Dynamic Soil Structure Interaction

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Chapter 4:

Dynamic Impedance Function and Foundation Input Motion

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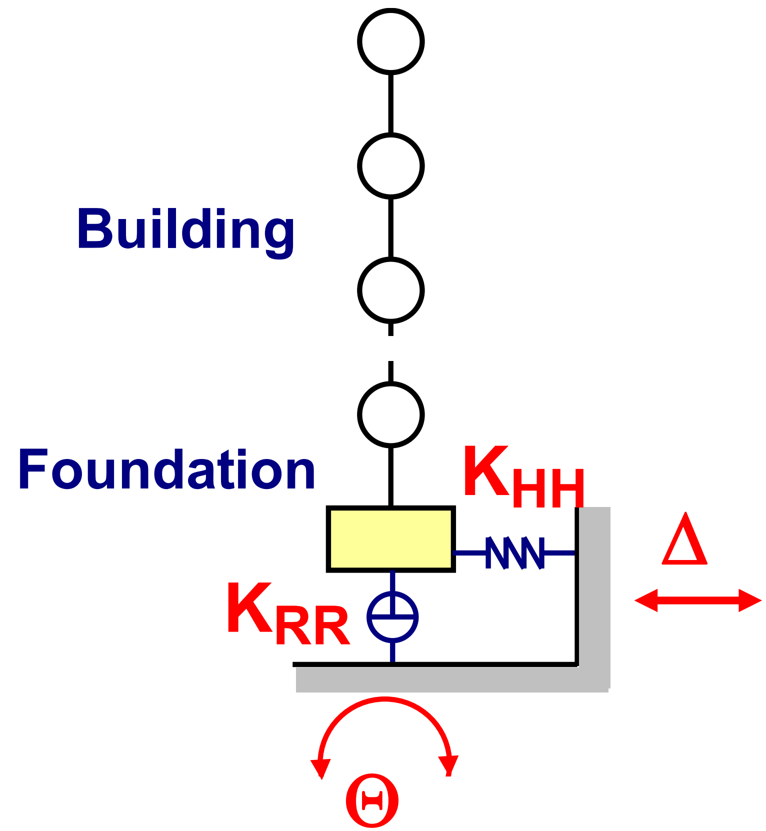
4. Dynamic Impedance Function and Foundation Input Motion

A **Sway-Rocking Model** is one of dynamic analysis methods for Soil-Structure Interaction (SSI) System.

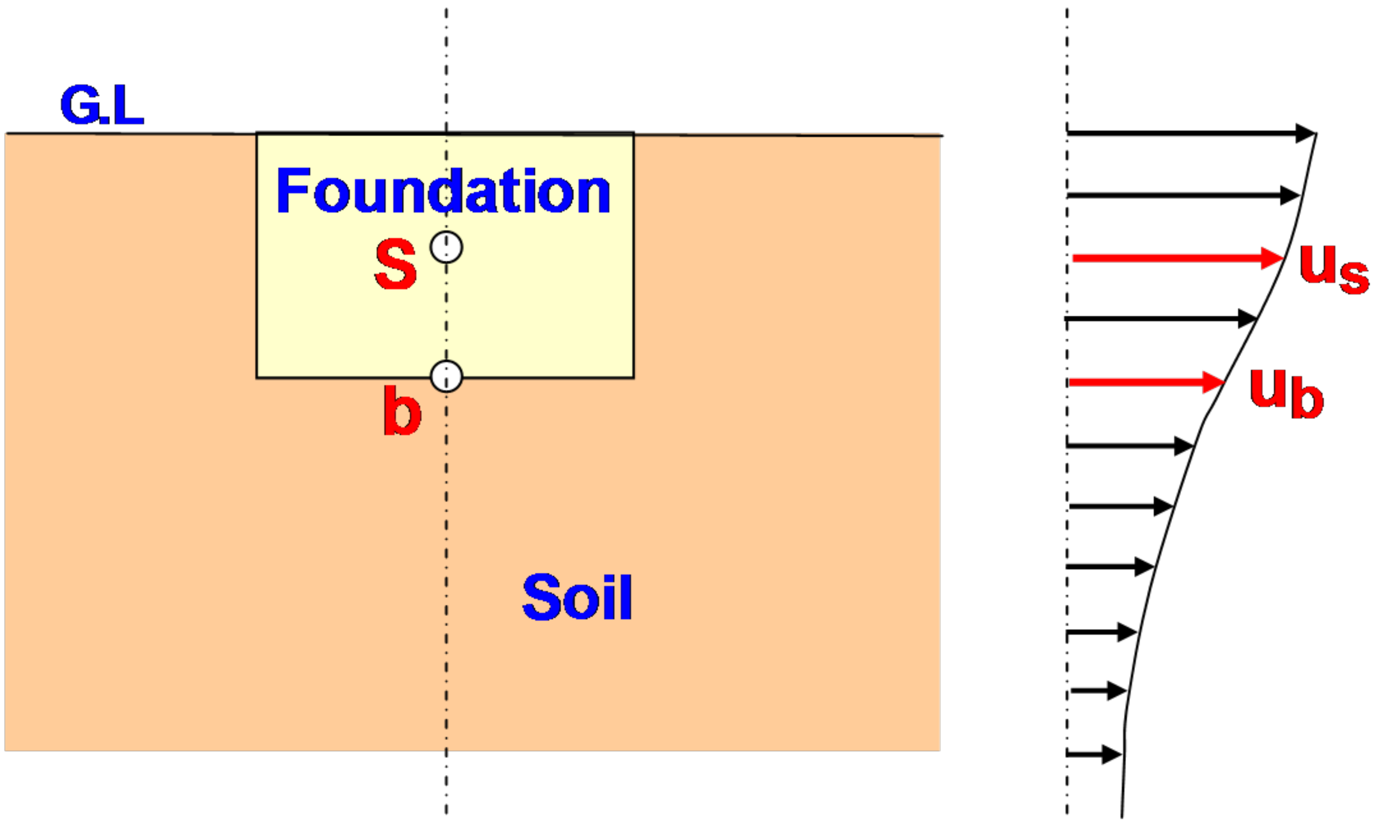
In this model, a basement or a foundation footing is assumed to be rigid and the interaction effect is incorporated through the dynamic soil springs called " **Dynamic Impedance Function**".

"**Foundation Input Motion**" due to earthquake disturbance is applied at the dynamic impedance functions.



Sway-Rocking Model

Here, we consider the physical meaning and the evaluation method of the Dynamic Impedance Function and the Foundation Input Motion using a rigid buried structure, which models the foundation as shown below.

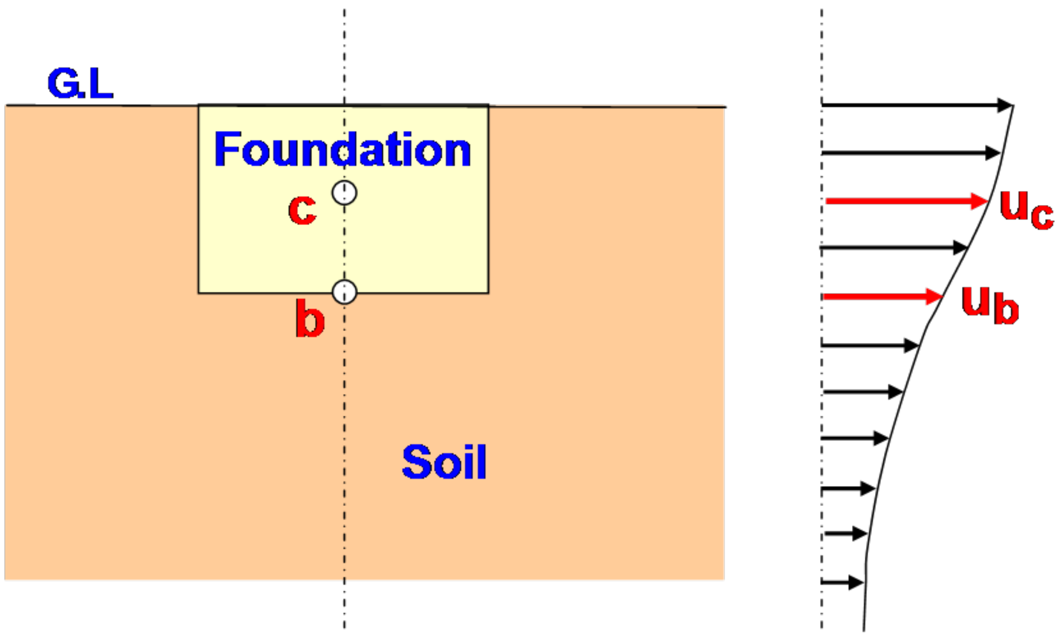


(a) Buried Foundation (b) Response of Free Field

For the simplicity, we assume the foundation is vibrating under the following assumptions:

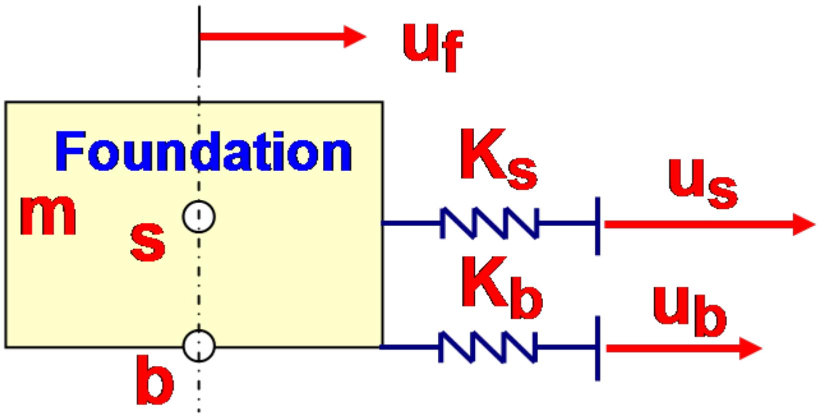
(1) The foundation is rigid and vibrating in the lateral direction without rotation.

(2) An earthquake disturbance is due to SH type incident earthquake wave propagating upward.



(a) Buried Foundation (b) Response of Free Field

(3) Soil springs attaching the foundation to the free field soil are K_S and K_b at the representative points.



Modeling of Buried Foundation

An equation of the motion for the foundation model is formulated by:

$$m\ddot{u}_f + K_S(u_f - u_S) + K_b(u_f - u_b) = 0 \qquad (1)$$

where,

- m : mass of foundation
- K_s, K_b : soil springs
- U_f : lateral displacement of foundation
- u_s, u_b : lateral displacement of free field soil

Eq.(1) can be rewritten into:

$$m\ddot{u}_f + (K_S + K_b)u_f = K_S u_S + K_b u_b \qquad (2)$$

Putting: $$K = K_S + K_b \tag{3}$$

Eq.(2) : $$m\ddot{u}_f + (K_S + K_b)u_f = K_S u_S + K_b u_b \tag{2}$$

can be rearranged as:

$$m\ddot{u}_f + K u_f = K_S u_S + K_b u_b = K\left(\frac{K_S u_S + K_b u_b}{K}\right) \tag{4}$$

Also, putting:

$$u_e = \frac{K_S u_S + K_b u_b}{K} \tag{5}$$

Eq.(4) can be rewritten as:

$$m\ddot{u}_f + K u_f = K u_e \tag{6}$$

Expressing u_f by:

$$u_f = x + u_e \qquad (7)$$

and substituting Eq.(7) into Eq.(6):

$$m\ddot{u}_f + Ku_f = Ku_e \qquad (6)$$

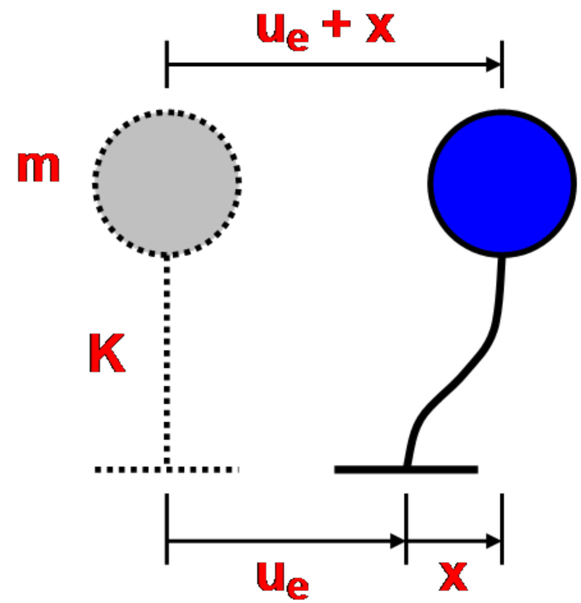
then, Eq.(8) can be obtained.

$$m(\ddot{x} + \ddot{u}_e) + K(x + u_e) = Ku_e$$

$$\therefore \quad m\ddot{x} + Kx = -m\ddot{u}_e \qquad (8)$$

$$m\ddot{x} + Kx = -m\ddot{u}_e \qquad (8)$$

Eq.(8) equals an equation of motion for a single degree of freedom system shown in the figure, which consists of a mass m, a spring K and an input disturbance u_e.



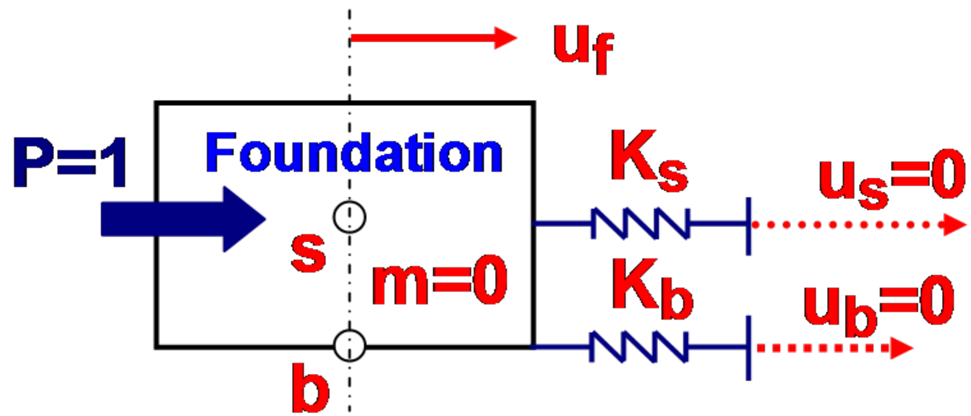
Single Degree of Freedom System

K and u_e are called "Dynamic Impedance Function" and "Foundation Input Motion".

You can understand easily the physical meaning of the dynamic impedance function and the foundation input motion.

Second, we consider how to calculate the dynamic impedance function and the foundation Input Motion.

Assuming a rigid massless buried foundation before an earthquake, that is:



$$m=0, \quad u_S=u_b=0 \qquad (9)$$

When this system is subject to an unit lateral force, the conditions of Eq.(9) are substituted into Eq.(1):

$$m\ddot{u}_f+K_S(u_f-u_S)+K_b(u_f-u_b)=0 \qquad (1)$$

then, we obtain Eq.(10): note: the external force term becomes the unite.

$$(K_S+K_b)\,u_f=1 \qquad (10)$$

and,

$$u_f=\frac{1}{K_S+K_b} \qquad (11)$$

Comparing definition of the dynamic impedance function (Eq.(3)):

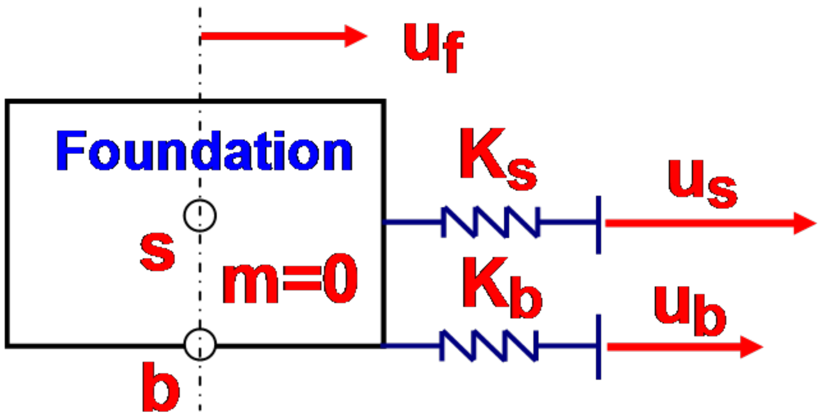
$$K = K_S + K_b \tag{3}$$

with Eq.(11):

$$u_f = \frac{1}{K_S + K_b} \tag{11}$$

We can understand that the inverse of displacement u_f coincides with the "Dynamic Impedance Function".

For the calculation of the foundation input motion, we also assume the massless foundation which is subjected to an earthquake disturbance.



The equation of motion for this foundation can be written as:

$$K_S(u_f - u_S) + K_b(u_f - u_b) = 0 \qquad (12)$$

and,

$$u_f = \frac{K_S u_S + K_b u_b}{K_S + K_b} = \frac{K_S u_S + K_b u_b}{K} \qquad (13)$$

Comparing Eq.(13) :

$$u_f = \frac{K_S u_S + K_b u_b}{K_S + K_b} = \frac{K_S u_S + K_b u_b}{K} \qquad (13)$$

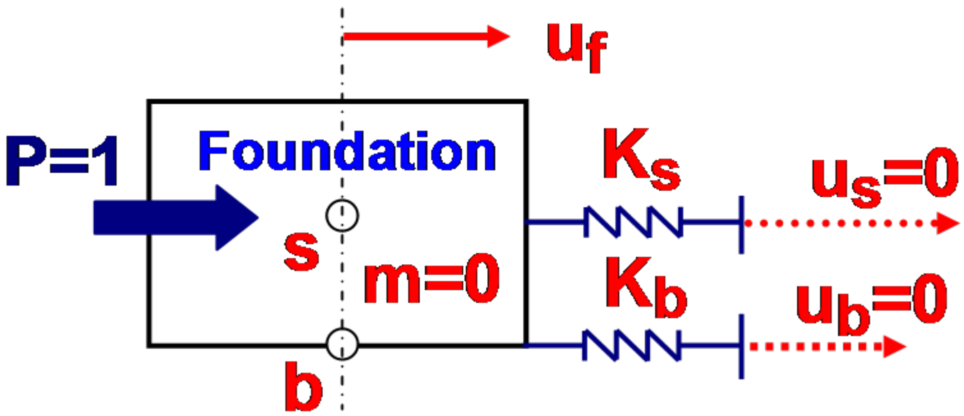
with Eq.(5) of the definition of the foundation input motion,

$$u_e = \frac{K_S u_S + K_b u_b}{K} \qquad (5)$$

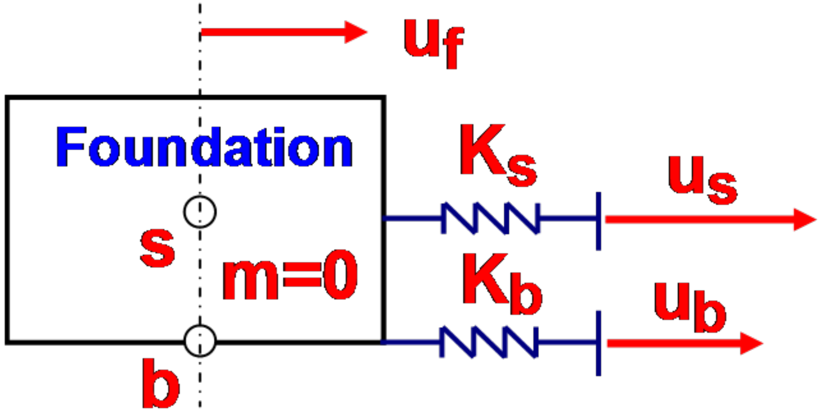
you can easily understand that u_f equals to the "Foundation Input Motion u_e".

When you want to calculate the dynamic impedance function and the foundation input motion, you may assume the massless foundation.

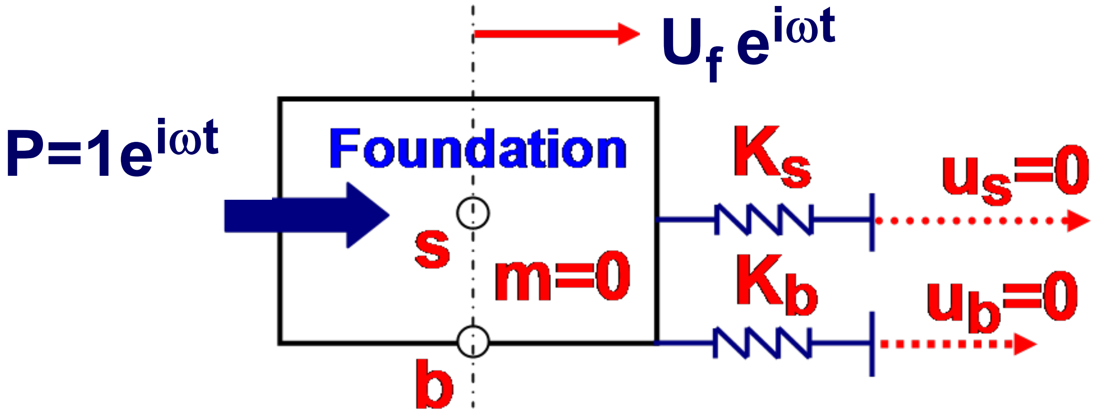
Calculating the displacement of the foundation due to a unit force, the inverse of the displacement equals to the dynamic impedance function.



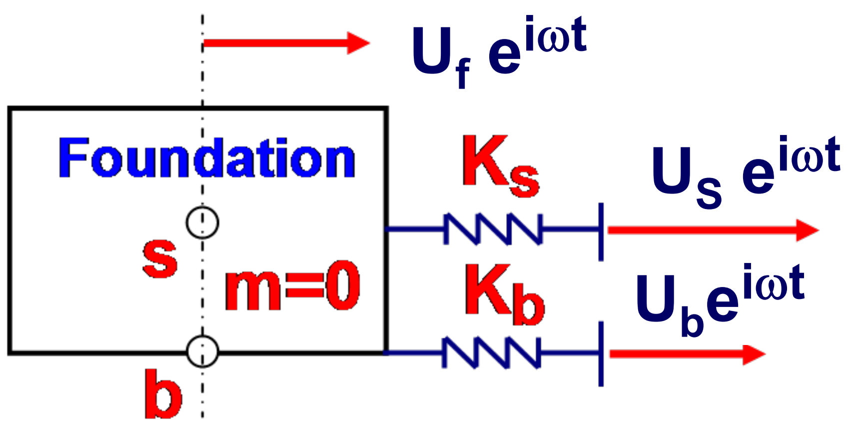
Calculating the displacement of the foundation due to an earthquake disturbance, this displacement is just the foundation input motion.



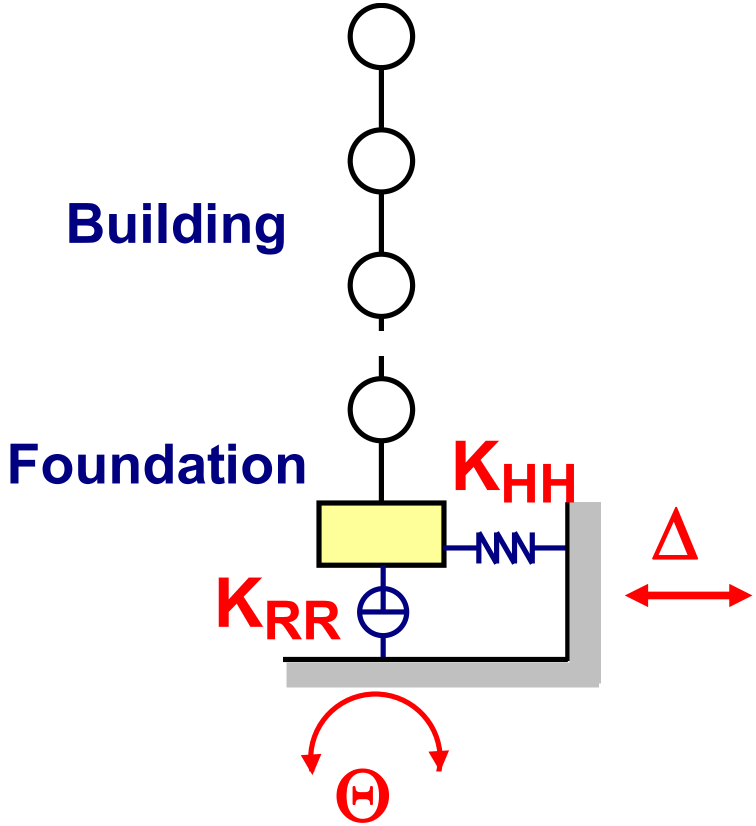
In frequency domain analysis, the applied unit force must be $1e^{i\omega t}$ for the calculation of the **dynamic impedance function**,



and the earthquake disturbance must be the response obtained by frequency response analysis of the free field soil for the calculation of the **foundation input motion**.



For a practical seismic design analysis of a building, the **Sway-Rocking Model** as shown in the figure have been being employed.



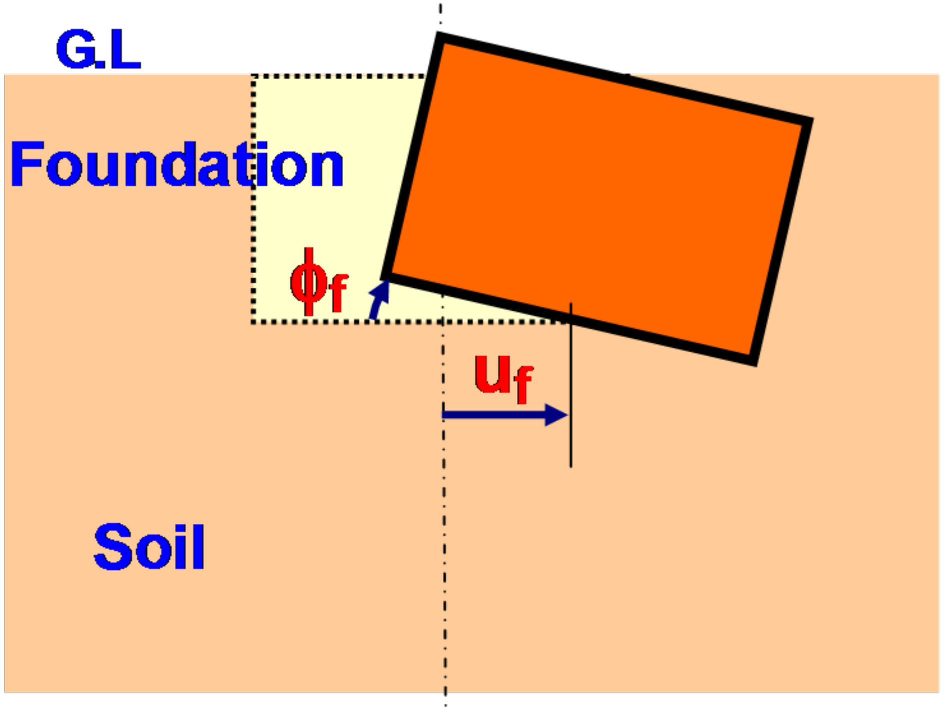
Sway-Rocking Model

In this figure,

K_{HH} and K_{RR} : Dynamic Impedance Function

Δ and Θ : Foundation Input Motion

When the foundation has two degrees of freedom (u_f,ϕ_f) shown in the figure, the equation of motion are expressed as:



$$\begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} \ddot{u}_f \\ \ddot{\phi}_f \end{Bmatrix} + \begin{bmatrix} K_{HH} & K_{HR} \\ K_{RH} & K_{RR} \end{bmatrix} \begin{Bmatrix} u_f \\ \phi_f \end{Bmatrix} = \begin{bmatrix} K_{HH} & K_{HR} \\ K_{RH} & K_{RR} \end{bmatrix} \begin{Bmatrix} \Delta \\ \Theta \end{Bmatrix} \qquad (14)$$

Where:

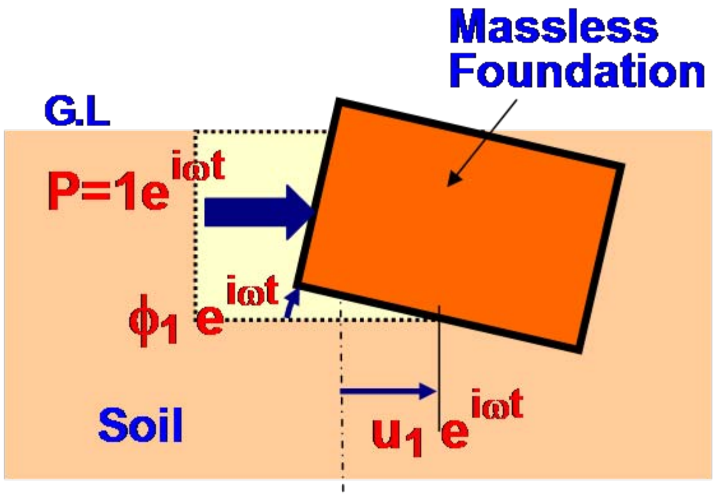
m and I : mass and inertia moment of foundation

K_{HH}, K_{RR}, K_{HR}=K_{RH} : dynamic impedance function

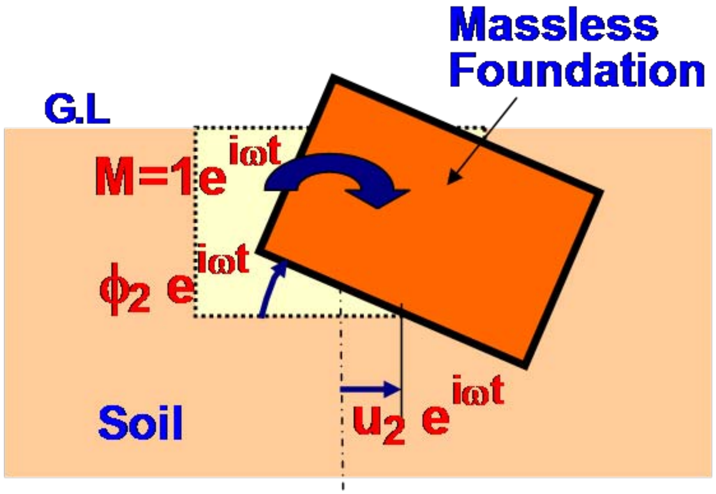
Δ , Θ : foundation input motion

A massless foundation is also assumed for calculation of the dynamic impedance function and the foundation input motion.

The lateral displacement $u_1 e^{i\omega t}$ and the rotational angle $\phi_1 e^{i\omega t}$ due to an unit lateral force $P=1e^{i\omega t}$ on the foundation are calculated.



And also, the lateral displacement $u_2 e^{i\omega t}$ and the rotational angle $\phi_2 e^{i\omega t}$ due to an unit moment $M= 1e^{i\omega t}$ on the foundation are calculated.



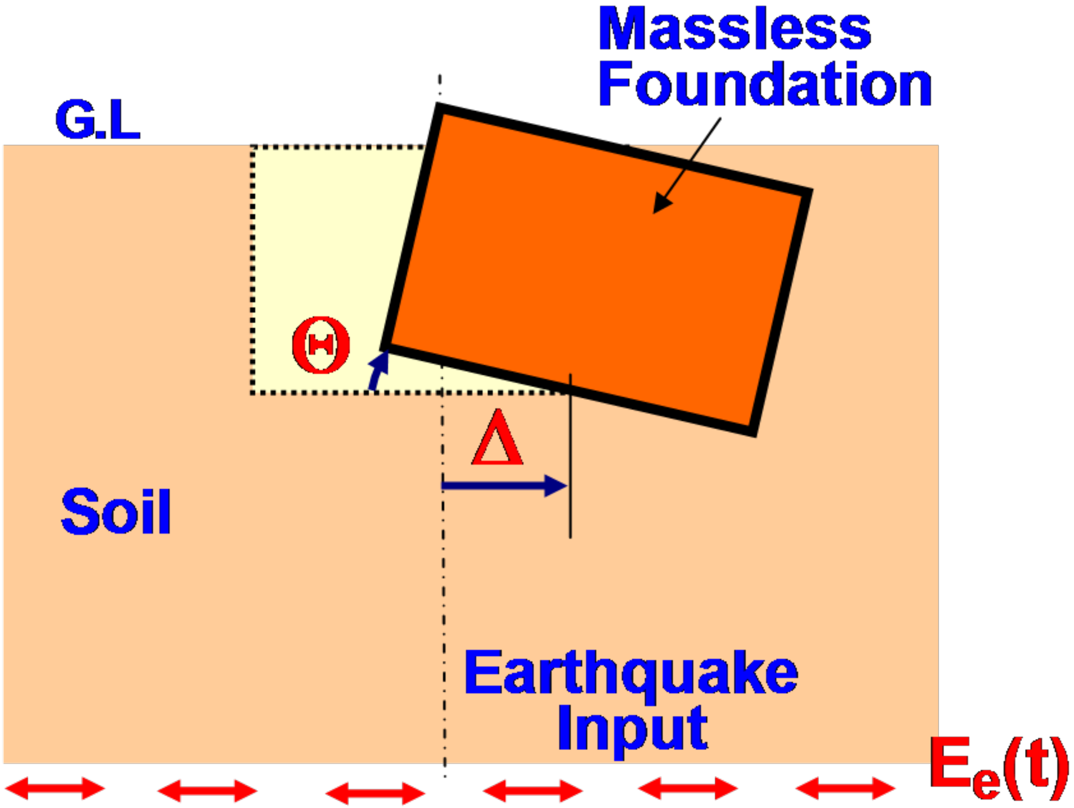
Then, the dynamic impedance matrix is given by:

$$\begin{bmatrix} K_{HH} & K_{HR} \\ K_{RH} & K_{RR} \end{bmatrix} = \begin{bmatrix} u_1 & u_2 \\ \phi_1 & \phi_2 \end{bmatrix}^{-1} \qquad (15)$$

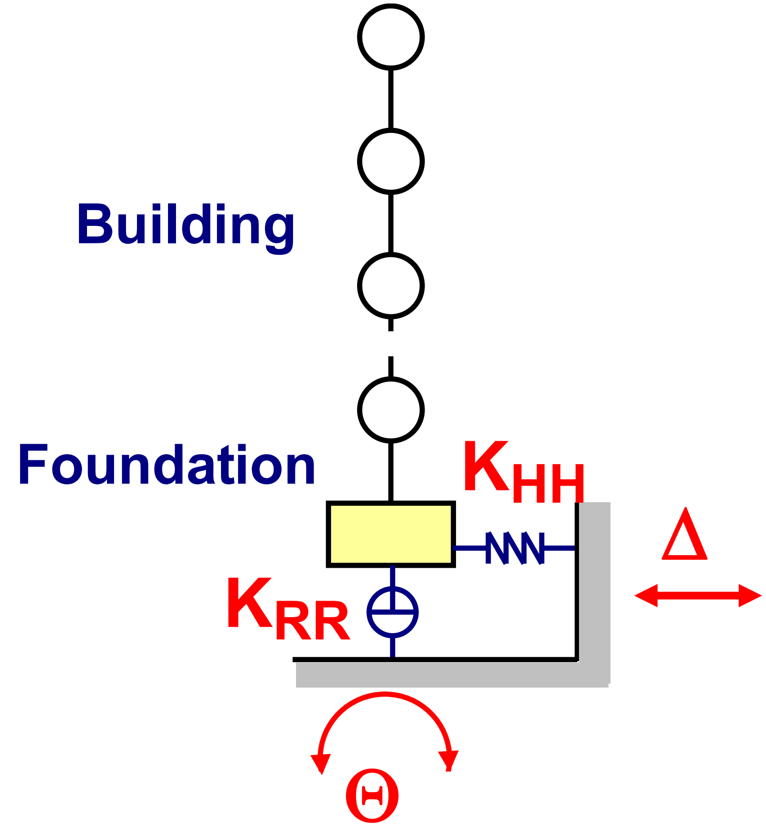
that is, the dynamic impedance matrix is given by inverse matrix of the displacement matrix.

The cross terms K_{HR} and K_{RH} in the dynamic impedance matrix are trivial for the foundation when the embedding depth is shallow.

The response of the massless foundation due to the earthquake disturbance is calculated, the lateral response Δ and the rotational angle Θ are the foundation input motion of the horizontal and the rotational component, respectively.



The figure shows the dynamic analysis model for the superstructure-foundation-soil interaction system (coupling system).



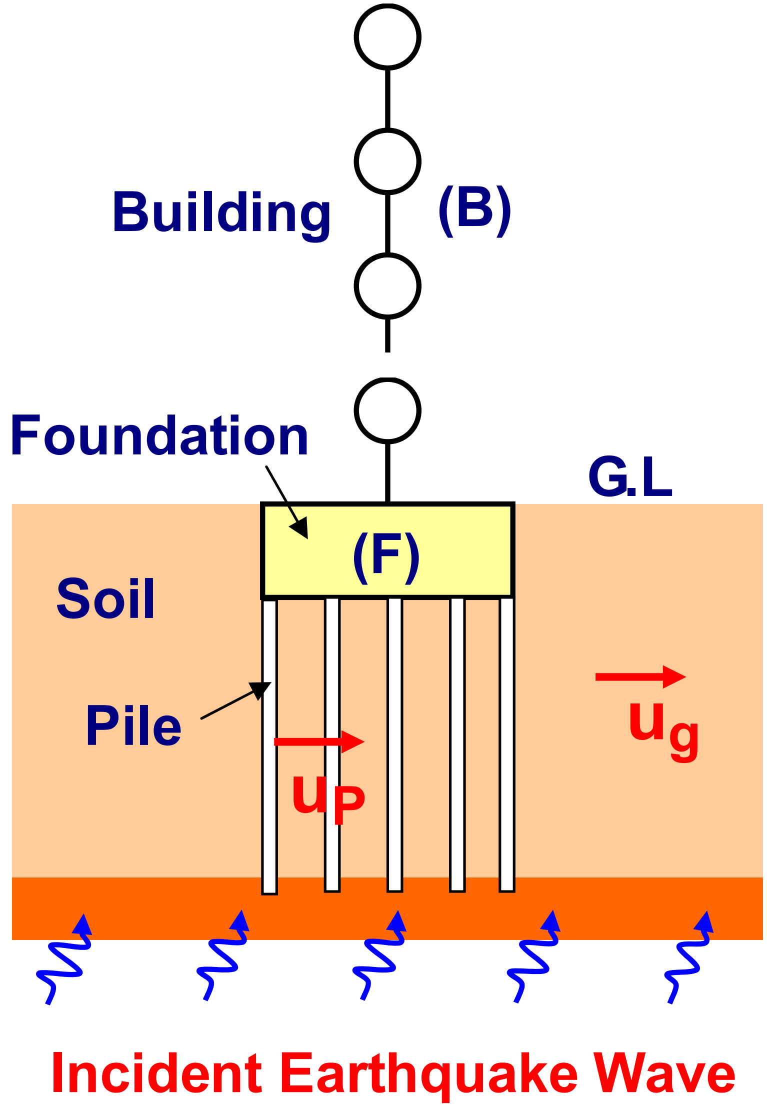
Sway-Rocking Model

In this model, the foundation assumed to be rigid is supported by the horizontal K_{HH} and the rotational K_{RR} dynamic impedance functions and the foundation input motions Δ and Θ are applied at the outside of the impedance functions.

We review the Sway-Rocking Model using the structure supported by pile group as shown in the figure.

For a seismic design analysis of a superstructure, a dynamic analysis using the sway-rocking model shown in the figure is enough.

However, a structure supported by pile foundation requires further calculation of dynamic behaviors of the pile group. The dynamic analysis must be performed as follows.

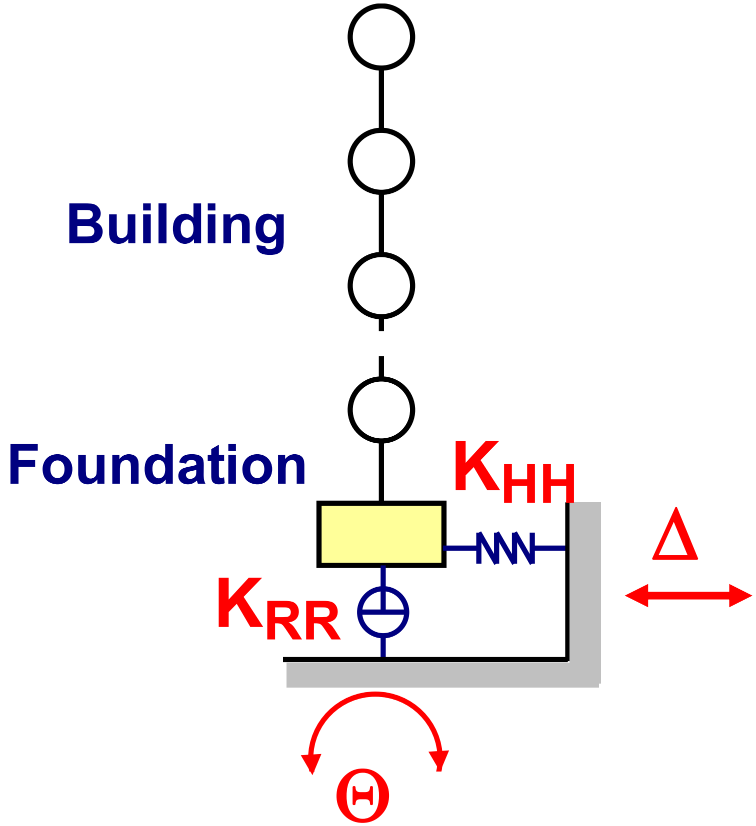


(a) Interaction System

The dynamic impedance functions and the foundation input motion for the pile group foundation are similarly calculated under the assumption that foundation footing or the basement on the piles is a massless rigid body.

Performing the dynamic analysis using the sway-rocking model shown in the figure, we obtain the total inertial force F and the total overturning moment M at the foundation bottom level,

which are equal respectively to the reaction forces of the horizontal K_{HH} and the rotational K_{RR} impedance functions.



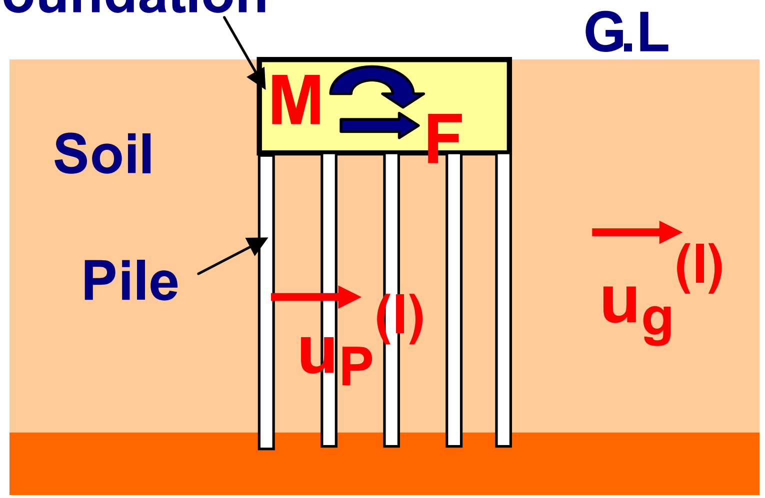
Sway-Rocking Model

At the first step, we consider interaction system called "Inertial Interaction"

where the massless rigid foundation is subjected to the total inertial force F and the total overturning moment M at the base.

F : Inertia Force of Building and Foundation

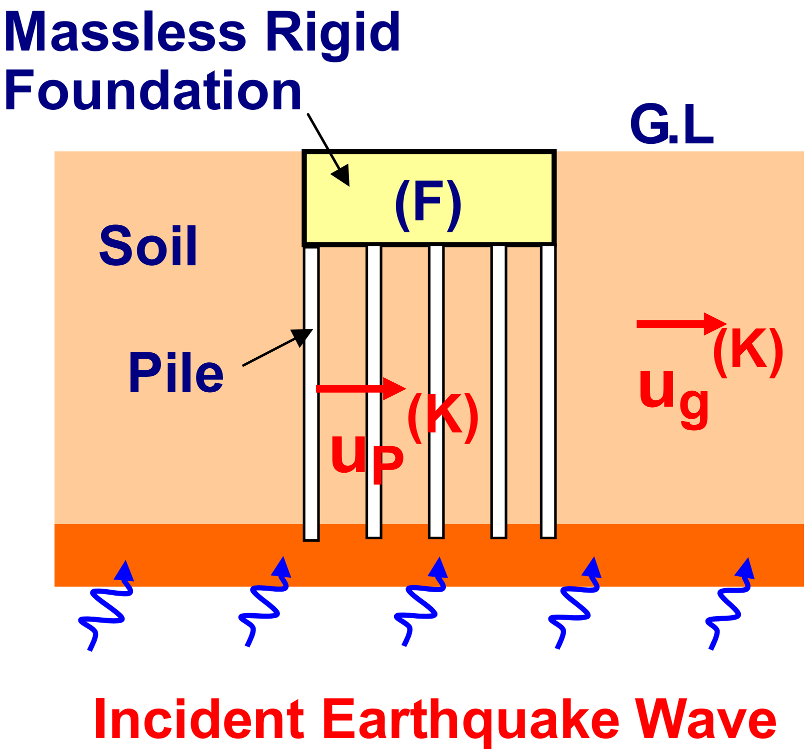
M : Overturning Moment



(b) Inertia Interaction

At the second step, we consider the interaction system called " Kinematic Interaction" ,

where the massless rigid foundation is subjected to the incident earthquake wave.



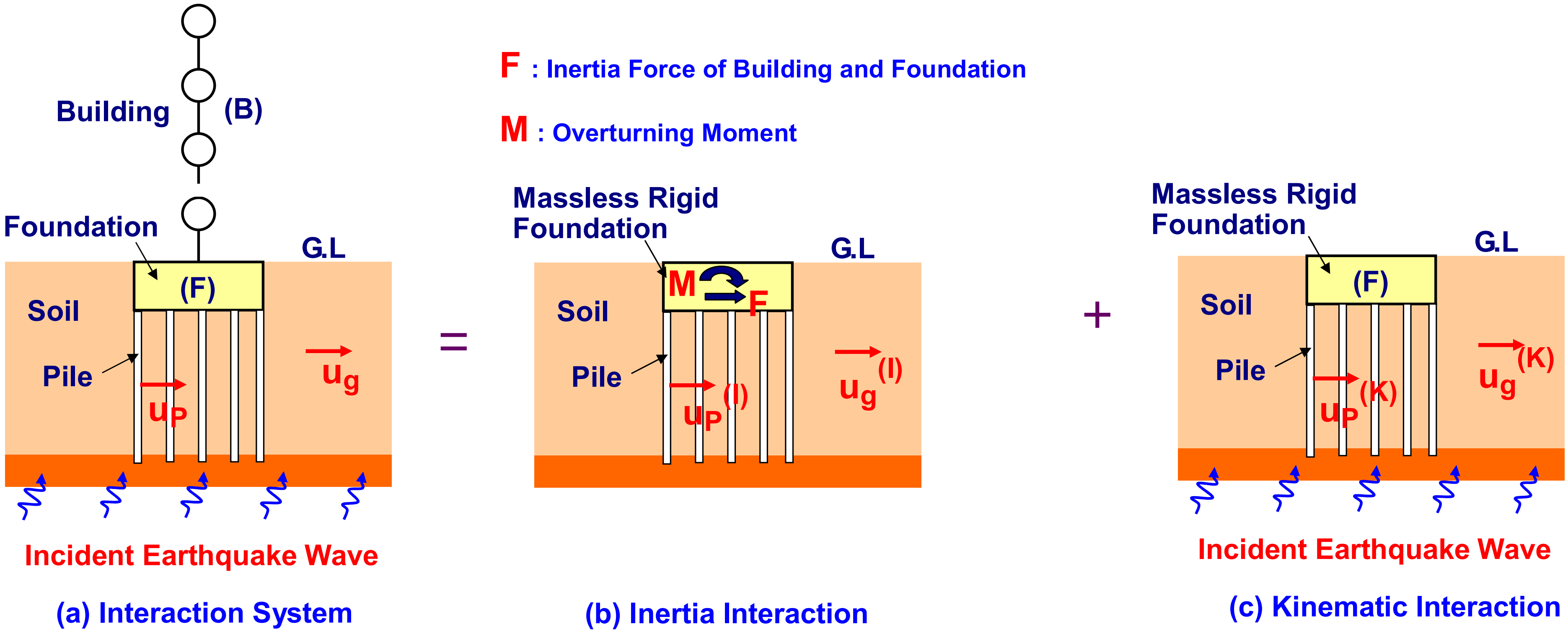
(c) Kinematic Interaction

The dynamic behaviors of the pile group foundation and the soil are the sum of those under the inertial and the kinematic interaction.

When we will require the earthquake responses of the pile group and the soil surrounding the structure, we have to analyze both of the interaction systems.

The concept of the kinematic and the inertial interaction is very useful, when you will employ FEM or FDM for the interaction analysis.

When you will performed an analysis using methods based on the elastic wave propagation theory, a sub-structural method described after will be more effective concept.



Inertial and Kinematic Interaction

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