sampling rate that can be prevented by considering the Nyquist frequency. Although the frequency band of interest to engineering seismologists is lower than 10 Hz, sampling with at least 100 Hz is recommended in order to obtain the value of the peak acceleration correctly.
(4) Electronic Noise: besides the AC noise at 50 or 60 Hz, low-frequency electric noise caused by fluctuation of temperature is present.
(5) Ambient noises that include a long-period microtremor and slow deformation of observatory building, *e. g.* , due to sunshine affects the integrated displacement seismogram significantly.

Their effects appear superposed over each other and the seismic signals in the seismograms and in the output of digital data processing. It is almost impossible to separate the signal from these noises in the time domain.

These cause the base line drift of accelerograms and absurd waveforms of velocity and displacement seismograms obtained by numerical integration. An example is shown below.

Fig. 56.1 shows the original accelerograms obtained at K-NET maintained by NIED, Japan; its station is KGS005 in the northwest part of Kagoshima pref. and the seismic event (M_{JMA} = 3.6) took place 21:26 on March 27, 1997. This is one of the aftershocks of Kagoshima-Ken Hokusei-bu Eq. (M = 6.3). These include the noises mentioned above, although there is no visible noise in the shown scale.

The acceleration amplitude spectra of the seismic event (time window; [12.0 s, 24.0 s]) shown in the top panels of Fig. 56.2 has a wide peak that corresponds to the seismic signals and a portion of noise that is more clearly shown in the low frequency band. The latter is emphasized in the displacement amplitude spectra shown in the bottom panels. Fig. 56.3 shows the amplitude spectra of the noise (time window; [0.0 s, 10.0 s]) recorded before the P-wave arrivals. A comparison of Fig. 56.3 with Fig. 56.2 shows the effects of the low-frequency noises in the frequency domain.

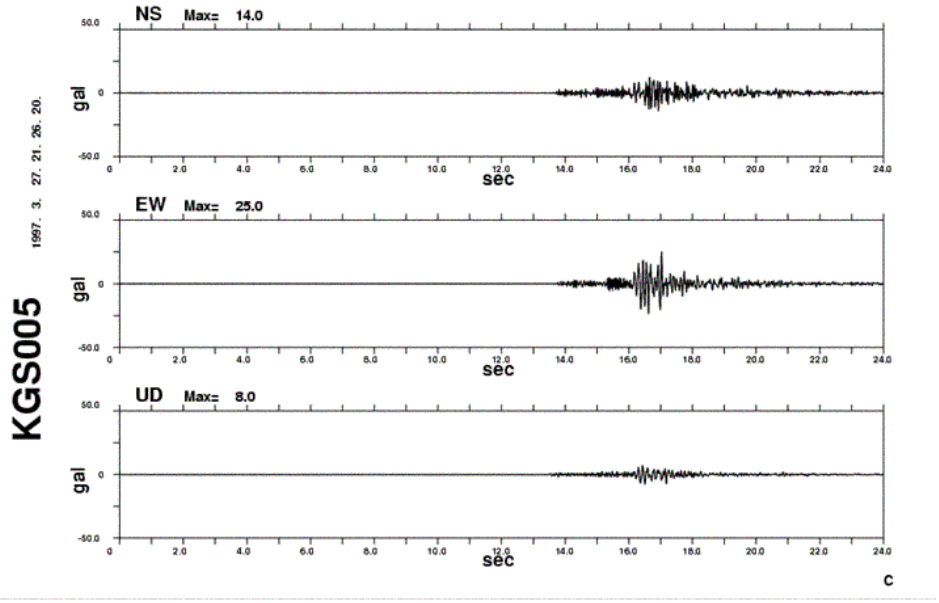


Fig. 56.1 Original Accelerograms. Plotted by *pltacc.exe*.

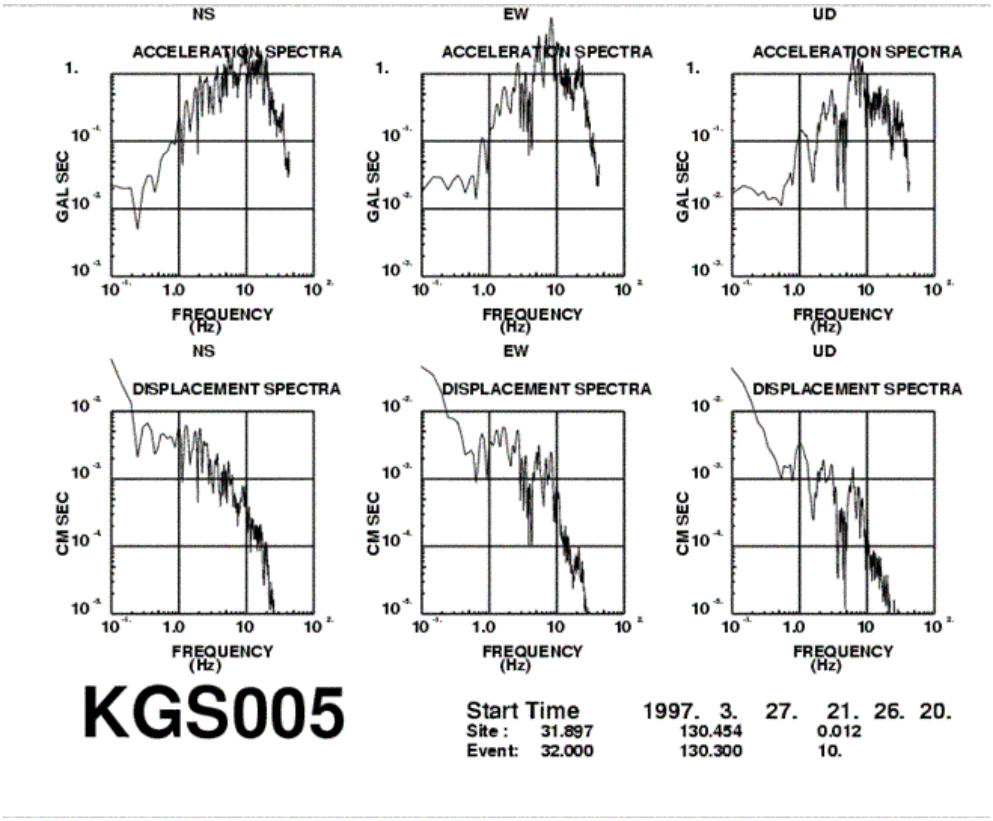


Fig. 56.2 Amplitude spectra of accelerograms shown in Fig. 56.1 for the time window [12.0 s, 24.0 s] that include the seismic event. The increasing peak at the frequency band lower than 0.4Hz is emphasized in the displacement amplitude spectra obtained by the division of acceleration spectra by $(2\pi f)^2$. Plotted by *splot.exe*.

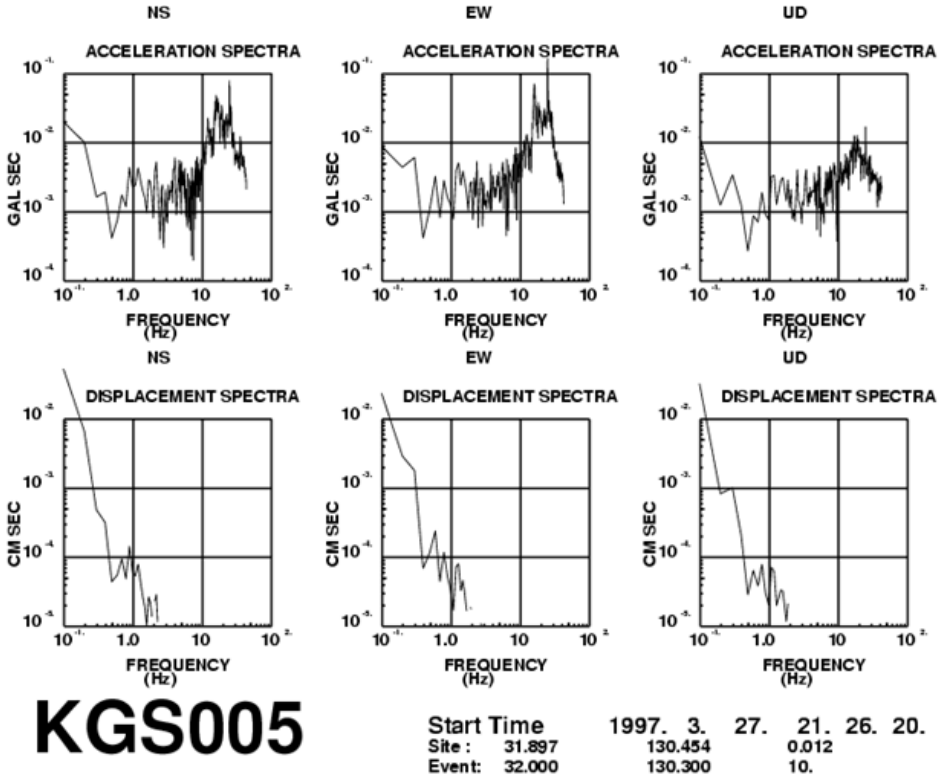


Fig. 56.3 Amplitude spectra of accelerograms shown in Fig. 56.1 for the time window [0.0 s, 10.0 s] that is the noise part. The increasing peak at the frequency band lower than 0.4 Hz is emphasized in the displacement amplitude spectra obtained by the division of acceleration spectra by $(2\pi f)^2$. Plotted by *splot.exe*.

The integration of the time series can be performed by recursive filtering as given by the following.

$$y_m = y_{m-1} + \Delta t \cdot x_m .$$

The velocity seismograms obtained by a direct integration of the accelerograms shown in Fig. 56.1 are shown in Fig. 56.4. These almost linearly increasing velocity seismograms show the effect of the constant offset included in the original accelerograms. The slightly observable curvature is due to the linear trend included in them.

The correction for the constant offset and linear trend in accelerograms is performed by the least square fitting as shown by Eq. (8). Then, the numerical integration is performed. The result is shown in Fig. 56.5. This is considerably improved in comparison with Fig. 56.4. The slight base line drift of the long period is supposed to be due to the influence of the low-frequency noise remaining in the corrected accelerograms.

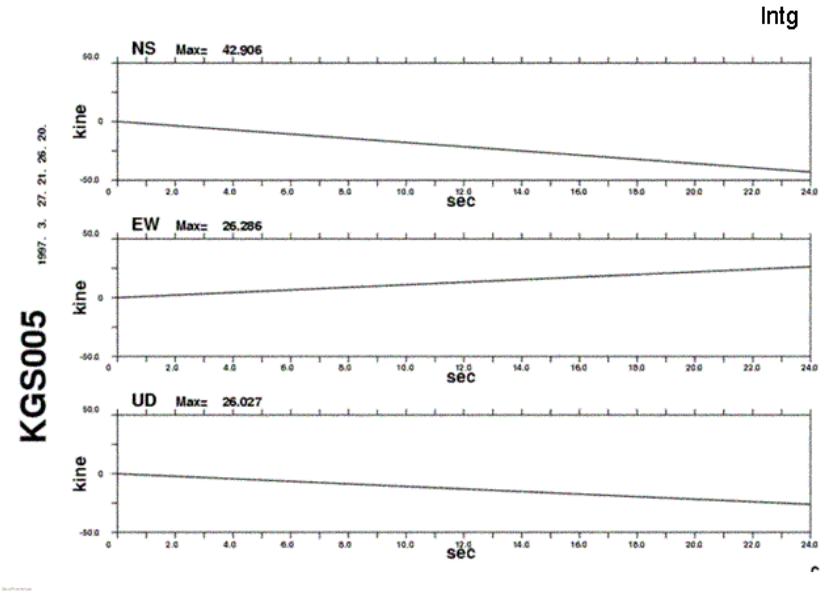


Fig. 56.4 Velocity seismograms obtained by a direct integration of the accelerograms shown in Fig. 56.1. The apparent linear trends are due to the influence of the constant drift in the accelerograms. Calculated by *acc2vel.exe* with comments for the offset-trend correction and band pass filtering, and then plotted by *pltvel.exe*.

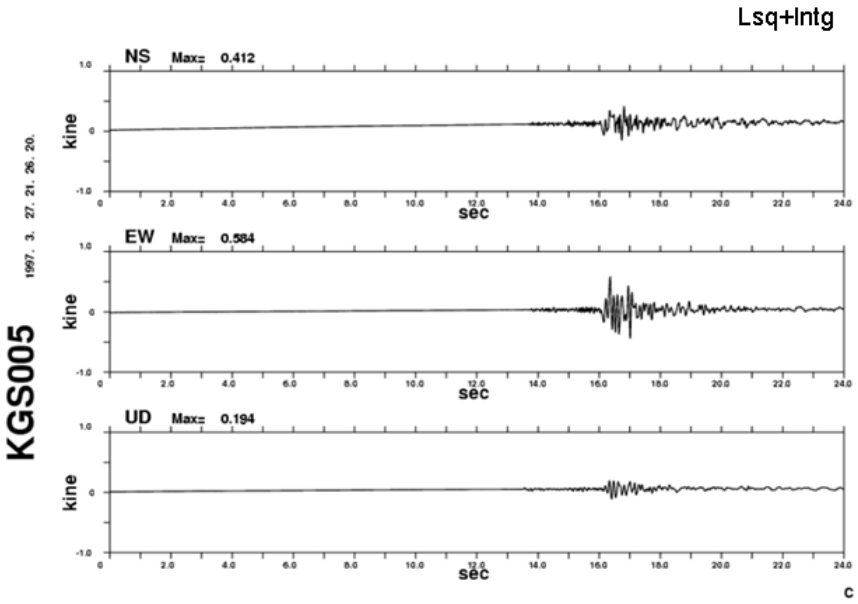


Fig. 56.5 The accelerograms shown in Fig. 56.1 are corrected by the least square fitting method and then integrated numerically. The slight drift in the base line is due to the influence of the low-frequency noise remaining in the corrected accelerograms. Calculated by *acc2vel.exe* with comments due to band pass filtering, and then plotted by *pltvel.exe*.

The remaining low-frequency noises can be removed by applying the high pass or band pass filter aimed to cut the low frequency component.

The example is shown in Fig. 56.6. The used values for the parameters of the band pass filter are as follows:.

fl, fh, fs/0.7, 25.0, 30.0/ ntype,nchara,ncausal/1,3,2/

f_l = 0.7 Hz corresponds to the limit frequency in which the seismic signals become dominant in comparison with noises (Fig. 56.2 *top panels*). f_h = 25.0 Hz and f_s = 30.0 Hz are selected in order to ensure the suppression of high-frequency noises. Ntype = 1 means Butterworth filter and nchara = 3 means band pass feature. ncause= 2 that means that a zero-phase filter is selected in order to maintain the waveform of seismic signals.

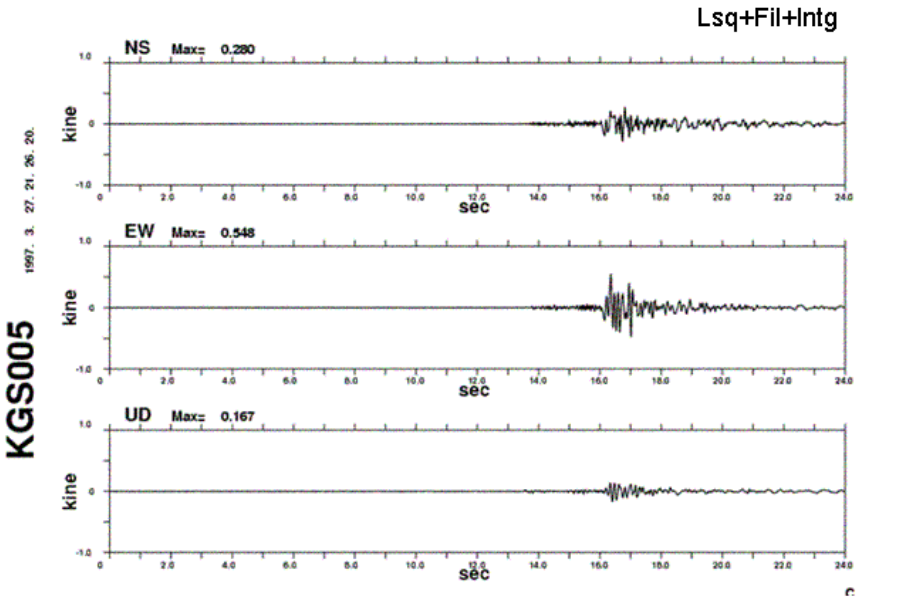


Fig. 56.6 The accelerograms shown in Fig. 56.1 are corrected by the least square fitting method, band pass filtered, and then integrated numerically. They were calculated by *acc2vel.exe* and plotted by *pltvel.exe*.

.

The conversion of the accelerogram to the displacement seismogram is tougher than that in the case of a velocity seismogram because of the double integrations. Fig. 56.7 shows the result of a direct integration of the velocity seismograms shown in Fig. 56.4, *i. e*., those of the double direct integration of the accelerograms shown in Fig. 56.1. The parabolic feature is due to the integration of the almost linear velocity seismograms (Fig. 56.4). Note the absurdly big values of the maximum value.

.

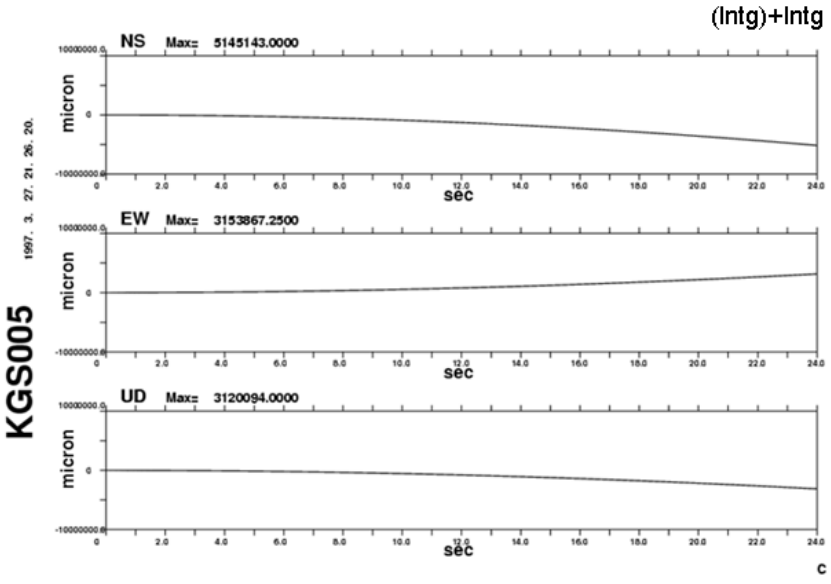


Fig. 56.7 Displacement seismogram obtained by a direct integration of the velocity seismograms shown in Fig. 56.4. Calculated by *acc2dis.exe* with comments on the offset-drift correction and band pass filtering for accelerograms and velocity seismograms; subsequently plotted by *pltdis.exe*.

Fig. 56.8 shows the displacement seismogram obtained by the least square correction and the integration of the velocity seismograms shown in Fig. 56.5. These show parabolic waveforms that are integrated low-frequency noises as well.

Fig. 56.9 Displacement seismogram obtained by a direct integration of the velocity seismograms shown in Fig. 56.6.

Fig. 56.10 Displacement seismogram obtained by the least square correction of the velocity seismograms shown in Fig. 56.6. This give a better performance than the others, but not sufficient. A slight trend is obvious in the NS component.

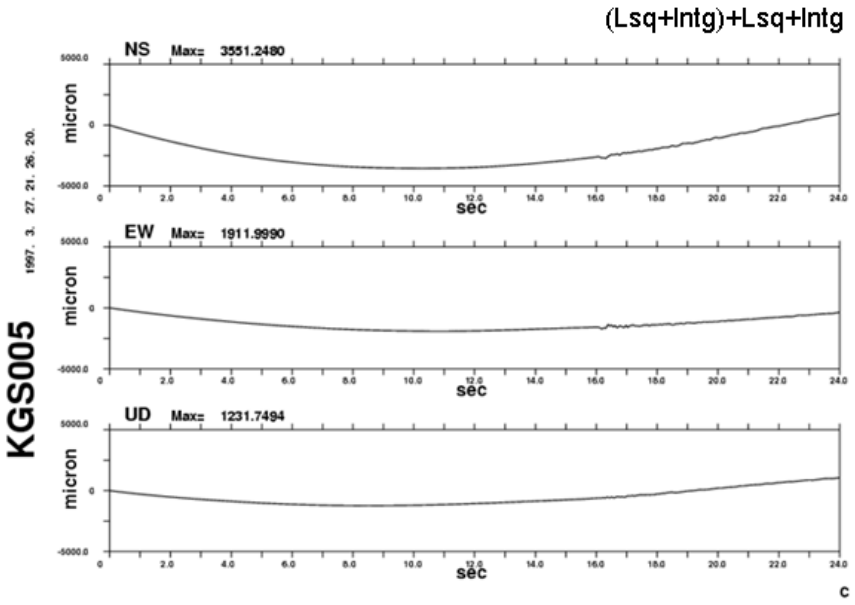


Fig. 56.8 Displacement seismogram obtained by the least square correction and integration of the velocity seismograms shown in Fig. 56.5. Calculated by *acc2dis.exe* with comments from the band pass filtering for accelerograms and velocity seismograms; subsequently plotted by *pltdis.exe*.

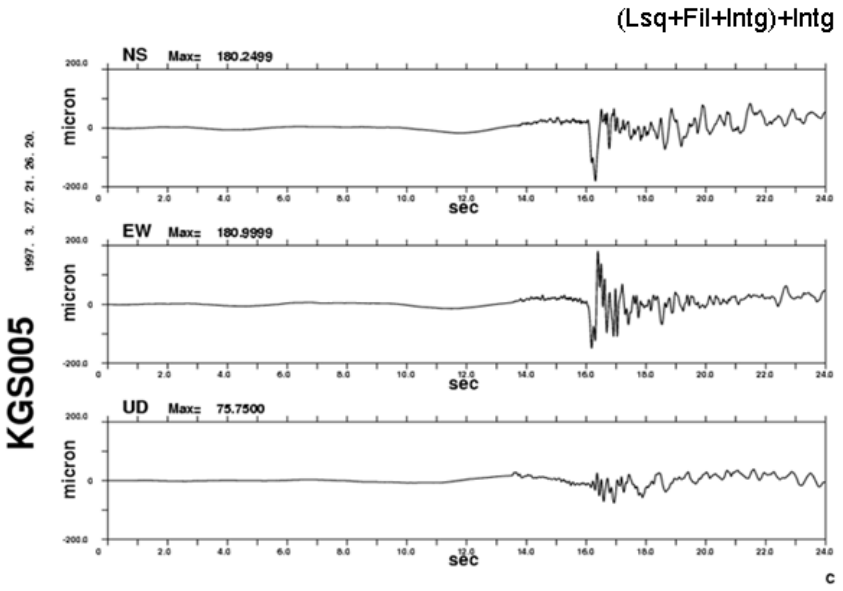


Fig. 56.9 Displacement seismogram obtained by a direct integration of the velocity seismograms shown in Fig. 56.6. Calculated by *acc2dis.exe* with comments from the offset-drift correction and band pass filtering for velocity seismograms; subsequently plotted by *pltdis.exe*.

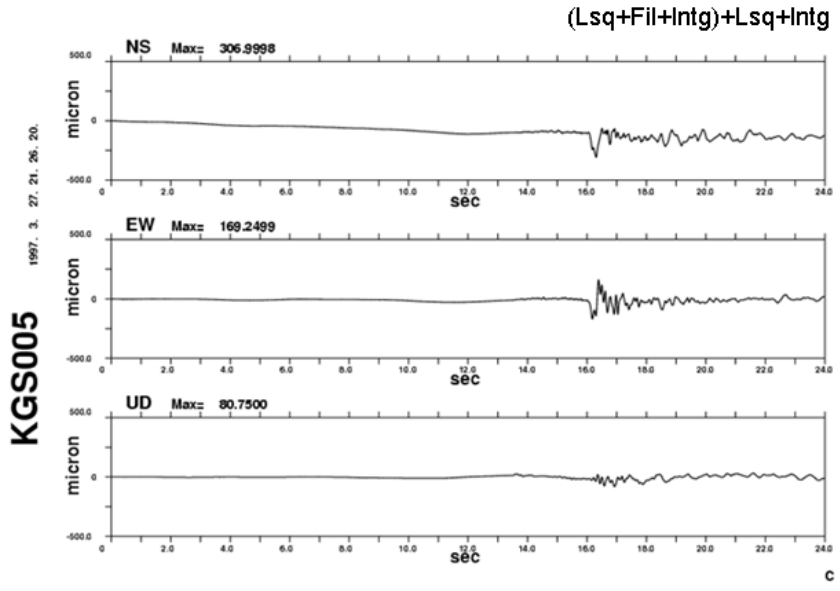


Fig. 56.10 Displacement seismogram obtained by the least square correction of the velocity seismograms shown in Fig. 56.6. Calculated by *acc2dis.exe* with comments from band pass filtering for velocity seismograms; subsequently plotted by *pltdis.exe*.

The attempts described above imply the necessity of an offset-drift correction and band pass filtering for accelerograms and velocity seismograms. Fig. 57.1 and Fig. 57.2 show stable results, i. e., displacement seismograms without a base line drift.

The difference between Fig. 57.1 and Fig. 57.2 is the value of the parameter f_l. In Fig. 57.1, f_l = 0.1 Hz, and in Fig. 57.2 f_l = 0.7 Hz. A ong-period ripple is observed before P-arrival in Fig. 57.1 due to the low-frequency noises that can be suppressed by selecting f_l = 0.7 Hz

The base line correction of the velocity and displacement seismograms integrated from observed an accelerogram is a very basic task for strong motion observation. The techniques of digital data processing described in this lecture note are necessary in totality.

The recommended way of integration is as follows.

(1) Calculate and plot the Fourier spectra of the observed accelerogram for both noise part and seismic event part.
(2) Determine the value of f_l that is the low frequency limit of the band such that the seismic signal is dominant in comparison with the noise level.
(3) Select the values of f_h and f_s, if the band pass filter is used. If a high pass filter is used, they are not necessary.
(4) Apply the offset-trend correction by the least square method and filtering to the observed accelerograms.
(5) Integrate the processed accelerograms; then the velocity seismograms are then obtained.
(6) Apply the offset-trend correction by the least square method and filtering to the obtained velocity seismograms.
(7) Integrate the processed velocity seismograms; then the displacement seismograms are obtained.
(8) If the obtained displacement seismograms have a long-period deviation before P-arrival time, return to (2) and change the value of f_l.

The result can be affected by the types of filter applied. Try to check the performance of various filters on your own.

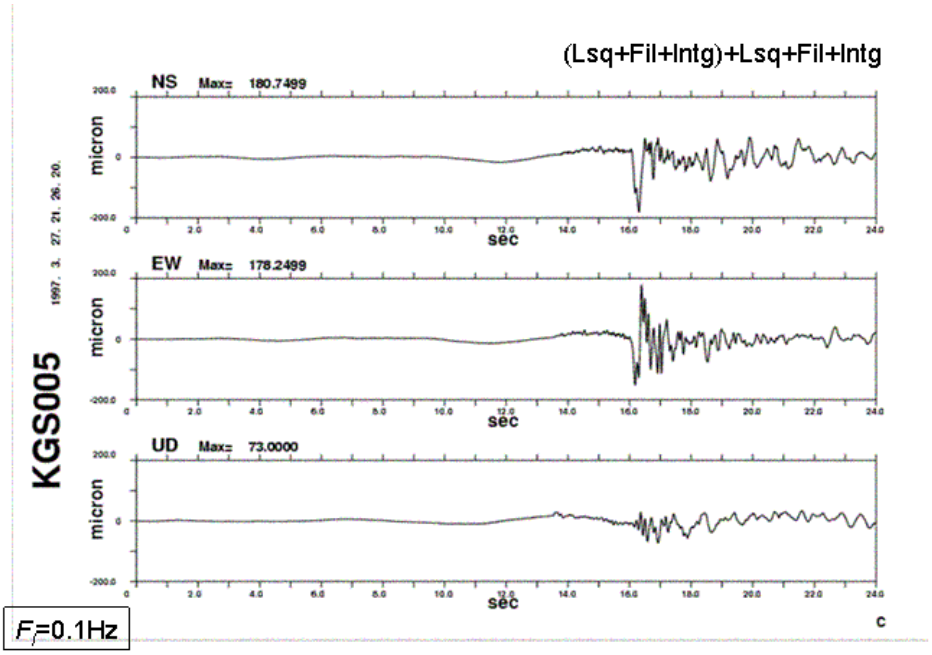


Fig. 57.1 Displacement seismogram obtained by the least square correction and band pass filtering of the velocity seismograms shown in Fig. 56.6. Long-period ripple observed before P-arrival is due to the low-frequency noises.

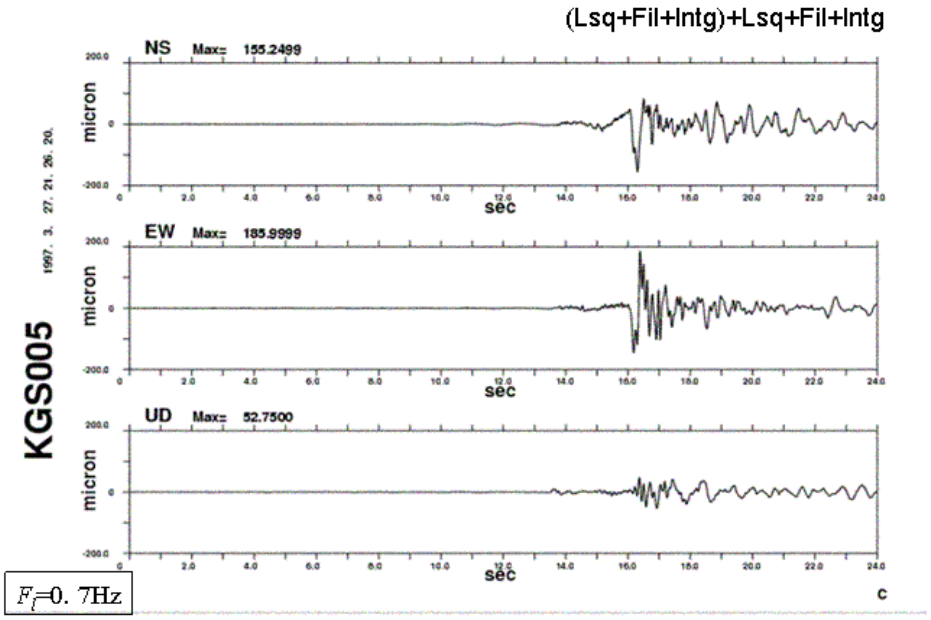


Fig. 57.2 Displacement seismogram obtained by the least square correction and band pass filtering of the velocity seismograms shown in Fig. 56.6. Long-period ripple observed before P-arrival in Fig. 57.1 is suppressed sufficiently.

CFFT.FOR

```
c*************************************************************
      SUBROUTINE CFFT(X,N,IND)
c.....................................................................
c Complex Fast Fourier Transform
c Coded by Y. OHSAKI in "Introduction to spectral analysis for
c                                   seismic waves"
c Typed by D.Suetsugu, IISEE
c Modified by H.Inoue, IISEE
c Usage.....    CALL FFT(N,X,IND)
c where
c X(N) In/Out Equi-spaced N complex data/Transformed values
c N    In     Number of data, MUST be a power of 2.
c IND In      -1=Fourier transform,   1=Inverse Fourier transform
c
c Note..... You must divide the output X() by N to get Fourier
c                   transform when IND=-1.
c.....................................................................
      complex x(n),temp,theta
c
      j=1
      do 140 i=1,n
        if(i.ge.j) goto 110
          temp=x(j)
          x(j)=x(i)
          x(i)=temp
  110   m=n/2
  120   if(j.le.m) goto 130
          j=j-m
          m=m/2
          if(m.ge.2) goto 120
  130   j=j+m
  140 continue
c
      kmax=1
      do 150 kkk=1,9999
        if(kmax.ge.n) return
        istep=kmax*2
        do 170 k=1,kmax
          theta=cmplx(0.0,3.141593*float(ind*(k-1))/float(kmax))
          do 160 i=k,n,istep
            j=i+kmax
            temp=x(j)*cexp(theta)
            x(j)=x(i)-temp
            x(i)=x(i)+temp
  160     continue
  170   continue
        kmax=istep
  150 continue
      end
```

TESTSIG.FOR

```
c*************************************************************
c     Make a test signal u(t)
c*************************************************************
      program main
      parameter (nmax=8192,pi=3.1415926536)
      real t(nmax),u(nmax)
      character yn*1,ofile*80
c
      if(nargs().le.1) then
        write(*,*)
        write(*,*) 'Usage: TESTSIG outfile'
        stop ' '
      end if
      call getarg(1,ofile,istatus)
c     ..................................... Input parameters
      write(*,*)
      write(*,*) 'Parameters for a test signal.'
      write(*,'(a$)') ' Sampling interval dT (sec) >>'
      read(*,*) dt
      write(*,'(a$)') ' Number of data points N      >>'
      read(*,*) ndata
      write(*,'(1x,a,f10.3,a)')'Total time is',dt*ndata,' sec.'
      write(*,'(1x,72a)') ('-',i=1,72)
      do i=1,ndata
        t(i)=(i-1)*dt
        u(i)=0.0
      end do
c     ..................................... Superposition loop
      do isum=1,999
        write(*,'(a$)') ' Cos/Sin=1, Const=2, Impulse=3,'//
     &      ' Step=4, Triangle=5, Square=6   >>'
        read(*,*) ichoice
        if(ichoice.eq.1) then
          write(*,'(a$)') ' Period of the cosine wave (sec)>>'
          read(*,*) period
          write(*,'(a$)') ' Amplitude                    >>'
          read(*,*) amp
          write(*,'(a$)') ' Phase (0-360 in degree)      >>'
          read(*,*) phase
          write(*,'(a$)') ' Damping factor               >>'
          read(*,*) damp
          do i=1,ndata
            arg=phase/180.*pi+2*pi/period*t(i)
            u(i)=u(i)+amp*cos(arg)*exp(-damp*t(i))
          end do
        else if(ichoice.eq.2) then
          write(*,'(a$)') ' The constant value           >>'
          read(*,*) const
          do i=1,ndata
            u(i)=u(i)+const
          end do
        else if(ichoice.eq.3) then
          write(*,'(a$)') ' Location of the impulse (sec)  >>'
          read(*,*) timpulse
          write(*,'(a$)') ' Amplitude of the impulse       >>'
          read(*,*) aimpulse
          do i=1,ndata
            if(t(i).ge.timpulse) then
              u(i)=u(i)+aimpulse
              exit
            end if
          end do
        else if(ichoice.eq.4) then
          write(*,'(a$)') ' Location of the step(sec)      >>'
          read(*,*) tstep
          write(*,'(a$)') ' Amplitude of the step          >>'
          read(*,*) astep
          do i=1,ndata
            if(t(i).ge.tstep) then
              u(i)=u(i)+astep
            end if
          end do
        else if(ichoice.eq.5) then
          write(*,'(a$)')'Period of the triangular wave(sec)>>'
          read(*,*) period
          write(*,'(a$)') ' Amplitude                    >>'
          read(*,*) amp
          write(*,'(a$)') ' Phase (0-360 in degree)      >>'
          read(*,*) phase
          do i=1,ndata
            half=period/2.
            tt=t(i)+phase/360.*period
            tmod=mod(tt,half)
            if(mod(int(tt/half),2).eq.0) then
              u(i)=u(i)+amp-amp*2*tmod/half
            else
              u(i)=u(i)-amp+amp*2*tmod/half
            end if
          end do
        else if(ichoice.eq.6) then
          write(*,'(a$)')'Period of the square wave(sec)   >>'
          read(*,*) period
          write(*,'(a$)') ' Amplitude                    >>'
          read(*,*) amp
          write(*,'(a$)') ' Phase (0-360 in degree)      >>'
          read(*,*) phase
          do i=1,ndata
            half=period/2
            tt=t(i)+phase/360*period+period/4.0
            if(mod(int(tt/half),2).eq.0) then
              u(i)=u(i)+amp
            else
              u(i)=u(i)-amp
            end if
          end do
        end if
        write(*,'(a$)') ' Add more? (y/n)   >>'
        read(*,'(a)') yn
        if(yn.ne.'y'.and.yn.ne.'Y') exit
      end do
      write(*,'(1x,72a)') ('-',i=1,72)
c     ..................................... Output Time Series
      open(10,file=ofile)
      write(10,*) ndata,' =N'
      write(10,*) dt,' =dT (sec)'
      do i=1,ndata
        write(10,*) u(i)
      end do
      close(10)
      write(*,*) 'Done'
c
      end
```

PTIME.FOR

```
c*************************************************************
c       Plot time series in a file
c*************************************************************
        program main
        parameter (nmax=8192)
        real t(nmax),u(nmax)
        character ifile*80
c       ..... ..... Input Time Series
        if(nargs().le.1) then
              write(*,*)
              write(*,*) 'Usage: PTIME infile'
              stop ' '
        end if
        call getarg(1,ifile,istatus)
        open(10,file=ifile,status='old')
        read(10,*) ndata
        read(10,*) dt
        do i=1,ndata
              read(10,*) u(i)
              t(i)=dt*(i-1)
        end do
        close(10)
c               ..... Plot Time Series
        umax=u(1)
        umin=u(1)
        do i=2,ndata
              if(u(i).gt.umax) umax=u(i)
              if(u(i).lt.umin) umin=u(i)
        end do
        ymax=max(abs(umax),abs(umin))*1.5
c
        call opengraphics
        call setwindow(1,0.,-ymax,dt*ndata,ymax)
        call setviewpt(1,0.3,0.3,1.1,0.7)
        call drawtitle('Time Series')
cc         call setline('width=2')
        call settext('size=0.8')
        call drawaxis('labelx=Time(s),labely=Amplitude')
        call drawline(0.,0.,dt*ndata,0.)
        call setmark('type=solid-square,size=0.5')
        call drawlines(ndata,t,u)
        call drawmarks(ndata,t,u)
        call closegraphics
        end
```

FFT.FOR

```
        include 'cfft.for'
c*************************************************************
c       Calculate Fourier Coefficients by FFT
c*************************************************************
        parameter(nmax=8192)
        real u(nmax)
        complex cf(nmax)
        character ifile*80,ofile*80
c
        if(nargs().le.2) then
              write(*,*)
              write(*,*) 'Usage: FFT infile outfile'
              stop ' '
        end if
        call getarg(1,ifile,istatus)
        call getarg(2,ofile,istatus)
c..... Read time series from file
        open(10,file=ifile,status='old')
        read(10,*) ndata
        read(10,*) dt
        do i=1,ndata
              read(10,*) u(i)
        end do
        close(10)
c..... Check if ndata=2**n
        npow=nint(log(float(ndata))/log(2.0))
        nn=2**npow
        if(ndata.ne.nn) then
          write(*,*) 'N=',ndata,',      Error: N must be a power of 2.'
stop
        end if
c..... FFT
        do i=1,nn
              cf(i)=cmplx(u(i),0.0)
        end do
        call cfft(cf,nn,-1)
        do i=1,nn
              cf(i)=cf(i)/nn
        end do
c..... Write Fourier Coeff. to file
        df=1.0/(dt*nn)
        open(20,file=ofile)
        write(20,*) nn,' =N'
        write(20,*) df,' =df'
        do i=1,nn
              write(20,*) cf(i)
        end do
        close(20)
        write(*,*) 'Done'
c.....
        end
```

PCFFT.FOR

```
c*************************************************************
c       Plot FFT Coefficients
c*************************************************************
        character*20 ifile,ofile
        parameter(nmax=8192,pi=3.1415926535,d2r=pi/180.)
        complex cf(nmax)
        real amp(nmax/2),phase(nmax/2),freq(nmax/2)
c.....Input from file
        if(nargs().le.1) then
           write(*,*)
           write(*,*) 'Usage: PFFT infile'
           stop ' '
        end if
        call getarg(1,ifile,istatus)
        open(10,file=ifile,status='old')
        read(10,*) nn
        read(10,*) df
        do i=1,nn
           read(10,*) cf(i)
        end do
        close(10)
        fmax=nn/2*df
c..... Plot spectrum
        i=0
        do k=nn/2+1,nn-1
           i=i+1
           freq(i)=(k-nn)*df
           amp(i)=cabs(cf(k+1))
           if(amp(i).ne.0.0) then
              phase(i)=atan2(imag(cf(k+1)),real(cf(k+1)))/d2r
           end if
        end do
        do k=0,nn/2
           i=i+1
           freq(i)=df*k
           amp(i)=cabs(cf(k+1))
           if(amp(i).ne.0.0) then
              phase(i)=atan2(imag(cf(k+1)),real(cf(k+1)))/d2r
           end if
        end do
c
        ampmax=amp(1)
        do i=2,nn
           if(amp(i).gt.ampmax) ampmax=amp(i)
        end do
        do i=1,nn
           if(amp(i)/ampmax.lt.1.e-5) then
              phase(i)=0.0
           end if
        end do
c
c.....  Graphics
        call opengraphics
        call setwindow(1,-fmax,0.0,fmax,ampmax*1.2)
        call setviewpt(1,0.3,0.45,1.1,0.8)
        call drawtitle('FFT Coefficients')
        call settext('size=0.8')
        call drawaxis('labely=Amplitude')
        call setwindow(2,-fmax,-180.,fmax,180.)
        call setviewpt(2,0.3,0.2,1.1,0.40)
        call drawaxis('labelx=Frequency(Hz),labely=Phase,'//
       &                'dy=90,ddy=90')
c..... Plot amplitude and phase spectrum
        call setmark('type=solid-square,size=0.7')
        call selectwindow(1)
        call drawlines(nn,freq,amp)
        call drawmarks(nn,freq,amp)
        call selectwindow(2)
        call drawlines(nn,freq,phase)
        call drawmarks(nn,freq,phase)
        call closegraphics
c.....
        end
```

PSPEC.FOR

```
c*************************************************************
c       Plot Fourier Spectrum
c*************************************************************
        character*20 ifile,ofile
```

```
      parameter(nmax=8192,pi=3.1415926535,d2r=pi/180.)
      complex cf(nmax)
      real amp(nmax/2),phase(nmax/2),freq(nmax/2)

c..................................................... Input from file
      if(nargs().le.1) then
            write(*,*)
            write(*,*) 'Usage: PSPEC infile'
            stop ' '
      end if
      call getarg(1,ifile,istatus)
      open(10,file=ifile,status='old')
      read(10,*) nn
      read(10,*) df
      do i=1,nn
            read(10,*) cf(i)
      end do
      close(10)
c..................................................... Plot spectrum
      do i=1,nn/2+1
            freq(i)=df*(i-1)
            amp(i)=cabs(cf(i))/df
      end do
      fmax=nn/2*df
c
      ampmax=amp(1)
      do i=2,nn/2+1
            if(amp(i).gt.ampmax) ampmax=amp(i)
      end do
      do i=1,nn
            if(amp(i)/ampmax.gt.1.e-5) then
                  phase(i)=atan2(imag(cf(i)),real(cf(i)))/d2r
            else
                  phase(i)=0.0
            end if
      end do
c
c.......................................................  Graphics
      call opengraphics
      call setwindow(1,0.0,0.0,fmax,ampmax*1.2)
      call setviewpt(1,0.3,0.45,1.1,0.8)
      call drawtitle('Fourier Spectrum')
      call settext('size=0.8')
cc       call setline('width=2.0')
      call drawaxis('labely=Amplitude')

      call setwindow(2,0.0,-180.,fmax,180.)
      call setviewpt(2,0.3,0.2,1.1,0.40)
      call drawaxis('labelx=Frequency(Hz),labely=Phase,'//
     &                  'dy=90,ddy=90')
c..................................... Plot amplitude and phase spectrum
      call setmark('type=solid-square,size=0.5')
      call selectwindow(1)
      call drawlines(nn/2+1,freq,amp)
      call drawmarks(nn/2+1,freq,amp)
      call selectwindow(2)
      call drawlines(nn/2+1,freq,phase)
      call drawmarks(nn/2+1,freq,phase)
      call closegraphics

c.....................................................
      end
```

IFFT.FOR

```
      include 'cfft.for'
c************************************************************
c     Inverse FFT
c************************************************************
      parameter(nmax=8192)
      complex cf(nmax)
      character ifile*80,ofile*80
c
      if(nargs().le.2) then
            write(*,*)
            write(*,*) 'Usage: IFFT infile outfile'
            stop ' '
      end if
      call getarg(1,ifile,istatus)
      call getarg(2,ofile,istatus)
c..................................................... Read Fourier Coeff.
      open(10,file=ifile,status='old')
      read(10,*) nn
      read(10,*) df
      do i=1,nn
            read(10,*) cf(i)
      end do
      close(10)
c..................................................... Inv FFT
      call cfft(cf,nn,1)
c..................................................... Write time series
      open(20,file=ofile)
      write(20,*) nn, '   =N'
      write(20,*) (1.0/df)/nn, '    =dt'
      do i=1,nn
            write(20,*) real(cf(i))
      end do
      close(20)
      write(*,*) 'Done'
c
      end
```

FFILT.FOR

```
c************************************************************
c     Designing filter in the frequency domain
c************************************************************
      parameter(nmax=8192)
      complex cf(nmax)
      character ofile*80
c
      write(6,*)'??? OUTPUT FILE NAME ???'
      read(5,'(a)') ofile
c..................................... Input parameters
      write(6,*)
      write(6,*) 'Parameters for a filter wavelet.'
      write(6,'(a)') ' Sampling interval dT (sec) >>'
      read(5,*) dt
      write(6,'(a)') ' Number of data points N       >>'
      read(5,*) ndata
      write(6,'(1x,a,f10.3,a)')'Total time is',dt*ndata,' sec.'
      write(6,'(1x,72a)') ('-',i=1,72)
c..................................... Check if ndata=2**n
      npow=nint(log(float(ndata))/log(2.0))
      nn=2**npow
      if(ndata.ne.nn) then
         write(6,*) 'N=',ndata,',     Error: N must be a power of 2.'
         stop
      end if
      td=dt*ndata
      df=1./td
      fnyq=0.5/dt
      goto 78
   77 continue
      write(6,*)'Try again!'
   78 continue
c.....................Read filter parameter (f1,f2,f3,f4)
      write(6,*)'                 ----------\                    '
      write(6,*)'              /                \                 '
      write(6,*)'            /                    \              '
      write(6,*)'          /                        \            '
      write(6,*)'        /                            \          '
      write(6,*)'      /                                \        '
      write(6,*)'----                                    ---------'
      write(6,*)'     f1   <   f2       <       f3   <   f4   <',fnyq,'Hz'
      read(5,*) f1,f2,f3,f4
c..................... Calculate Fourier Coefficients of filter
      if(f4.ge.fnyq) then
        write(6,*)' f4 must be less than the Nyquist frequency.'
        goto 77
      endif
      if(f1.ge.f2) then
        write(6,*)' f1 must be less than f2.'
        goto 77
      endif
      if(f2.ge.f3) then
        write(6,*)' f2 must be less than f3.'
        goto 77
      endif
      if(f3.ge.f4) then
        write(6,*)' f3 must be less than f4.'
        goto 77
      endif
      n1=int(f1/df)+1
      n2=int(f2/df+0.5)
      n3=int(f3/df+0.5)
      n4=int(f4/df)
      do i=1,n1
         cf(i)=cmplx(0.0,0.0)
      end do
      do i=n1+1,n2
         cf(i)=cmplx(real(i-n1)*df/(f2-f1)/td,0.0)
      enddo
      do i=n2+1,n3
         cf(i)=cmplx(1./td,0.0)
      enddo
      do i=n3+1,n4
         cf(i)=cmplx((1.0-real(i-n3)*df/(f4-f3))/td,0.0)
      enddo
      do i=n4+1,nn/2
         cf(i)=cmplx(0.0,0.0)
      enddo
```

```
      do i=nn/2+1,nn
         cf(i)=conjg(cf(nn-i+1))
      enddo
c..................................... Write Fourier Coeff. to file
      open(20,file=ofile)
      write(20,*) nn,' =N'
      write(20,*) df,' =df'
      do i=1,nn
         write(20,*) cf(i)
      end do
      close(20)
      write(6,*) 'It is assumed that the phase spectra are zero.'
      write(6,*) 'Done'
c.....................................................
      end
```

FPRDCT.FOR

```
c**********************************************************
c     Product of filter spectra and input time series spectra
c**********************************************************
      character*20 ifile,ofile
      parameter(nmax=8192)
      complex cf1(nmax),cf2(nmax)
c.....................................................Input from file
   10 write(6,*)'??? INPUT FILE NAME (FIlter Spectra) ???'
      read(5,'(a)') ifile
      open(10,file=ifile,status='old')
      read(10,*) nn
      read(10,*) df
      do i=1,nn
        read(10,*) cf1(i)
      end do
      close(10)
      write(6,*)'??? INPUT FILE NAME (Input Signal Spectra) ???'
      read(5,'(a)') ifile
      open(10,file=ifile,status='old')
      read(10,*) nn1
      read(10,*) df1
      do i=1,nn
         read(10,*) cf2(i)
      end do
      close(10)
      if(nn1.ne.nn.or.df1.ne.df) then
        write(6,*)'Data number or frequency interval unmatched.'
        goto 10
      endif
      fmax=nn/2*df
      td=1./df
c..................................... Product
      do i=1,nn/2
        cf1(i)=cf1(i)*cf2(i)*td
      enddo
      do i=nn/2+1,nn
        cf1(i)=conjg(cf1(nn-i+1))
      enddo
c..................................... Write Fourier Coeff. to file
      write(6,*)'??? OUTPUT FILE NAME ???'
      read(5,'(a)') ofile
      open(20,file=ofile)
      write(20,*) nn,' =N'
      write(20,*) df,' =df'
      do i=1,nn
         write(20,*) cf1(i)
      end do
      close(20)
      write(6,*)'Done'
      end
```

FWVLET.FOR

```
c**********************************************************
c     Make filter wavelet of a filter from the inverse
c     Fourier Transform of given amplitude spectra
c**********************************************************
      program main
      parameter (nmax=8192)
      real u(nmax),wt(nmax)
      character ifile*80,ofile*80
c     ..................... ........ Input Time Series
      write(6,*)'??? INPUT FILE NAME'
      read(5,'(a)') ifile
      write(6,*)'??? OUTPUT FILE NAME'
      read(5,'(a)') ofile
      open(10,file=ifile,status='old')
      read(10,*) ndata
      read(10,*) dt
      do i=1,ndata
        read(10,*) u(i)
      end do
      close(10)
c     ..................... make filter wavelet
   11 write(6,*)'Causal (1) or Zero phase (2) ???'
      read(5,*) iflg
      if(iflg.eq.1) then
        write(6,*)
     *'How many points will you use for the weighted moving
average?'
        read(5,*) np
        do i=1,np
          wt(i)=u(i)
        enddo
      else if(iflg.eq.2) then
        write(6,*)
     *'How many points will you use for the weighted moving
average?',
     *'The number must be odd. '
        read(5,*) np
        nph=(np-1)/2
        do i=1,nph
          wt(i)=u(ndata-nph+i)
        enddo
        do i=nph+1,np
          wt(i)=u(i-nph)
        enddo
      else
        goto 11
      endif
      do i=1,np
        write(6,*) i,wt(i)
      enddo
      open(10,file=ofile,status='unknown')
      write(10,*) np,iflg
      write(10,*) dt,' =dT (sec)'
      do i=1,np
         write(10,*) wt(i)
      end do
      close(10)
      end
```

FCONV.FOR

```
c**********************************************************
c    Filtering by convolution in the time domain
c**********************************************************
      program main
      parameter (nmax=8192)
      real wt(nmax),v(nmax),u(nmax)
      character*80 ifile,ofile
c     ..................... ........ Input Time Series
      write(6,*)'??? INPUT FILE NAME (Filter Wavelet)???'
      read(5,'(a)') ifile
      open(10,file=ifile,status='old')
      read(10,*) np,iflg
      read(10,*) dt
      do i=1,np
        read(10,*) wt(i)
      end do
      close(10)
      write(6,*)'??? INPUT FILE NAME (Input Signal)???'
      read(5,'(a)') ifile
      open(10,file=ifile,status='old')
      read(10,*) ndata
      read(10,*) dt1
      if(dt1.ne.dt) then
        write(6,*)'Time interval unmatched!'
        stop
      endif
      do i=1,ndata
        read(10,*) u(i)
      end do
      close(10)
c..................................... Convolution
c Reverse filter time series
c                          ---> Coefficient of weighted moving average
      if(iflg.eq.1) then
        write(6,*)'Causal Filtering'
        do n=1,np
          v(n)=0.0
          do m=1,n
            v(n)=v(n)+wt(n-m)*u(m)*dt
          enddo
        enddo
        do n=np+1,ndata
          v(n)=0.0
          do m=n-np,n
            v(n)=v(n)+wt(n-m)*u(m)*dt
          enddo
        enddo
      else if(iflg.eq.2) then
        nph=(np-1)/2
        write(6,*)'Zero phase filtering'
```

```
      do n=1,nph
        v(n)=0.0
        do m=1,n+nph
          v(n)=v(n)+wt(n-m+nph+1)*u(m)*dt
        enddo
      enddo
      do n=nph+1,ndata
        v(n)=0.0
        do m=n-nph,n+nph
          v(n)=v(n)+wt(n-m+nph+1)*u(m)*dt
        enddo
      enddo
    endif
c   ..... Output Time Series
    write(6,*)'??? OUTPUT FILE NAME ???'
    read(5,'(a)') ofile
    open(10,file=ofile,status='unknown')
    write(10,*) ndata,' =N'
    write(10,*) dt,' =dT (sec)'
    do i=1,ndata
      write(10,*) v(i)
    end do
    close(10)
    write(6,*) 'Done'
    stop
    end
```

BANDP1

```
    subroutine bandp1(x,n,dt,fl,fh,fs,ap,as,ntype,nchara,ncausal)
    real*4 x(n),y(65536),f(128)
c   write(6,*)' Filter coefficients calculation started'
    flu=fl*dt
    fhu=fh*dt
    fsu=fs*dt
c.....Calculation of filter coefficients
    if(ntype.eq.1) then
c     Butterworth Filter
      if(nchara.eq.1) then
c       Low-Cut (High-Pass) Filter
        call buthip(f,mf,gn,nchb,flu,fsu,ap,as)
      else if(nchara.eq.2) then
c       High-Cut (Low-Pass) Filter
c         write(6,*)' high cut'
        call butlop(f,mf,gn,nchb,fhu,fsu,ap,as)
      else if(nchara.eq.3) then
c       Band-Pass Filter
        call butpas(f,mf,gn,nchb,flu,fhu,fsu,ap,as)
      else
c       Band-Stop Filter
        call butstp(f,mf,gn,nchb,flu,fhu,fsu,ap,as)
      endif
    else if(ntype.eq.2) then
c     Chevyshev-I FIlter
      if(nchara.eq.1) then
c       Low-Cut (High-Pass) Filter
        call chbhip(f,mf,gn,nchb,eps,fhu,fsu,ap,as)
      else if(nchara.eq.2) then
c       High-Cut (Low-Pass) Filter
        call chblop(f,mf,gn,nchb,eps,flu,fsu,ap,as)
      else if(nchara.eq.3) then
c       Band-Pass Filter
        call chbpas(f,mf,gn,nchb,eps,flu,fhu,fsu,ap,as)
      else
c       Band-Stop Filter
        call chbstp(f,mf,gn,nchb,eps,flu,fhu,fsu,ap,as)
      endif
    else if(ntype.eq.3) then
c     Chevyshev-II FIlter
      if(nchara.eq.1) then
c       Low-Cut (High-Pass) Filter
        call tchhip(f,mf,gn,nchb,eps,fhu,fsu,ap,as)
      else if(nchara.eq.2) then
c       High-Cut (Low-Pass) Filter
        call tchlop(f,mf,gn,nchb,eps,flu,fsu,ap,as)
      else if(nchara.eq.3) then
c       Band-Pass Filter
        call tchpas(f,mf,gn,nchb,eps,flu,fhu,fsu,ap,as)
      else
c       Band-Stop Filter
        call tchstp(f,mf,gn,nchb,eps,flu,fhu,fsu,ap,as)
      endif
    else
c     Elliptic Filter
      if(nchara.eq.1) then
c       Low-Cut (High-Pass) Filter
        call equhip(f,mf,gn,nchb,eps,ek,er,fhu,fsu,ap,as)
      else if(nchara.eq.2) then
c       High-Cut (Low-Pass) Filter
        call equlop(f,mf,gn,nchb,eps,ek,er,flu,fsu,ap,as)
      else if(nchara.eq.3) then
c       Band-Pass Filter
        call equpas(f,mf,gn,nchb,eps,ek,er,flu,fhu,fsu,ap,as)
      else
c       Band-Stop Filter
        call equstp(f,mf,gn,nchb,eps,ek,er,flu,fhu,fsu,ap,as)
      endif
    endif
c
c     write(6,*)' Filter coefficients are calculated.'
c     write(6,'(2x,a3,i3,2x,a3,f10.6)')'mf=',mf,'gn=',gn
c     write(6,'(2x,i2,f10.6)') (i,f(i),i=1,mf)
c.....Filtering subroutine
    call gnf(x,y,n,f,mf,gn,ncausal)
    return
    end
```

TRFILT.FOR

```
c*********************************************************
c     Filtering in the time domain (Recursive filter)
c*********************************************************
    include 'rfilter.for'
C
    program main
    parameter (nmax=8192)
    real t(nmax),u(nmax)
    character ifile*80,ofile*80
c..... Input Time Series
    write(6,*)'??? INPUT FILE NAME ???'
        read(5,'(a)') ifile
        write(6,*)'??? OUTPUT FILE NAME ???'
        read(5,'(a)') ofile
    open(10,file=ifile,status='old')
    read(10,*) ndata
    read(10,*) dt
    do i=1,ndata
      read(10,*) u(i)
      t(i)=dt*real(i-1)
    end do
    close(10)
C..... Filtering
 10 write(6,*) ' ??? Filter Type ???'
    write(6,*) '      Butterworth   type                ==> 1'
    write(6,*) '      Chebyshev-I   type                ==> 2'
    write(6,*) '      Chebyshev-II type                 ==> 3'
    write(6,*) '      Elliptic      type                ==> 4'
    read(5,*) ntype
    if(ntype.GT.4.OR.ntype.LT.1) GOTO 10
 20 write(6,*) ' ??? CHARACTERISTICS OF FILTER ???'
    write(6,*) '      Low-Cut   (High-Pass) Filter ==> 1'
    write(6,*) '      High-cut (Low-Pass)   Filter ==> 2'
    write(6,*) '      Band-Pass                Filter ==> 3'
    write(6,*) '      Band-Stop                Filter ==> 4'
    read(5,*) nchara
    if(nchara.GT.4.OR.nchara.LT.1) GOTO 20
    if(nchara.eq.1) then
c     Low-Cut (High-Pass) Filter
      write(6,*) '              FL                '
      write(6,*) '             /------------'
      write(6,*) '            /                  '
      write(6,*) '           /                   '
      write(6,*) '          /                    '
      write(6,*) '------/                        '
      write(6,*) '      FS                       '
      write(6,*) ' ??? FS,FL,AP,AS ???'
      write(6,*) '      AP,AS: Parameter defining the ripple '
      write(6,*) '             in pass band and stop band.   '
      write(6,*) '             Use AP=0.1, AS=10.0, if you do'
      write(6,*) '             not like to think.            '
      read(5,*) fs,fl,ap,as
    else if(nchara.eq.2) then
c     High-Cut (Low-Pass) Filter
```

```
            write(6,*) '         FH                          '
            write(6,*) '------\                          '
            write(6,*) '       \                         '
            write(6,*) '        \                        '
            write(6,*) '         \                       '
            write(6,*) '          \------------'
            write(6,*) '          FS                '
            write(6,*) ' ??? FH,FS,AP,AS ???'
            write(6,*) '         AP,AS: Parameter defining the ripple '
            write(6,*) '                in pass band and stop band.     '
            write(6,*) '                Use AP=0.1, AS=10.0, if you do'
            write(6,*) '                not like to think.                '
            read(5,*) fh,fs,ap,as
         else if(nchara.eq.3) then
c           Band Pass Filter
            write(6,*) '            FL          FH              '
            write(6,*) '            /--------\              '
            write(6,*) '           /          \             '
            write(6,*) '          /            \            '
            write(6,*) '         /              \           '
            write(6,*) '------/                \------'
            write(6,*) '                        FS          '
            write(6,*) ' ??? FL,FH,FS,AP,AS ???'
            write(6,*) '         AP,AS: Parameter defining the ripple '
            write(6,*) '                in pass band and stop band.     '
            write(6,*) '                Use AP=0.1, AS=10.0, if you do'
            write(6,*) '                not like to think.                '
            read(5,*) fl,fh,fs,ap,as
         else if(nchara.eq.4) then
c           Band Stop Filter
            write(6,*) '         FL                      FH              '
            write(6,*) '------\                  /-----------'
            write(6,*) '       \                /                  '
            write(6,*) '        \              /                   '
            write(6,*) '         \            /                    '
            write(6,*) '          \------/                     '
            write(6,*) '                  FS                        '
            write(6,*) ' ??? FL,FS,FH,AP,AS ???'
            write(6,*) '         AP,AS: Parameter defining the ripple '
            write(6,*) '                in pass band and stop band.     '
            write(6,*) '                Use AP=0.1, AS=10.0, if you do'
            write(6,*) '                not like to think.                '
            read(5,*) fl,fs,fh,ap,as
         endif
c        write(6,*) fl,fh,fs,ap,as
         write(6,*) ' ??? Causal or Zero-Phase ???'
         write(6,*) '         Causal         FIlter ==> 1'
         write(6,*) '         Zero-Phase Filter ==> 2'
         read(5,*) ncausal
         call bandp1(u,ndata,dt,fl,fh,fs,ap,as,ntype,nchara,ncausal)
c.....................................................Output Time Series
         open(10,file=ofile)
         write(10,*) ndata,' =N'
         write(10,*) dt,' =dT (sec)'
         do i=1,ndata
            write(10,*) u(i)
         end do
         close(10)
         write(*,*) 'Done'
         stop
         end
```

RFILTER.FOR

```
c*************************************************************
c        slave routine for trfilt.for
c*************************************************************
         include 'buthip.f'
         include 'butlop.f'
         include 'butpas.f'
         include 'butstp.f'
         include 'chbhip.f'
         include 'chblop.f'
         include 'chbpas.f'
         include 'chbstp.f'
         include 'tchhip.f'
         include 'tchlop.f'
         include 'tchpas.f'
         include 'tchstp.f'
         include 'equhip.f'
         include 'equlop.f'
         include 'equpas.f'
         include 'equstp.f'
         include 'equpol.f'
         include 'celin1.f'
         include 'snthet.f'
         include 'tandem.f'
         include 'recfil.f'
C-------------------------------------------------------------------------.
         subroutine bandp(x,n,dt,fl,fh,fs,ap,as,ntype,nchara,ncausal)
         real*4 x(n),y(8192),f(128)
c        write(6,*)' Filter coefficients calculation started'
         flu=fl*dt
         fhu=fh*dt
         fsu=fs*dt
c.....................................Calculation of filter coefficients
         if(ntype.eq.1) then
c           Butterworth Filter
            if(nchara.eq.1) then
c              Low-Cut (High-Pass) Filter
               call buthip(f,mf,gn,nchb,flu,fsu,ap,as)
            else if(nchara.eq.2) then
c              High-Cut (Low-Pass) Filter
               write(6,*)' high cut'
               call butlop(f,mf,gn,nchb,fhu,fsu,ap,as)
            else if(nchara.eq.3) then
c              Band-Pass Filter
               call butpas(f,mf,gn,nchb,flu,fhu,fsu,ap,as)
            else
c              Band-Stop Filter
               call butstp(f,mf,gn,nchb,flu,fhu,fsu,ap,as)
            endif
         else if(ntype.eq.2) then
c           Chevyshev-I FIlter
            if(nchara.eq.1) then
c              Low-Cut (High-Pass) Filter
               call chbhip(f,mf,gn,nchb,eps,fhu,fsu,ap,as)
            else if(nchara.eq.2) then
c              High-Cut (Low-Pass) Filter
               call chblop(f,mf,gn,nchb,eps,flu,fsu,ap,as)
            else if(nchara.eq.3) then
c              Band-Pass Filter
               call chbpas(f,mf,gn,nchb,eps,flu,fhu,fsu,ap,as)
            else
c              Band-Stop Filter
               call chbstp(f,mf,gn,nchb,eps,flu,fhu,fsu,ap,as)
            endif
         else if(ntype.eq.3) then
c           Chevyshev-II FIlter
            if(nchara.eq.1) then
c              Low-Cut (High-Pass) Filter
               call tchhip(f,mf,gn,nchb,eps,fhu,fsu,ap,as)
            else if(nchara.eq.2) then
c              High-Cut (Low-Pass) Filter
               call tchlop(f,mf,gn,nchb,eps,flu,fsu,ap,as)
            else if(nchara.eq.3) then
c              Band-Pass Filter
               call tchpas(f,mf,gn,nchb,eps,flu,fhu,fsu,ap,as)
            else
c              Band-Stop Filter
               call tchstp(f,mf,gn,nchb,eps,flu,fhu,fsu,ap,as)
            endif
         else
c           Elliptic Filter
            if(nchara.eq.1) then
c              Low-Cut (High-Pass) Filter
               call equhip(f,mf,gn,nchb,eps,ek,er,fhu,fsu,ap,as)
            else if(nchara.eq.2) then
c              High-Cut (Low-Pass) Filter
               call equlop(f,mf,gn,nchb,eps,ek,er,flu,fsu,ap,as)
            else if(nchara.eq.3) then
c              Band-Pass Filter
               call equpas(f,mf,gn,nchb,eps,ek,er,flu,fhu,fsu,ap,as)
            else
c              Band-Stop Filter
               call equstp(f,mf,gn,nchb,eps,ek,er,flu,fhu,fsu,ap,as)
            endif
         endif
c
         write(6,*)' Filter coefficients are calculated.'
         write(6,'(2x,a3,i3,2x,a3,f10.6)')'mf=',mf,'gn=',gn
         write(6,'(2x,i2,f10.6)') (i,f(i),i=1,mf)
c.....................................................Filtering subroutine
         call gnf(x,y,n,f,mf,gn,ncausal)
         return
```

```
      end
C-------------------------------------------------------------------------.
      subroutine gnf(x,y,nn,f,mf,gn,ncausal)
      real*4 x(nn),y(8192),f(4)
      if(ncausal.eq.2) then
         call tandem(x,y,nn,f,mf,-1)
         write(6,*)' TANDEM-1'
         do 813 i=1,nn
  813       x(i)=y(i)*gn
      endif
      call tandem(x,y,nn,f,mf, 1)
      write(6,*)' TANDEM 1'
      do 815 i=1,nn
  815    x(i)=y(i)*gn
      return
      end
```

DSEISM.FOR

```
      program dseism
c*********************************************************
c digital filter equivarent to displacement seismometer
c*********************************************************
      parameter (nmax=8192)
      real v(nmax),u(nmax)
      character*80 ifile,ofile
      data pai/3.1415926535/
      write(6,*)' Natural Period:T0, Damping Facter:h'
      write(6,*)' Sampling interval:dt, Gain:G0'
      read(5,*) t0,h,dt,g0
      omg0=2.*pai/t0
      write(6,*) omg0
      omg0=(2./dt)*tan(omg0*dt/2.)
      qq=tan(omg0*dt/2.)
      qq2=qq*qq
      a0= 1.0
      a1=-2.0
      a2= 1.0
      b0= 1.0+2.*h*qq+qq2
      b1=-2.0+2.*qq2
      b2= 1.0-2.*h*qq+qq2
      write(6,*)' Numerator    =',a0,a1,a2
      write(6,*)' Denominator=',b0,b1,b2
c.....................................................input
      write(6,*)'??? INPUT FILE NAME (Input Signal)???'
      read(5,'(a)') ifile
      open(10,file=ifile,status='old')
      read(10,*) ndata
      read(10,*) dt1
      if(dt1.ne.dt) then
        write(6,*)'Time interval unmatched!'
        stop
      endif
      do i=1,ndata
        read(10,*) u(i)
      end do
      close(10)
c.....................................Recursive Filtering
        write(6,*)'Causal Filtering'
        v(1)= a0*u(1)/b0
        v(2)=(a0*u(2)+a1*u(1)-b1*v(1))/b0
        do i=3,ndata
          v(i)=(a0*u(i)+a1*u(i-1)+a2*u(i-2)
     *                    -b1*v(i-1)-b2*v(i-2))/b0
        enddo
c     ..................................... Output Time Series
      write(6,*)'??? OUTPUT FILE NAME ???'
      read(5,'(a)') ofile
      open(10,file=ofile,status='unknown')
      write(10,*) ndata,' =N'
      write(10,*) dt,' =dT (sec)'
      do i=1,ndata
        write(10,*) v(i)
      end do
      close(10)
      write(6,*) 'Done'
      stop
      end
```

FWARP.FOR

```
c*********************************************************
c Calculate and Plot warped angular frequency
c*********************************************************
      program main
      parameter (nmax=8192)
      real x(nmax),y(nmax)
      data pai/3.1415926535/
c     ..................................... ..... Input
      write(6,*)'??? Dt, Ndata'
          read(5,*) dt, ndata
      td=ndata*dt
      domg=2.*pai/td
      omgnyq=pai/dt
      do i=1,ndata
        x(i)=real(i-1)*domg
        y(i)=(2./dt)*tan(x(i)*dt/2.)
      end do
c               ..................................... Plot Time Series
      umax=y(1)
      umin=y(1)
      do i=2,ndata
         if(y(i).gt.umax) umax=y(i)
         if(y(i).lt.umin) umin=y(i)
      end do
cc    ymax=max(abs(umax),abs(umin))*1.5
      ymax=pai/dt
c
      call opengraphics
      call setwindow(1,0.,0.,omgnyq*2,ymax)
      call setviewpt(1,0.3,0.3,1.0,0.5)
      call drawtitle('Warped Angular Frequency')
      call setline('width=2')
      call settext('size=0.8')
      call drawaxis('labelx=omg,labely=omg(k)')
      call drawline(omgnyq,0.,omgnyq,ymax)
c     call setmark('type=solid-square,size=0.5')
      call drawlines(ndata,x,y)
c     call drawmarks(ndata,x,y)
      call drawtext(omgnyq*2*1.01,ymax,'pai/dt')
      call settext('alignment=center-bottom')
      call drawtext(omgnyq,0.0,'pai/dt')
      call closegraphics
      end
```

DSEISMO.FOR

```
      program dseism
c digital filter equivarent to displacement seismometer
      parameter (nmax=8192)
      real v(nmax),u(nmax)
      character*80 ifile,ofile
      data pai/3.1415926535/
      write(6,*)' Natural Period:T0, Damping Facter:h'
      write(6,*)' Sampling interval:dt, Gain:G0'
      read(5,*) t0,h,dt,g0
      omg0=2.*pai/t0
      write(6,*) omg0
      qq=tan(omg0*dt/2.)
      qq2=qq*qq
      a0= 1.0
      a1=-2.0
      a2= 1.0
      b0= 1.0+2.*h*qq+qq2
      b1=-2.0+2.*qq2
      b2= 1.0-2.*h*qq+qq2
      write(6,*)' Numerator    =',a0,a1,a2
      write(6,*)' Denominator=',b0,b1,b2
c.....................................................input
      write(6,*)'??? INPUT FILE NAME (Input Signal)???'
      read(5,'(a)') ifile
      open(10,file=ifile,status='old')
      read(10,*) ndata
      read(10,*) dt1
      if(dt1.ne.dt) then
        write(6,*)'Time interval unmatched!'
        stop
      endif
      do i=1,ndata
        read(10,*) u(i)
      end do
      close(10)
c.....................................Recursive Filtering
        write(6,*)'Causal Filtering'
        v(1)= a0*u(1)/b0
        v(2)=(a0*u(2)+a1*u(1)-b1*v(1))/b0
        do i=3,ndata
          v(i)=(a0*u(i)+a1*u(i-1)+a2*u(i-2)
     *                    -b1*v(i-1)-b2*v(i-2))/b0
        enddo
c     ..................................... Output Time Series
      write(6,*)'??? OUTPUT FILE NAME ???'
      read(5,'(a)') ofile
```

```
      open(10,file=ofile,status='unknown')
      write(10,*) ndata,' =N'
      write(10,*) dt,' =dT (sec)'
      do i=1,ndata
        write(10,*) v(i)
      end do
      close(10)
      write(6,*) 'Done'
      stop
      end
```

VSEISMO.FOR

```
      program vseism
c digital filter equivarent to displacement seismometer
      parameter (nmax=8192)
      real v(nmax),u(nmax)
      character*80 ifile,ofile
      data pai/3.1415926535/
      write(6,*)' Natural Period:T0, Damping Facter:h'
      write(6,*)' Sampling interval:dt, Gain:G0'
      read(5,*) t0,h,dt,g0
      omg0=2.*pai/t0
      write(6,*) omg0
      qq=tan(omg0*dt/2.)
      qq2=qq*qq
      a0= 1.0*g0
      a1=-3.0*g0
      a2= 3.0*g0
      a3=-1.0*g0
      b0=( 1.0+2.*h*qq+      qq2)*dt/2.
      b1=(-1.0+2.*h*qq+3.*qq2)*dt/2.
      b2=(-1.0-2.*h*qq+3.*qq2)*dt/2.
      b3=( 1.0-2.*h*qq+      qq2)*dt/2.
      write(6,*)' Numerator    =',a0,a1,a2,a3
      write(6,*)' Denominator=',b0,b1,b2,b3
c.....................................................input
      write(6,*)'??? INPUT FILE NAME (Input Signal)???'
      read(5,'(a)') ifile
      open(10,file=ifile,status='old')
      read(10,*) ndata
      read(10,*) dt1
      if(dt1.ne.dt) then
        write(6,*)'Time interval unmatched!'
        stop
      endif
      do i=1,ndata
        read(10,*) u(i)
      end do
      close(10)
c.....................................Recursive Filtering
        write(6,*)'Causal Filtering'
        v(1)= a0*u(1)/b0
        v(2)=(a0*u(2)+a1*u(1)-b1*v(1))/b0
        v(3)=(a0*u(3)+a1*u(2)+a2*u(1)-b1*v(2)-b2*v(1))/b0
        do i=4,ndata
          v(i)=(a0*u(i)+a1*u(i-1)+a2*u(i-2)+a3*u(i-3)
     *                    -b1*v(i-1)-b2*v(i-2)-b3*v(i-3))/b0
        enddo
c       ..... Output Time Series
  100   write(6,*)'??? OUTPUT FILE NAME ???'
      read(5,'(a)') ofile
      open(10,file=ofile,status='unknown')
      write(10,*) ndata,' =N'
      write(10,*) dt,' =dT (sec)'
      do i=1,ndata
        write(10,*) v(i)
      end do
      close(10)
      write(6,*) 'Done'
      stop
      end
```

ISEISMO.FOR

```
      program iseism
c digital inverse filter to obtain ground displacement from velocity
seismograms
      parameter (nmax=8192)
      real v(nmax),u(nmax)
      character*80 ifile,ofile
      data pai/3.1415926535/
      write(6,*)' Natural Period:T0, Damping Facter:h'
      write(6,*)' Sampling interval:dt, Gain:G0'
      read(5,*) t0,h,dt,g0
      omg0=2.*pai/t0
      write(6,*) omg0
      qq=tan(omg0*dt/2.)
      qq2=qq*qq
      b0= 1.0*g0
      b1=-3.0*g0
      b2= 3.0*g0
      b3=-1.0*g0
      a0=( 1.0+2.*h*qq+      qq2)*dt/2.
      a1=(-1.0+2.*h*qq+3.*qq2)*dt/2.
      a2=(-1.0-2.*h*qq+3.*qq2)*dt/2.
      a3=( 1.0-2.*h*qq+      qq2)*dt/2.
      write(6,*)' Numerator    =',a0,a1,a2,a3
      write(6,*)' Denominator=',b0,b1,b2,b3
c.....................................................input
      write(6,*)'??? INPUT FILE NAME (Input Signal)???'
      read(5,'(a)') ifile
      open(10,file=ifile,status='old')
      read(10,*) ndata
      read(10,*) dt1
      if(dt1.ne.dt) then
        write(6,*)'Time interval unmatched!'
        stop
      endif
      do i=1,ndata
        read(10,*) u(i)
      end do
      close(10)
c.....................................Recursive Filtering
        write(6,*)'Causal Filtering'
        v(1)= a0*u(1)/b0
        v(2)=(a0*u(2)+a1*u(1)-b1*v(1))/b0
        v(3)=(a0*u(3)+a1*u(2)+a2*u(1)-b1*v(2)-b2*v(1))/b0
        do i=4,ndata
          v(i)=(a0*u(i)+a1*u(i-1)+a2*u(i-2)+a3*u(i-3)
     *                    -b1*v(i-1)-b2*v(i-2)-b3*v(i-3))/b0
        enddo
c       ..... Output Time Series
  100   write(6,*)'??? OUTPUT FILE NAME ???'
      read(5,'(a)') ofile
      open(10,file=ofile,status='unknown')
      write(10,*) ndata,' =N'
      write(10,*) dt,' =dT (sec)'
      do i=1,ndata
        write(10,*) v(i)
      end do
      close(10)
      write(6,*) 'Done'
      stop

      end
```

CSEISMO.FOR

```
      program cseism
c digital filter to convert the longer natural period seismometer
      parameter (nmax=8192)
      real v(nmax),u(nmax)
      character*80 ifile,ofile
      data pai/3.1415926535/
      write(6,*)' Original Seismometer'
      write(6,*)' Natural Period:T0, Damping Facter:h0'
      write(6,*)' Sampling interval:dt, Gain:G0'
      read(5,*) t0,h0,dt0,g0
      omg0=2.*pai/t0
      write(6,*) omg0
      qq=tan(omg0*dt0/2.)
      qq2=qq*qq
      a0= 1.0+2.*h0*qq+qq2
      a1=-2.0+2.*qq2
      a2= 1.0-2.*h0*qq+qq2
      write(6,*)' Simulated Seismometer'
      write(6,*)' Natural Period:T1, Damping Facter:h1'
      write(6,*)' Sampling interval:dt, Gain:G1'
      read(5,*) t1,h1,dt1,g1
      omg1=2.*pai/t1
      write(6,*) omg1
```

```
      qq=tan(omg1*dt1/2.)
      qq2=qq*qq
      b0=( 1.0+2.*h1*qq+qq2)*g0/g1
      b1=(-2.0          +2.*qq2)*g0/g1
      b2=( 1.0-2.*h1*qq+qq2)*g0/g1
      write(6,*)' Numerator    =',a0,a1,a2
      write(6,*)' Denominator=',b0,b1,b2
c.....................................................input
      write(6,*)'??? INPUT FILE NAME (Input Signal)???'
      read(5,'(a)') ifile
      open(10,file=ifile,status='old')
      read(10,*) ndata
      read(10,*) dt
      if(dt1.ne.dt.or.dt0.ne.dt) then
        write(6,*)'Time interval unmatched!'
        stop
      endif
      do i=1,ndata
        read(10,*) u(i)
      end do
      close(10)
c.....................................Recursive Filtering
        write(6,*)'Causal Filtering'
        v(1)= a0*u(1)/b0
        v(2)=(a0*u(2)+a1*u(1)-b1*v(1))/b0
        do i=3,ndata
          v(i)=(a0*u(i)+a1*u(i-1)+a2*u(i-2)
     *                    -b1*v(i-1)-b2*v(i-2))/b0
        enddo
c       ..... Output Time Series
      write(6,*)'??? OUTPUT FILE NAME ???'
      read(5,'(a)') ofile
      open(10,file=ofile,status='unknown')
      write(10,*) ndata,' =N'
      write(10,*) dt,' =dT (sec)'
      do i=1,ndata
        write(10,*) v(i)
      end do
      close(10)
      write(6,*) 'Done'
      stop

      end
```

PLTACC.FOR

```
      include 'pltwv2.for'
      include 'timesft.for'
      include 'timecrt.for'
      program pltacc
c     k-net data viewer (raw acceleration data)
      parameter(nmax=50000,nch=3)
      integer ix(nmax)
      real*4 x(nch,nmax),xx(nmax),amaxacc(nch)
      character cid*6,filen*47,dummy*18,cdir*3,cmemo(nch)*80,
     *          cstcode*6,cdrv*9,ccomp(3)*2
      data aym,x0,yl,DTL/16.0,2.0,2.0,1.0/ fac/1./
      data ccomp/'NS','EW','UD'/ tst,ted/0.01,500./
      open(10,file='stlist.txt',status='old')
      read(10,*) cdrv,nsite
      do 2000 isite=1,nsite
      read(10,*) cid,tst
c     write(6,'(A)') cdrv//cid//'.**'
      filen=cdrv//cid//'.ns'
      open(1,file=filen,status="old")
      write(6,'(a)') filen
      read(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2)')
     *     dummy,irgyear,irgmonth,irgday,irghour,irgmin
      read(1,'(a18,f5.1)')dummy,orglat
      read(1,'(a18,f5.1)')dummy,orglong
      read(1,'(a18,i4)')  dummy,idepth
      orgdepth=real(idepth)
      read(1,'(a18,f4.1)')dummy,amag
      read(1,'(a18,a6)')dummy,cstcode
      read(1,'(a18,f8.4)')dummy,stlat
      read(1,'(a18,f8.4)')dummy,stlong
      read(1,'(a18,i4)')  dummy,idepth
      stheight=real(idepth)
      read(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2,1x,i2)')
     *     dummy,ircyear,ircmonth,ircday,irchour,ircmin,ircsec
      call timecrt(ircyear,ircmonth,ircday,irchour,ircmin,ircsec)
      read(1,'(a18,i3)')dummy,iratio
      sratio=real(iratio)
      dt=1.0/sratio
      read(1,'(a18,i3)')dummy,idur
      ndata=idur*100
      duration=real(idur)
      read(1,'(a18,A3)')dummy,cdir
      read(1,'(a18,10x,i7)')dummy,iscale
      scale=2000./real(iscale)
      read(1,'(a18,i4)')dummy,imaxacc
      amaxacc(1)=real(imaxacc)
      read(1,'(a18)')dummy
      read(1,'(a80)')cmemo(1)
      read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
      do 10 i=1,ndata
10      x(1,i)=real(ix(i))*scale
      close(1)
      filen=cdrv//cid//'.ew'
      open(1,file=filen,status="old")
      write(6,'(a)') filen
      do 20 kk=1,14
20      read(1,'(a18)')dummy
      read(1,'(a18,i4)')dummy,imaxacc
      amaxacc(2)=real(imaxacc)
      read(1,'(a18)')dummy
      read(1,'(a80)')cmemo(2)
      read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
      do 30 i=1,ndata
30      x(2,i)=real(ix(i))*scale
      close(1)
      filen=cdrv//cid//'.ud'
      open(1,file=filen,status="old")
      write(6,'(a)') filen
      do 40 kk=1,14
40      read(1,'(a18)')dummy
      read(1,'(a18,i4)')dummy,imaxacc
      amaxacc(3)=real(imaxacc)
      read(1,'(a18)')dummy
      read(1,'(a80)')cmemo(3)
      read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
      do 50 i=1,ndata
50      x(3,i)=real(ix(i))*scale
      close(1)
      ntst=int(tst)
      nst=int(tst/dt+0.5)
      ned=int(ted/dt+0.5)
      call timesft(ntst,ircyear,ircmonth,ircday,
     *                                  irchour,ircmin,ircsec)
      filen=cdrv//cid//'ac.ps'
      open(25,file=filen,status='unknown')
      write(6,'(a)') filen
      CALL PLOTS(25)
      call yoko(25)
      call symbol(1.5,5.0,0.8,cid,90.0,6)
      call number(1.5,11.00,0.25,real(ircyear),90.0,0)
      call number(1.5,12.25,0.25,real(ircmonth),90.0,0)
      call number(1.5,13.00,0.25,real(ircday),90.0,0)
      call number(1.5,13.75,0.25,real(irchour),90.0,0)
      call number(1.5,14.50,0.25,real(ircmin),90.0,0)
      call number(1.5,15.25,0.25,real(ircsec),90.0,0)
      CALL FACTOR(FAC)
      amaxx=max1(amaxacc(1),amaxacc(2),amaxacc(3))
      if(amaxx.lt.2000.) xxmax=2000.
      if(amaxx.lt.1000.) xxmax=1000.
      if(amaxx.lt. 500.) xxmax= 500.
      if(amaxx.lt. 200.) xxmax= 200.
      if(amaxx.lt. 100.) xxmax= 100.
      if(amaxx.lt.  50.) xxmax=  50.
      if(amaxx.lt.  20.) xxmax=  20.
      if(amaxx.lt.  10.) xxmax=  10.
      if(amaxx.lt.   5.) xxmax=   5.
      if(amaxx.lt.   2.) xxmax=   2.
      if(amaxx.lt.   1.) xxmax=   1.
      call plot(x0,0.,-3)
      CALL NEWPEN(1)
      ay=aym
```

```
      ndatamax=min1(real(ned),real(ndata),24.*dtl/dt)+nst-1
      if(ndatamax.lt.ndata) call symbol(26.,0.5,0.3,'C',0.0,1)
      do 1000 kk=1,3
        call symbol(2.5,ay+yl+0.3,0.4,ccomp(kk),0.0,2)
        call symbol(4.0,ay+yl+0.3,0.3,'Max=',0.0,4)
        call number(5.5,ay+yl+0.3,0.3,amaxacc(kk),0.0,1)
        do 500 i=1,ndata
  500     xx(i)=x(kk,i+nst-1)
        sum=xx(1)
        do 600 i=2,ndata
  600   sum=sum+xx(i)
        ave=sum/real(ndata)
        do 610 i=1,ndata
  610     xx(i)=xx(i)-ave
        CALL PLOT( 2., AY,-3)
        call pltwv2(xx,ndatamax-nst+1,dt,dtl,xxmax,yl,1)
        CALL PLOT(-2.,-AY,-3)
        ay=ay-3.*yl
 1000 continue
      call plote(25)
      close(25)
 2000 continue
      close(10)
      stop
      end
```

PLTWV2.FOR

```
      subroutine pltwv2(d,n,dt,dtl,YMAX,yl,ind)
      REAL*4 d(n)
      data htitle,hcha,htick/0.4,0.2,0.2/
      tlong=real(int(real(n)*dt/dtl)+1)
      ntlong=int(tlong)
      dsize=dt/dtl
      IF(YMAX.LT.1.0E-30) YMAX=1.0E-30
      call plot(     0.,-yl,3)
      call plot(tlong,-yl,2)
      call plot(     0., yl,3)
      call plot(tlong, yl,2)
      call symbol(0.5*tlong-htitle*1.5,
     *
-(yl+htick*2.+htitle+0.1),htitle,'sec',0.0,3)
      do 10 i=0,ntlong
        call plot(real(i),-yl,3)
   10   call plot(real(i),-yl-htick,2)
      do 11 i=0,ntlong
        call plot(real(i),yl,3)
   11   call plot(real(i),yl+htick,2)
      do 12 i=0,ntlong,2
   12   call number(real(i)-hcha*2.,-(yl+htick+hcha+0.1),hcha,
     *                real(i)*dtl,0.0,1)
      DYL=YMAX/YL
      call plot(0.,-yl,3)
      call plot(0., yl,2)
      if(ind.eq.1)
     *call symbol(-(htick*2.+hcha*3.5),-htitle*1.5,
     *                                          htitle,'gal',90.0,3)
      if(ind.eq.2)
     *call symbol(-(htick*2.+hcha*3.5),-htitle*1.5,
     *
htitle,'kine',90.0,4)
      if(ind.eq.3)
     *call symbol(-(htick*2.+hcha*3.5),-htitle*1.5,
     *
htitle,'micron',90.0,6)
      do 110 i=-int(yl+0.5),int(yl+0.5)
        call plot(-htick,real(i),3)
        call plot(     0.,real(i),2)
  110 continue
      if(ymax.ge.   1.) then
       ncfra=4
       if(ymax.ge. 10.) then
        ncfra=5
        if(ymax.ge.100.) then
         ncfra=6
         if(ymax.ge.1000.) ncfra=7.
        endif
       endif
      endif
      xx=-(htick+hcha*real(ncfra)+0.1)
      yy=yl+hcha*0.5
      call number(xx,-yy,hcha,-ymax,0.0,1)
      call number(-hcha*3.25,-hcha*0.5,hcha,0.0,0.0,1)
      call number(xx, yy,hcha,ymax,0.0,1)
      call plot(tlong, yl,3)
      call plot(tlong,-yl,2)
      call plot(0.,d(1)/DYL,3)
      do   130   i=2,n
      Y=D(I)/DYL
      call plot((i-1)*dsize,Y,2)
  130 continue
      return
      end
```

TIMECRT.FOR

```
      subroutine timecrt(ircyear,ircmonth,ircday,
     *                                        irchour,ircmin,ircsec)
      if(ircsec.ge.15) then
       ircsec=ircsec-15
      else
       if(ircmin.ge.1) then
        ircmin=ircmin-1
        ircsec=ircsec+45
       else
        if(irchour.ge.1) then
         irchour=irchour-1
         ircmin =59
         ircsec=ircsec+45
        else
         ircday =ircday-1
         irchour=23
         ircmin =59
         ircsec=ircsec+45
        endif
       endif
      endif
      return
      end
```

TIMESFT.FOR

```
      subroutine timesft(ntst,ircyear,ircmonth,ircday,
     *                                        irchour,ircmin,ircsec)
      if(ircsec+ntst.lt.60) then
       ircsec=ircsec+ntst
      else
       if(ircmin.lt.59) then
        ircmin=ircmin+1
        ircsec=ircsec-60+ntst
       else
        if(irchour.lt.23) then
         irchour=irchour+1
         ircmin =0
         ircsec=ircsec-60+ntst
        else
         ircday =ircday+1
         irchour=0
         ircmin =0
         ircsec=ircsec-60+ntst
        endif
       endif
      endif
      return
      end
```

SPLOT.FOR

```
      include 'cfft.for'
      include 'timecrt.for'
      include 'timesft.for'
      parameter(nmax=50000,nch=3)
      complex c(nmax*2)
      integer ix(nmax)
      real*4
x(nch,nmax),xx(nmax),amaxacc(nch),fr(nmax),yy(nmax),
     *          amaxdsp(nch),y(nch,nmax)
      character
cid*6,filen*47,dummy*18,cdir*3,cmemo(nch)*80,cstcode*6,
     *               cdrv*9,ccomp(3)*2
      data yl/2.0/ fac/0.7/
      data ccomp/'NS','EW','UD'/ tst,ted/ 0.01,70./ asmthf/0.02/
      data pai,c0/3.141592,(0.0,0.0)/  ircdelay/0.0/
```

```
      data NBX,NBY,ABX,ABY/3,3,2.5,2.5/ HCH,HTP/0.4,0.1/
FMIN/0.1/
      open(10,file='stlist.txt',status='old')
      read(10,*) cdrv,nsite
      do 2000 isite=1,nsite
      read(10,*) cid,tst,ted
      write(6,'(a)')cdrv//cid//'.**'
      filen=cdrv//cid//'.ns'
      open(1,file=filen,status="old")
      read(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2)')
     *     dummy,irgyear,irgmonth,irgday,irghour,irgmin
      read(1,'(a18,f5.1)')dummy,orglat
      read(1,'(a18,f5.1)')dummy,orglong
      read(1,'(a18,i4)')  dummy,idepth
      orgdepth=real(idepth)
      read(1,'(a18,f4.1)')dummy,amag
      read(1,'(a18,a6)')dummy,cstcode
      read(1,'(a18,f8.4)')dummy,stlat
      read(1,'(a18,f8.4)')dummy,stlong
      read(1,'(a18,i4)')  dummy,idepth
      stheight=real(idepth)
      read(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2,1x,i2)')
     *     dummy,ircyear,ircmonth,ircday,irchour,ircmin,ircsec
      call timecrt(ircyear,ircmonth,ircday,irchour,ircmin,ircsec)
      read(1,'(a18,i3)')dummy,iratio
      sratio=real(iratio)
      dt=1.0/sratio
      read(1,'(a18,i3)')dummy,idur
      ndata=idur*100
      duration=real(idur)
      read(1,'(a18,A3)')dummy,cdir
      read(1,'(a18,10x,i7)')dummy,iscale
      scale=2000./real(iscale)
      read(1,'(a18,i4)')dummy,imaxacc
      amaxacc(1)=real(imaxacc)
      read(1,'(a18)')dummy
      read(1,'(a80)')cmemo(1)
      read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
      do 10 i=1,ndata
   10   x(1,i)=real(ix(i))*scale
      close(1)
      filen=cdrv//cid//'.ew'
      open(1,file=filen,status="old")
      do 20 kk=1,14
   20   read(1,'(a18)')dummy
      read(1,'(a18,i4)')dummy,imaxacc
      amaxacc(2)=real(imaxacc)
      read(1,'(a18)')dummy
      read(1,'(a80)')cmemo(2)
      read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
      do 30 i=1,ndata
   30   x(2,i)=real(ix(i))*scale
      close(1)
      filen=cdrv//cid//'.ud'
      open(1,file=filen,status="old")
      do 40 kk=1,14
   40   read(1,'(a18)')dummy
      read(1,'(a18,i4)')dummy,imaxacc
      amaxacc(3)=real(imaxacc)
      read(1,'(a18)')dummy
      read(1,'(a80)')cmemo(3)
      read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
      do 50 i=1,ndata
   50   x(3,i)=real(ix(i))*scale
      close(1)
      ncomp=nch
      ntst=int(tst)
      nst=int(tst/dt+0.5)
      ned=int(ted/dt+0.5)
c     write(6,*)nst,ned
c     ned=ndata
      call timesft(ntst,ircyear,ircmonth,ircday,
     *                                  irchour,ircmin,ircsec)
      nn=ned-nst+1
      n2e=int(alog10(real(nn))/0.30103+0.5)
      mm=2**n2e
      mm2=mm*2
      mwin=int(0.1*real(mm))
      datal=dt*real(mm)
c     write(6,*)datal
      do 106 i=1,mm
  106  fr(i)=real(i-1)/datal
C     FMAX=FMIN*(10**REAL(NBX))
      FMAX=45.
      NMX=INT(FMAX*DATAL)+1
      NMN=INT(FMIN*DATAL)+1
      do 525 mcomp=1,ncomp
       do 207 i=1,nn
  207   xx(i)=x(mcomp,i+nst-1)
c linear trend fitting
       b1=xx(1)
       do 392 i=2,nn
  392   b1=b1+real(i)*xx(i)
       b2=xx(1)
       do 394 i=1,nn
  394    b2=b2+xx(i)
       a0=real(nn*(nn+1)*(2*nn+1))/6.*dt
       b0=real(nn*(nn+1))/2.
       c0=b0*dt
       d0=real(nn)
       f0=a0*d0-b0*c0
       aa=( d0*b1-b0*b2)/f0
       bb=(-c0*b1+a0*b2)/f0
       do 396 i=1,nn
  396    xx(i)=xx(i)-aa*real(i)*dt-bb
      do 409 i=1,mwin
  409   xx(i)=xx(i)*SIN(PAI/2.*REAL(I-1)/REAL(MWIN))
      do 412 i=nn-mwin,nn
  412   xx(i)=xx(i)*SIN(PAI/2.*REAL(nn-i)/REAL(MWIN))
      do 413 i=nn+1,mm
  413   xx(i)=0.0
      do 414 i=1,mm
  414   c(i)=cmplx(xx(i))
      write(6,*) mm
      CALL cfft(c,mm,-1)
           do 518 i=2,mm
  518  xx(i)=cabs(c(i))*dt/(2.*PAI*FR(I))**2
      do 519 i=2,mm
  519  yy(i)=cabs(c(i))*dt
      amaxdsp(mcomp)=abs(xx(nmn))
      do 521 i=nmn,nmx
  521  if(abs(xx(i)).GT.amaxdsp(mcomp))
amaxdsp(mcomp)=abs(xx(i))
      do 522 i=2,mm
  522  x(mcomp,i)=xx(i)
      amaxacc(mcomp)=abs(yy(nmn))
      do 523 i=nmn,nmx
  523  if(abs(yy(i)).GT.amaxacc(mcomp))
amaxacc(mcomp)=abs(yy(i))
      do 524 i=2,mm
  524  y(mcomp,i)=yy(i)
  525 continue
      filen=cdrv//cid//'sp.ps'
      write(6,*) filen
      open(25,file=filen,status='unknown')
      CALL PLOTS(25)
      call yoko(25)
      smaxx=amax1(amaxacc(1),amaxacc(2),amaxacc(3))
      if(smaxx.lt.1000.          ) sxmax=1000.
      if(smaxx.lt. 100.          ) sxmax= 100.
      if(smaxx.lt.   10.         ) sxmax=   10.
      if(smaxx.lt.    1.         ) sxmax=    1.
      if(smaxx.lt.    0.1        ) sxmax=    0.1
      if(smaxx.lt.    0.01       ) sxmax=    0.01
      if(smaxx.lt.    0.001      ) sxmax=    0.001
      if(smaxx.lt.    0.0001     ) sxmax=    0.0001
      if(smaxx.lt.    0.00001    ) sxmax=   0.00001
      if(smaxx.lt.    0.000001   ) sxmax=0.000001
      sminacc=sxmax*10**real(-nby-1)
      smaxx=amax1(amaxdsp(1),amaxdsp(2),amaxdsp(3))
      if(smaxx.lt. 100.          ) sxmax= 100.
      if(smaxx.lt.   10.         ) sxmax=   10.
      if(smaxx.lt.    1.         ) sxmax=    1.
      if(smaxx.lt.    0.1        ) sxmax=    0.1
      if(smaxx.lt.    0.01       ) sxmax=    0.01
      if(smaxx.lt.    0.001      ) sxmax=    0.001
```

```
      if(smaxx.lt.    0.0001    ) sxmax=     0.0001
      if(smaxx.lt.    0.00001 ) sxmax=    0.00001
      if(smaxx.lt.    0.000001) sxmax= 0.000001
      smindsp=sxmax*10**real(-nby-1)
      call plot(2.0,1.65,-3)
      call symbol( 0.0,0.5,1.2,cid,0.0,6)
      call symbol(10.0,0.0,0.3,'Event:',0.0,6)
      call number(11.8,0.0,0.3, orglat,0.0,3)
      call number(15.4,0.0,0.3,orglong,0.0,3)
      call number(19.0,0.0,0.3,orgdepth,0.0,0)
      call symbol(10.0,0.5,0.3,'Site :',0.0,6)
      call number(11.8,0.5,0.3, stlat,0.0,3)
      call number(15.4,0.5,0.3,stlong,0.0,3)
      call number(19.0,0.5,0.3,stheight/1000.,0.0,3)
      call symbol(10.0,1.0,0.4,'Start Time ',0.0,11)
      call plot(14.4,0.0,-3)
      call number( 0.0,1.0,0.4,real(ircyear) ,0.0,0)
      call number( 2.0,1.0,0.4,real(ircmonth),0.0,0)
      call number( 3.2,1.0,0.4,real(ircday)  ,0.0,0)
      call number( 4.8,1.0,0.4,real(irchour) ,0.0,0)
      call number( 6.0,1.0,0.4,real(ircmin)  ,0.0,0)
      call number( 7.2,1.0,0.4,real(ircsec)+real(ircdelay),0.0,0)
      call plot(-14.4,0.5,-3)
      CALL FACTOR(FAC)
      CALL NEWPEN(1)
      call plot(2.,5.5,-3)
      xl=abx*real(nbx)+4.0
      yl=aby*real(nby)+2.0
      do 800 kk=1,3
        call symbol(abx+0.5,yl,0.4,ccomp(kk),0.0,2)
        do 790 i=1,mm
  790   xx(i)=x(kk,i)
        CALL
ESMTHF(XX(nmn),XX(nmn),FR(nmn),NMX,asmthf)
        CALL
PLTLOG(NBX,NBY,ABX,ABY,HCH,HTP,FMIN,SMINDSP,1)
        CALL NEWPEN(1)
        CALL
PTLINE(FR,XX,ABX,ABY,FMIN,SMINDSP,NMAX,NMN,NMX)
        CALL NEWPEN(1)
        CALL PLOT(xl,0.,-3)
  800 continue
      call plot(-xl*3.,12.5,-3)
      do 1000 kk=1,3
       call symbol(abx+0.5,yl,0.4,ccomp(kk),0.0,2)
      do 900 i=1,mm
  900   yy(i)=y(kk,i)
      CALL ESMTHF(yy(nmn),yy(nmn),FR(nmn),NMX,asmthf)
      CALL
PLTLOG(NBX,NBY,ABX,ABY,HCH,HTP,FMIN,SMINACC,3)
      CALL NEWPEN(1)
      CALL
PTLINE(FR,yy,ABX,ABY,FMIN,SMINACC,NMAX,NMN,NMX)
      CALL NEWPEN(1)
      CALL PLOT(xl,0.,-3)
 1000 continue
      Call plot(0.,-12.,-3)
      call plote(25)
 2000       continue
      stop
      end
C+++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE
PLTLOG(N,M,DX,DY,HEIT,TIP,XMIN,YMIN,IDX)
C
C       N,M              : BLOCK NUMBER OF X-AXIS & Y-AXIS
C       DX,DY            : BLOCK LENGTH OF X-AXIS & Y-AXIS
(CM)
C       TIP              : LENGTH OF SCALE TIP IN AXISES
(CM)
C       HEIT             : HEIGHT OF CHARACTER
(CM)
C       XMIN,YMIN        : (XMIN,YMIN) REFERENCE POINT
DATA
C
      CHARACTER
XUNIT*7,YUNIT*13,XNAME*10,YNAME*20
      CHARACTER YAX(5)*8,YNA(5)*20
      DATA YAX/' CM SEC ','CM/S SEC','GAL SEC ','          ','
'/
      DATA YNA/'DISPLACEMENT SPECTRA',' VELOCITY
SPECTRA    ',
     *          'ACCELERATION SPECTRA','SPECTRUM
RATIO (M/A)',
     *          'SPECTRUM RATIO (S/M)'/
      XUNIT=' (Hz) '
      YUNIT='      '//YAX(IDX)//' '
      XNAME='FREQUENCY '
      YNAME=YNA(IDX)
      CALL NEWPEN(3)
      DO 100 I=0,N
       XL=XMIN*10**I
       IX=NINT(ALOG10(XL)+0.1)
       IF(IX.GE.2.OR.IX.LE.-1) GO TO 400
       IF(IX.EQ.1) THEN
        CALL SYMBOL(I*DX-HEIT,-0.3-HEIT,HEIT,'10',0.0,2)
       ELSE
        CALL
NUMBER(I*DX-2*HEIT,-HEIT-0.3,HEIT,10.**IX+0.05,0.0,1)
       ENDIF
       GO TO 100
  400  CALL SYMBOL(I*DX-HEIT,-0.3-HEIT,HEIT,'10',0.0,2)
       CALL
NUMBER(I*DX+HEIT,-0.3       ,HEIT*0.5,REAL(IX),0.0,0)
  100  CONTINUE
      DO 101 I=0,M
       YL=YMIN*10**I
       IY=NINT(ALOG10(YL)+0.1)
       IF(IY.GE.2.OR.IY.LE.-1) GO TO 401
       IF(IY.EQ.1) THEN
        CALL SYMBOL(-0.3-3*HEIT,I*DY,HEIT,'10',0.0,2)
       ELSE
        CALL
NUMBER(-0.3-4*HEIT,I*DY,HEIT,10.**IY+0.05,0.0,0)
       ENDIF
       GO TO 101
  401  CALL SYMBOL(-0.3-3*HEIT,I*DY,HEIT,'10',0.0,2)
       CALL
NUMBER(-0.3-HEIT,I*DY+HEIT,HEIT*0.5,REAL(IY),0.0,0)
  101  CONTINUE
       CALL
SYMBOL(N*DX/2-2.5*HEIT,-1.-2.7*HEIT,HEIT*1.,XUNIT,0., 7)
       CALL
SYMBOL(N*DX/2-3.8*HEIT,-1.-1.5*HEIT,HEIT*1.,XNAME,0.,10)
       CALL
SYMBOL(-4*HEIT,N*DX/2-6*HEIT,HEIT*1.,YUNIT,90.,13)
       CALL
SYMBOL(N*DX/2-7.5*HEIT,M*DY+1.5*HEIT,HEIT*1.,YNAME,0.,
20)
       CALL NEWPEN(2)
       DO 1 I=0,N
        CALL PLOT(I*DX, 0.0,3)
        CALL PLOT(I*DX,M*DY,2)
        IF(I.EQ.N) GO TO 1
        DO 11 J=2,9
         CALL PLOT(I*DX+ALOG10(REAL(J))*DX,0.0,3)
   11    CALL PLOT(I*DX+ALOG10(REAL(J))*DX,TIP,2)
        CALL PLOT(I*DX+ALOG10(5.)*DX,0.0,3)
        CALL PLOT(I*DX+ALOG10(5.)*DX,TIP*1.5,2)
    1  CONTINUE
       DO 2 I=0,M
        CALL PLOT(0.0,I*DY,3)
        CALL PLOT(N*DX,I*DY,2)
        IF(I.EQ.M) GO TO 2
        DO 12 J=2,9
         CALL PLOT(0.0,I*DY+ALOG10(REAL(J))*DY,3)
   12    CALL PLOT(TIP,I*DY+ALOG10(REAL(J))*DY,2)
        CALL PLOT(0.0,I*DY+ALOG10(5.)*DY,3)
        CALL PLOT(TIP*1.5,I*DY+ALOG10(5.)*DY,2)
    2  CONTINUE
       CALL NEWPEN(1)
       RETURN
       END
C+++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE
PTLINE(F,D,DX,DY,XMIN,YMIN,NN,NMIN,NMAX)
```

```
      REAL*4 X(4096),Y(4096),D(NN),F(NN)
      DO 80 I=NMIN,NMAX
        IF(F(I).LT.1.E-10.OR.D(I).LT.1.E-10) GO TO 80
        X(I)=DX*ALOG10(F(I)/XMIN)
        Y(I)=DY*ALOG10(D(I)/YMIN)
   80   CONTINUE
      CALL PLOT(X(NMIN),Y(NMIN),3)
      DO 300 I=NMIN+1,NMAX
        IF(Y(I-1).GE.0.0.AND.Y(I).GT.0.0) THEN
          CALL PLOT(X(I),Y(I),2)
        ELSE
          CALL PLOT(X(I),Y(I),3)
        ENDIF
  300 CONTINUE
  301 RETURN
      END
C+++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE ESMTHF(AI,AO,F,NF,SMTHF)
C * ALISAR ESPECTRO
      REAL*4 AI(NF),AO(NF),F(NF),MADD
      IF(SMTHF.LE.0.0) GO TO 100
      AO(1)=AI(1)
      DO 35 I=2,NF
        F1=(1.0-0.5*SMTHF)*F(I)
        F2=(1.0+0.5*SMTHF)*F(I)
        IF(F2.LE.F(NF)) GO TO 31
        MNF=I-1
        GO TO 36
   31   SUM=0.0
        MADD=0.0
        DO 32 K=1,NF
          IF(F(K).LT.F1) GO TO 32
          IF(F(K).GT.F2) GO TO 33
          SUM=SUM+AI(K)
          MADD=MADD+1
   32   CONTINUE
   33   CONTINUE
        AO(I)=SUM/MADD
   35 CONTINUE
   36 CONTINUE
      NF=MNF
      RETURN
  100 CONTINUE
      DO 101 I=1,NF
  101 AO(I)=AI(I)
      RETURN
      END
```

ACC2VEL.FOR

```
      program acc2vel
c     k-net data integrater (convert to velocity data)
      parameter(nmax=65536,nch=3)
      integer     ix(nmax),imaxacc(nch),imaxvel(nch)
      real*4      x(nch,nmax),xx(nmax),amaxacc(nch),amaxvel(nch)
      complex     cx(nmax)
      character cid*6,filen(nch)*20,dummy(16)*18,cdir(nch)*3,
     *
cmemo(nch)*80,cstcode*6,cdrv*9,ccomp(nch)*2,filenm*18
      data nst/1/           ntype,nchara,ncausal/1,3,2/
ccomp/'NS','EW','UD'/
      data ap,as/0.1,10.0/
      ned=nmax
      open(10,file='stlist.txt',status='old')
      read(10,*) cdrv,nsite
      do 5000 isite=1,nsite
      read(10,*) cid,tst,ted,fl,fh,fs
      write(6,'(a)')cdrv//cid//'.**'
      filen(1)=cdrv//cid//'.ns'
      open(1,file=filen(1),status="old")
      read(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2)')
     *       dummy(1),irgyear,irgmonth,irgday,irghour,irgmin
      read(1,'(a18,f5.1)')dummy(2),orglat
      read(1,'(a18,f5.1)')dummy(3),orglong
      read(1,'(a18,i4)')  dummy(4),idepth
      orgdepth=real(idepth)
      read(1,'(a18,f4.1)')dummy(5),amag
      read(1,'(a18,a6)')dummy(6),cstcode
      read(1,'(a18,f8.4)')dummy(7),stlat
      read(1,'(a18,f8.4)')dummy(8),stlong
      read(1,'(a18,i4)')  dummy(9),istdepth
      stheight=real(istdepth)
      read(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2,1x,i2)')
     *       dummy(10),ircyear,ircmonth,ircday,irchour,ircmin,ircsec
      read(1,'(a18,i3)')dummy(11),iratio
      sratio=real(iratio)
      dt=1.0/sratio
      read(1,'(a18,i3)')dummy(12),idur
      ndata=idur*100-8
      nst=int(tst/dt+0.5)*0.0
c     ned=min(int(ted/dt+0.5),ndata)
      ned=ndata
      duration=real(idur)
      read(1,'(a18,A3)')dummy(13),cdir(1)
      read(1,'(a18,10x,i7)')dummy(14),iscale
      scale=2000./real(iscale)
      read(1,'(a18,i4)')dummy(15),imaxacc(1)
      amaxacc(1)=real(imaxacc(1))
      read(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2,1x,i2)')
     *       dummy(16),icryear,icrmonth,icrday,icrhour,icrmin,icrsec
      read(1,'(a80)')cmemo(1)
      read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
            do 5 i=1,nmax
    5   cx(i)=(0.0,0.0)
      do 10 i=1,ndata
   10   x(1,i)=real(ix(i))*scale
      close(1)
      filen(2)=cdrv//cid//'.ew'
      open(1,file=filen(2),status="old")
      do 20 kk=1,14
   20   read(1,'(a18)')dummy(kk)
      cdir(2)='EW'
      read(1,'(a18,i4)')dummy(15),imaxacc(2)
      amaxacc(2)=real(imaxacc(2))
      read(1,'(a18)')dummy(16)
      read(1,'(a80)')cmemo(2)
      read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
      do 30 i=1,ndata
   30   x(2,i)=real(ix(i))*scale
      close(1)
      filen(3)=cdrv//cid//'.ud'
      open(1,file=filen(3),status="old")
      do 40 kk=1,14
   40   read(1,'(a18)')dummy(kk)
      cdir(3)='UD'
      read(1,'(a18,i4)')dummy(15),imaxacc(3)
      amaxacc(3)=real(imaxacc(3))
      read(1,'(a18)')dummy(16)
      read(1,'(a80)')cmemo(3)
      read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
      do 50 i=1,ndata
   50   x(3,i)=real(ix(i))*scale
      close(1)
      nn=ned-nst+1
      idur=int(real(nn)*dt)
      td   =real(idur)
      nn=td/dt
      ntst=int(tst)
      do 500 kk=1,3
        do 180 i=1,nn
  180     xx(i)=x(kk,i+nst-1)
c linear trend fitting
        b1=xx(1)
        do 192 i=2,nn
  192     b1=b1+real(i)*xx(i)
        b2=xx(1)
        do 194 i=1,nn
  194     b2=b2+xx(i)
        a=real(nn*(nn+1)*(2*nn+1))/6.*dt
        b=real(nn*(nn+1))/2.
        c=b*dt
        d=real(nn)
        f=a*d-b*c
        aa=( d*b1-b*b2)/f
        bb=(-c*b1+a*b2)/f
        do 196 i=1,nn
  196     xx(i)=xx(i)-aa*real(i)*dt-bb
c band pass filtering
```

```
          call bandp1(xx,nn,dt,fl,fh,fs,ap,as,ntype,nchara,ncausal)
c integration
   200    xx(1)=xx(1)*dt
          do 300 i=2,nn
   300      xx(i)=xx(i-1)+xx(i)*dt
          amaxvel(kk)=abs(xx(1))
          do 400 i=2,nn
   400    if(amaxvel(kk).lt.abs(xx(i))) amaxvel(kk)=abs(xx(i))
          imaxvel(kk)=int(amaxvel(kk)*1000.+0.5)
          do 410 i=1,nn
   410      x(kk,i)=xx(i)
   500 continue
        do 600 kk=1,nch
         filenm=filen(kk)(1:15)//'VL.'
         filen(kk)=filenm//ccomp(kk)
   600   write(6,'(1x,a23)') filen(kk)
c    25 micro kine per digit
        scale=2.5e-5
        iscale=int(200/scale+0.5)
c
        do 1000 kk=1,nch
         do 1010 i=1,nn
  1010    ix(i)=int(x(kk,i)/scale)
        open(1,file=filen(kk),status='unknown')
        write(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2)')
       *                                dummy(1),
       *
irgyear,irgmonth,irgday,irghour,irgmin
        write(1,'(a18,f5.1)')dummy(2),orglat
        write(1,'(a18,f5.1)')dummy(3),orglong
        write(1,'(a18,i4)')   dummy(4),idepth
        write(1,'(a18,f4.1)')dummy(5),amag
        write(1,'(a18,a6)')   dummy(6),cstcode
        write(1,'(a18,f8.4)')dummy(7),stlat
        write(1,'(a18,f8.4)')dummy(8),stlong
        write(1,'(a18,i4)')   dummy(9),istdepth
        write(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2,1x,i2)')
       *                                dummy(10),
       *
ircyear,ircmonth,ircday,irchour,ircmin,ircsec
        write(1,'(a18,i3)')   dummy(11),iratio
        write(1,'(a18,i3)')   dummy(12),idur
        write(1,'(a18,A3)')   dummy(13),cdir(kk)
        write(1,'(a18,a10,i7)')dummy(14),'200(kine)/',iscale
        write(1,'(a18,i8)')   'Max. Vel.(mkine)    ',imaxvel(kk)
        write(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2,1x,i2)')
       *                                dummy(16),
       *
icryear,icrmonth,icrday,icrhour,icrmin,icrsec
        write(1,'(a80)')         cmemo(kk)
        write(1,'(8(i8,1x))')(ix(i),i=1,nn)
        close(1)
 1000 continue
 5000 continue
        close(10)
        stop
        end
```

PLTVEL.FOR

```
        include 'pltwv2.for'
        include 'timesft.for'
        include 'timecrt.for'
        program pltvel
c       k-net data viewer (integrated velocity data)
        parameter(nmax=50000,nch=3)
        integer ix(nmax)
        real*4 x(nch,nmax),xx(nmax),amaxvel(nch)
        character
cid*6,filen*47,dummy*18,cdir*3,cmemo(nch)*80,cstcode*6,
       *                cdrv*9,ccomp(nch)*2
        data aym,x0,yl,DTL/16.0,2.0,2.0,1.0/ fac/1./
        data ccomp/'NS','EW','UD'/ tst,ted/0.01,500./
        open(10,file='stlist.txt',status='old')
        read(10,*)cdrv,nsite
        do 5000 isite=1,nsite
        read(10,*) cid,tst
        filen=cdrv//cid//'vl.ns'
        write(6,'(a)')cdrv//cid//'vl.**'
        open(1,file=filen,status="old")
        read(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2)')
       *       dummy,irgyear,irgmonth,irgday,irghour,irgmin
        read(1,'(a18,f5.1)')dummy,orglat
        read(1,'(a18,f5.1)')dummy,orglong
        read(1,'(a18,i4)')   dummy,idepth
        orgdepth=real(idepth)
        read(1,'(a18,f4.1)')dummy,amag
        read(1,'(a18,a6)')dummy,cstcode
        read(1,'(a18,f8.4)')dummy,stlat
        read(1,'(a18,f8.4)')dummy,stlong
        read(1,'(a18,i4)')   dummy,idepth
        stheight=real(idepth)
        read(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2,1x,i2)')
       *       dummy,ircyear,ircmonth,ircday,irchour,ircmin,ircsec
        call timecrt(ircyear,ircmonth,ircday,irchour,ircmin,ircsec)
        read(1,'(a18,i3)')dummy,iratio
        sratio=real(iratio)
        dt=1.0/sratio
        read(1,'(a18,i3)')dummy,idur
        ndata=idur*100
        duration=real(idur)
        read(1,'(a18,A3)')dummy,cdir
        read(1,'(a18,10x,i7)')dummy,iscale
        scale=200./real(iscale)
        read(1,'(a18,i8)')dummy,imaxvel
        amaxvel(1)=real(imaxvel)
        read(1,'(a18)')dummy
        read(1,'(a80)')cmemo(1)
        read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
        do 10 i=1,ndata
   10     x(1,i)=real(ix(i))*scale
        close(1)
        ntst=int(tst)
        nst=int(tst/dt+0.5)
        ned=int(ted/dt+0.5)
        call timesft(ntst,ircyear,ircmonth,ircday,
       *                                 irchour,ircmin,ircsec)
        filen=cdrv//cid//'vl.ew'
        open(1,file=filen,status="old")
        do 20 kk=1,14
   20     read(1,'(a18)')dummy
        read(1,'(a18,i8)')dummy,imaxvel
        amaxvel(2)=real(imaxvel)
        read(1,'(a18)')dummy
        read(1,'(a80)')cmemo(2)
        read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
        do 30 i=1,ndata
   30     x(2,i)=real(ix(i))*scale
        close(1)
        filen=cdrv//cid//'vl.ud'
        open(1,file=filen,status="old")
        do 40 kk=1,14
   40     read(1,'(a18)')dummy
        read(1,'(a18,i8)')dummy,imaxvel
        amaxvel(3)=real(imaxvel)
        read(1,'(a18)')dummy
        read(1,'(a80)')cmemo(3)
        read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
        do 50 i=1,ndata
   50     x(3,i)=real(ix(i))*scale
        close(1)
        filen=cdrv//cid//'vl.ps'
        write(6,'(A)')filen
        open(25,file=filen,status='unknown')
        CALL PLOTS(25)
        call yoko(25)
        call symbol(1.5,5.0,0.8,cid,90.0,6)
        call number(1.5,11.0,0.25,real(ircyear),90.0,0)
        call number(1.5,12.25,0.25,real(ircmonth),90.0,0)
        call number(1.5,13.00,0.25,real(ircday),90.0,0)
        call number(1.5,13.75,0.25,real(irchour),90.0,0)
        call number(1.5,14.50,0.25,real(ircmin),90.0,0)
        call number(1.5,15.25,0.25,real(ircsec),90.0,0)
        CALL FACTOR(FAC)
        ndatamax=min1(real(ndata-nst+1),24.*dtl/dt)+nst-1
        do 60 kk=1,nch
         amaxvel(kk)=x(kk,nst)
         do 60 i=2,ndatamax-nst+1
```

```
            if(amaxvel(kk).lt.abs(x(kk,nst+i-1))) then
                amaxvel(kk)     =abs(x(kk,nst+i-1))
            endif
   60 continue
      amaxx=amax1(amaxvel(1),amaxvel(2),amaxvel(3))
      if(amaxx.lt. 500.) xxmax= 500.
      if(amaxx.lt. 200.) xxmax= 200.
      if(amaxx.lt. 100.) xxmax= 100.
      if(amaxx.lt.  50.) xxmax=  50.
      if(amaxx.lt.  20.) xxmax=  20.
      if(amaxx.lt.  10.) xxmax=  10.
      if(amaxx.lt.   5.) xxmax=   5.
      if(amaxx.lt.   2.) xxmax=   2.
      if(amaxx.lt.   1.) xxmax=   1.
      if(amaxx.lt.0.5)   xxmax=   0.5
      if(amaxx.lt.0.2)   xxmax=   0.2
      if(amaxx.lt.0.1)   xxmax=   0.1
      if(amaxx.lt.0.05)  xxmax=   0.05
      if(amaxx.lt.0.02)  xxmax=   0.02
      if(amaxx.lt.0.01)  xxmax=   0.01
      call plot(x0,0.,-3)
      CALL NEWPEN(1)
      ay=aym
      if(ndatamax.lt.ndata) call symbol(26.,0.5,0.3,'C',0.0,1)
      do 1000 kk=1,3
        call symbol(2.5,ay+yl+0.3,0.4,ccomp(kk),0.0,2)
        call symbol(4.0,ay+yl+0.3,0.3,'Max=',0.0,4)
        call number(5.5,ay+yl+0.3,0.3,amaxvel(kk),0.0,3)
        do 100 i=1,ndata
  100     xx(i)=x(kk,i+nst-1)
        CALL PLOT( 2., AY,-3)
        call pltwv2(xx,ndatamax-nst+1,dt,dtl,xxmax,yl,2)
        CALL PLOT(-2.,-AY,-3)
        ay=ay-3.*yl
 1000 continue
      call plote(25)
      close(25)
 5000       continue
      close(10)
      stop
      end
```

ACC2DIS.FOR

```
      program acc2dis
c     k-net data integrater (convert to displacement data)
      parameter(nmax=65536,nch=3)
      integer     ix(nmax),imaxacc(nch),imaxdsp(nch)
      real*4      x(nch,nmax),xx(nmax),amaxacc(nch),amaxdsp(nch)
      character
cid*6,filen(nch)*20,filein*15,dummy(16)*18,cdir(nch)*3,
     *
cmemo(nch)*80,cstcode*6,cdrv*9,ccomp(nch)*2,filenm*18
      data nst/1/           ntype,nchara,ncausal/1,3,2/
ccomp/'NS','EW','UD'/
      data ap,as/0.1,10.0/
      ned=nmax
      open(10,file='stlist.txt',status='old')
      read(10,*) cdrv,nsite
      do 5000 isite=1,nsite
      read(10,*) cid,tst,ted,fl,fh,fs
      filein=cdrv//cid
      filen(1)=filein//'.ns'
      write(6,'(a)')filein//'.**'
      open(1,file=filen(1),status="old")
      read(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2)')
     *       dummy(1),irgyear,irgmonth,irgday,irghour,irgmin
      read(1,'(a18,f5.1)')dummy(2),orglat
      read(1,'(a18,f5.1)')dummy(3),orglong
      read(1,'(a18,i4)')   dummy(4),idepth
      orgdepth=real(idepth)
      read(1,'(a18,f4.1)')dummy(5),amag
      read(1,'(a18,a6)')dummy(6),cstcode
      read(1,'(a18,f8.4)')dummy(7),stlat
      read(1,'(a18,f8.4)')dummy(8),stlong
      read(1,'(a18,i4)')   dummy(9),idepth
      stheight=real(idepth)
      read(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2,1x,i2)')
     *       dummy(10),ircyear,ircmonth,ircday,irchour,ircmin,ircsec
      read(1,'(a18,i3)')dummy(11),iratio
      sratio=real(iratio)
      dt=1.0/sratio
      read(1,'(a18,i3)')dummy(12),idur
      ndata=idur*100-8
      nst=int(tst/dt+0.5)*0.0
c       ned=min(int(ted/dt+0.5),ndata)
      ned=ndata
      duration=real(idur)
      read(1,'(a18,A3)')dummy(13),cdir(1)
      read(1,'(a18,10x,i7)')dummy(14),iscale
      scale=2000./real(iscale)
      read(1,'(a18,i4)')dummy(15),imaxacc(1)
      amaxacc(1)=real(imaxacc(1))
      read(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2,1x,i2)')
     *       dummy(16),icryear,icrmonth,icrday,icrhour,icrmin,icrsec
      read(1,'(a80)')cmemo(1)
      read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
      do 10 i=1,ndata
   10   x(1,i)=real(ix(i))*scale
      close(1)
      filen(2)=filein//'.ew'
      open(1,file=filen(2),status="old")
      do 20 kk=1,14
   20   read(1,'(a18)')dummy(kk)
      cdir(2)='EW'
      read(1,'(a18,i4)')dummy(15),imaxacc(2)
      amaxacc(2)=real(imaxacc(2))
      read(1,'(a18)')dummy(16)
      read(1,'(a80)')cmemo(2)
      read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
      do 30 i=1,ndata
   30   x(2,i)=real(ix(i))*scale
      close(1)
      filen(3)=filein//'.ud'
      open(1,file=filen(3),status="old")
      do 40 kk=1,14
   40   read(1,'(a18)')dummy(kk)
      cdir(3)='UD'
      read(1,'(a18,i4)')dummy(15),imaxacc(3)
      amaxacc(3)=real(imaxacc(3))
      read(1,'(a18)')dummy(16)
      read(1,'(a80)')cmemo(3)
      read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
      do 50 i=1,ndata
   50   x(3,i)=real(ix(i))*scale
      close(1)
      nn=ned-nst+1
      idur=int(real(nn)*dt)
      td    =real(idur)
      nn=td/dt
      ntst=int(tst)
      do 500 kk=1,3
        do 180 i=1,nn
  180     xx(i)=x(kk,i+nst-1)
c linear trend fitting
        b1=xx(1)
        do 192 i=2,nn
  192     b1=b1+real(i)*xx(i)
        b2=xx(1)
        do 194 i=1,nn
  194     b2=b2+xx(i)
        a=real(nn*(nn+1)*(2*nn+1))/6.*dt
        b=real(nn*(nn+1))/2.
        c=b*dt
        d=real(nn)
        f=a*d-b*c
        aa=( d*b1-b*b2)/f
        bb=(-c*b1+a*b2)/f
        do 186 i=1,nn
  186     xx(i)=xx(i)-aa*real(i)*dt-bb
c band pass filtering
        call bandp1(xx,nn,dt,fl,fh,fs,ap,as,ntype,nchara,ncausal)
c integration
  888   xx(1)=xx(1)*dt
        do 200 i=2,nn
  200     xx(i)=xx(i-1)+xx(i)*dt
c linear trend fitting
        b1=xx(1)
```

```
            do 292 i=2,nn
  292         b1=b1+real(i)*xx(i)
            b2=xx(1)
            do 294 i=1,nn
  294         b2=b2+xx(i)
            a=real(nn*(nn+1)*(2*nn+1))/6.*dt
            b=real(nn*(nn+1))/2.
            c=b*dt
            d=real(nn)
            f=a*d-b*c
            aa=( d*b1-b*b2)/f
            bb=(-c*b1+a*b2)/f
            do 286 i=1,nn
  286       xx(i)=xx(i)-aa*real(i)*dt-bb
c band pass filtering
            call bandp1(xx,nn,dt,fl,fh,fs,ap,as,ntype,nchara,ncausal)
c integration
            xx(1)=xx(1)*dt
            do 300 i=2,nn
  300         xx(i)=xx(i-1)+xx(i)*dt
            amaxdsp(kk)=abs(xx(1))
            do 400 i=2,nn
  400       if(amaxdsp(kk).lt.abs(xx(i))) amaxdsp(kk)=abs(xx(i))
            imaxdsp(kk)=int(amaxdsp(kk)*10000.+0.5)
            do 410 i=1,nn
  410         x(kk,i)=xx(i)
  500 continue
          do 600 kk=1,nch
            filenm=filen(kk)(1:15)//'DS.'
            filen(kk)=filenm//ccomp(kk)
            write(6,'(1x,a23)') filen(kk)
  600 continue
c   0.25 micron per digit
          scale=2.5e-5
          iscale=int(25./scale+0.5)
c
          do 1000 kk=1,nch
          do 1010 i=1,nn
 1010       ix(i)=int(x(kk,i)/scale)
          open(1,file=filen(kk),status='unknown')
          write(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2)')
         *                              dummy(1),
         *
irgyear,irgmonth,irgday,irghour,irgmin
          write(1,'(a18,f5.1)')dummy(2),orglat
          write(1,'(a18,f5.1)')dummy(3),orglong
          write(1,'(a18,i4)')   dummy(4),idepth
          write(1,'(a18,f4.1)')dummy(5),amag
          write(1,'(a18,a6)')   dummy(6),cstcode
          write(1,'(a18,f8.4)')dummy(7),stlat
          write(1,'(a18,f8.4)')dummy(8),stlong
          write(1,'(a18,i4)')   dummy(9),idepth
          write(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2,1x,i2)')
         *                              dummy(10),
         *
ircyear,ircmonth,ircday,irchour,ircmin,ircsec
          write(1,'(a18,i3)')   dummy(11),iratio
          write(1,'(a18,i3)')   dummy(12),idur
          write(1,'(a18,A3)')   dummy(13),cdir(kk)
          write(1,'(a18,a10,i7)')dummy(14),'25( cm    )/',iscale
          write(1,'(a18,i8)')   'Max.Disp.(micron) ',imaxdsp(kk)
          write(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2,1x,i2)')
         *                              dummy(16),
         *
icryear,icrmonth,icrday,icrhour,icrmin,icrsec
          write(1,'(a80)')         cmemo(kk)
          write(1,'(8(i8,1x))')(ix(i),i=1,nn)
          close(1)
 1000 continue
 5000 continue
          close(10)
          stop
          end
```

PLTDIS.FOR

```
          include 'pltwv2.for'
          include 'timesft.for'
          include 'timecrt.for'
          program pltdis
c         k-net data viewer (integrated displacement data)
          parameter(nmax=50000,nch=3)
          integer ix(nmax)
          real*4 x(nch,nmax),xx(nmax),amaxvel(nch)
          character
cid*6,filen*47,dummy*18,cdir*3,cmemo(nch)*80,cstcode*6,
         *                cdrv*9,ccomp(nch)*2
          data aym,x0,yl,DTL/16.0,2.0,2.0,1.0/ fac/1./
          data ccomp/'NS','EW','UD'/    tst,ted/0.01,500./
          open(10,file='stlist.txt',status='old')
          read(10,*)cdrv,nsite
                do 5000 isite=1,nsite
          read(10,*) cid,tst
          filen=cdrv//cid//'ds.ns'
          write(6,'(a)')cdrv//cid//'ds.**'
          open(1,file=filen,status="old")
          read(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2,1x,i2)')
         *          dummy,irgyear,irgmonth,irgday,irghour,irgmin
          read(1,'(a18,f5.1)')dummy,orglat
          read(1,'(a18,f5.1)')dummy,orglong
          read(1,'(a18,i4)')    dummy,idepth
          orgdepth=real(idepth)
          read(1,'(a18,f4.1)')dummy,amag
          read(1,'(a18,a6)')dummy,cstcode
          read(1,'(a18,f8.4)')dummy,stlat
          read(1,'(a18,f8.4)')dummy,stlong
          read(1,'(a18,i4)')    dummy,idepth
          stheight=real(idepth)
          read(1,'(a18,i4,1x,i2,1x,i2,1x,i2,1x,i2,1x,i2,1x,i2)')
         *          dummy,ircyear,ircmonth,ircday,irchour,ircmin,ircsec
          call timecrt(ircyear,ircmonth,ircday,irchour,ircmin,ircsec)
          read(1,'(a18,i3)')dummy,iratio
          sratio=real(iratio)
          dt=1.0/sratio
          read(1,'(a18,i3)')dummy,idur
          ndata=idur*100
          duration=real(idur)
          read(1,'(a18,A3)')dummy,cdir
          read(1,'(a18,10x,i7)')dummy,iscale
          scale=25./real(iscale+0.5)
          read(1,'(a18,i8)')dummy,imaxvel
          amaxvel(1)=real(imaxvel)
          read(1,'(a18)')dummy
          read(1,'(a80)')cmemo(1)
          read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
          do 10 i=1,ndata
   10       x(1,i)=real(ix(i))*scale
          close(1)
          filen=cdrv//cid//'ds.ew'
          open(1,file=filen,status="old")
          do 20 kk=1,14
   20       read(1,'(a18)')dummy
          read(1,'(a18,i8)')dummy,imaxvel
          amaxvel(2)=real(imaxvel)
          read(1,'(a18)')dummy
          read(1,'(a80)')cmemo(2)
          read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
          do 30 i=1,ndata
   30       x(2,i)=real(ix(i))*scale
          close(1)
          filen=cdrv//cid//'ds.ud'
          open(1,file=filen,status="old")
          do 40 kk=1,14
   40       read(1,'(a18)')dummy
          read(1,'(a18,i8)')dummy,imaxvel
          amaxvel(3)=real(imaxvel)
          read(1,'(a18)')dummy
          read(1,'(a80)')cmemo(3)
          read(1,'(8(i8,1x))')(ix(i),i=1,ndata)
          do 50 i=1,ndata
   50       x(3,i)=real(ix(i))*scale
          close(1)
          ntst=int(tst)
          nst=int(tst/dt+0.5)
          ned=int(ted/dt+0.5)
          call timesft(ntst,ircyear,ircmonth,ircday,
         *                                  irchour,ircmin,ircsec)
          filen=cdrv//cid//'ds.ps'
```

```
      write(6,*) filen
      open(25,file=filen,status='unknown')
      CALL PLOTS(25)
      call yoko(25)
      call symbol(1.5,5.0,0.8,cid,90.0,6)
      call number(1.5,11.0,0.25,real(ircyear),90.0,0)
      call number(1.5,12.25,0.25,real(ircmonth),90.0,0)
      call number(1.5,13.00,0.25,real(ircday),90.0,0)
      call number(1.5,13.75,0.25,real(irchour),90.0,0)
      call number(1.5,14.50,0.25,real(ircmin),90.0,0)
      call number(1.5,15.25,0.25,real(ircsec),90.0,0)
      CALL FACTOR(FAC)
      ndatamax=min1(real(ndata-nst+1),24.*dtl/dt)+nst-1
      do 60 kk=1,nch
        amaxvel(kk)=x(kk,nst)
        do 60 i=2,ndatamax-nst+1
          if(amaxvel(kk).lt.abs(x(kk,nst+i-1))) then
                amaxvel(kk)      =abs(x(kk,nst+i-1))
          endif
   60 continue
      amaxx=amax1(amaxvel(1),amaxvel(2),amaxvel(3))
c     write(6,*)amaxvel(1),amaxvel(2),amaxvel(3)
      if(amaxx.lt.500000.) xxmax=500000.
      if(amaxx.lt.200000.) xxmax=200000.
      if(amaxx.lt.100000.) xxmax=100000.
      if(amaxx.lt.50000.) xxmax=50000.
      if(amaxx.lt.20000.) xxmax=20000.
      if(amaxx.lt.10000.) xxmax=10000.
      if(amaxx.lt.5000.) xxmax=5000.
      if(amaxx.lt.2000.) xxmax=2000.
      if(amaxx.lt.1000.) xxmax=1000.
      if(amaxx.lt. 500.) xxmax= 500.
      if(amaxx.lt. 200.) xxmax= 200.
      if(amaxx.lt. 100.) xxmax= 100.
      if(amaxx.lt.  50.) xxmax=  50.
      if(amaxx.lt.  20.) xxmax=  20.
      if(amaxx.lt.  10.) xxmax=  10.
      if(amaxx.lt.   5.) xxmax=   5.
      if(amaxx.lt.   2.) xxmax=   2.
      if(amaxx.lt.   1.) xxmax=   1.
      if(amaxx.lt.0.5)   xxmax=   0.5
      if(amaxx.lt.0.2)   xxmax=   0.2
      if(amaxx.lt.0.1)   xxmax=   0.1
      if(amaxx.lt.0.05)  xxmax=   0.05
      if(amaxx.lt.0.02)  xxmax=   0.02
      if(amaxx.lt.0.01)  xxmax=   0.01
      if(amaxx.lt.0.005) xxmax=   0.005
      if(amaxx.lt.0.002) xxmax=   0.002
      if(amaxx.lt.0.001) xxmax=   0.001
      xxmax=xxmax*10000.
      call plot(x0,0.,-3)
      CALL NEWPEN(1)
      ay=aym
      if(ndatamax.lt.ndata) call symbol(26.,0.5,0.3,'C',0.0,1)
      do 1000 kk=1,3
        call symbol(2.5,ay+yl+0.3,0.4,ccomp(kk),0.0,2)
        call symbol(4.0,ay+yl+0.3,0.3,'Max=',0.0,4)
        call number(5.5,ay+yl+0.3,0.3,amaxvel(kk)*10000.,0.0,4)
        do 100 i=1,ndata
  100     xx(i)=x(kk,i+nst-1)*10000.
        CALL PLOT( 2., AY,-3)
        call pltwv2(xx,ndatamax-nst+1,dt,dtl,xxmax,yl,3)
        CALL PLOT(-2.,-AY,-3)
        ay=ay-3.*yl
 1000 continue
      call plote(25)
      close(25)
 5000 continue
      close(10)
      stop
      end
```

Reference for further reading

Chiu, H-C (1997): Stable Correction of Digital Strong-Motion Data, *Bull. Seism. Soc. Am.*, Vol. 87, 932-944.

Graeme, J. G., G. E. Tubey and L. P. Huelsman (1971): Operational Amplifiers Design and Applications, McGraw-Hill.

Minami, S. (1986): Waveform Data Processing for Scientific Measurements, CQ-Publishing (in Japanese).

Ohsaki, Y. (1976): Introduction to the spectral analysis of seismic motion, Kajima (in Japanese).

Papoulis, A. (1984): Signal Analysis, Mcgraw-Hill .

Press, W. H., S. A. Teukolsky, W. T. Vetterling and B. P. Flannery (1992): Numerical Recipes in FORTRAN, The Art of Scientific Computing Second Edition, Cambridge University Press.

Rikitake, T., R. Sato and Y. Hagiwara (1980): Applied Mathematics -for Geosciences- Vol. I Basics, Center for Academic Publications Japan (in Japanese).

Robinson, A. and S. Treitel (1980): Geophysical Signal Analysis, Prentice-Hall.

Saito, M. (1978):An automatic Design Algorithm for Band Selective Recursive Digital Filters, BUTURI-TANSA, Vol. 31, No. 4, pp112-135 (in Japanese).

Sherbaum, F. (1994): Basic Concepts in Digital Signal Processing for Seismologists, Springer-Verlag.

Sherbaum, F. and J. Johnson (1992): Programmable Interactive Toolbox for Seismological Analysis (PITSA), International Association of Seismology and Physics of the Earth's Interior.

Sherbaum, F. (1996): Of Poles and Zeros, Kluwer Academic Publishers.

Silvia, M. T. and E. A. Robinson (1979): Deconvolution of Geophysical Time Series in the Exploration for Oil and Natural Gas, Elsevier.

Vich, R. (1987): Z-transform Theory and Applications, D. Reidel Publishing Company.

Yanagisawa, T and B. Kanemitu (1980): Design of Active Filter, Sanpo-Publishing (in Japanese).

Yilmaz, Ö (1994): Seismic Data Processing, Society of Exploration Geophysicists.