1. Overview

(1) Introduction

The seismic response of a building structure depends on its dynamic properties and characteristics of input ground motions. Nowadays, the dynamic properties of structures can be easily calculated using computer software. However, various uncertain factors disturb the exact estimation of the dynamic properties. For example, influence of non-structural elements is usually left out of account in the computation. Effect of the soil-structure interaction is one of the important yet hard factors. Under such a situation, it is important to investigate the dynamic properties of actual structures and discuss the validity of the computation.

There are some methods of investigating the dynamic properties of the building structures. Basically, we need to measure the response of the structure to external force. Table 1 explains some methods to examine the dynamic properties. The method using artificial external force is dealt with as dynamic testing hereinafter.

Method	External force	Amplitude
Micro-tremor measurement	Ambient vibrations and/or wind	Very small
Strong motion observation	Earthquake motions	Small to large
Dynamic testing	Artificial forces	Small to large

Table 1 Methods to investigate dynamic properties of structures

Dynamic testing can be classified into three types, i.e. shaking table test, vibration generator test and free vibration test, as shown in Table 2. We need to use some equipment to apply external force to the target structure. The response of the structure is measured as displacement, velocity and/or acceleration time histories. Through the analysis of the response, the dynamic properties, represented by natural frequencies, damping ratios, and vibration modes, can be estimated.

Table 2	Dynamic	testing methods	
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Test	External force	Result
Shaking table	1- to 3-D random wave (white noise,	Elastic resonance curve Time history of elastic or elasto-plastic response
Vibration generator	1-D harmonic wave	Resonance curve
Free vibration	Initial movement	Time history of free vibration

(2) Shaking Table Test

The shaking table test is realistic and clear when the response of a structure during an earthquake is discussed. A specimen is set up on the table which can be driven by actuators as shown in Fig. 1. Test specimens are usually manufactured for the shaking table test; therefore a destructive test can be performed. Miniaturized specimens are sometime used because of the capacity limitation of the table.

BRI has a one-dimensional middle-sized shaking table in the Structural Testing Laboratory. Recently constructed shaking tables can be driven in two- or three-dimensions. Consequently, those can reproduce actual earthquake motions more faithfully than one-dimensional tables.

Periodic waves, such as sine, rectangular and triangular, and random waves, such as white noise, artificial earthquake motions and observed earthquake motions, can be chosen as the input motion.

Steady-state shaking using harmonic waves with various frequencies is usually made to grasp general dynamic properties of the specimen. Amplitudes of the input harmonic waves are low and response of the specimen remains in the elastic range. Random shaking using white noise is utilized to check the dynamic properties in the elastic range as well.

Random shaking using observed strong motions can simulate actual situation during an earthquake. The amplitude of the strong motion record is adjusted according to the seismic capacity of the specimen and the testing purpose. You can damage and destroy the specimen by the earthquake motion. Artificial earthquake motions may be used if strong motion records with desired characteristics cannot be found out.

By the other hand, shaking table test is generally costly to execute. The construction of shaking table facilities would be a big project. Specimens must be newly built only for testing. In addition, there are limitations of size and weight due to the shaking table capacity. It is not easy to reproduce an actual situation.

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Figure 1 Shaking table test

(3) Vibration Generator Test

In the case of actual buildings, a vibration generator is often used to apply harmonic external forces. A vibration generator is usually set up on the top of the target structure as shown in Fig. 2. A simple vibration generator consists of two sets of weights rotating in the opposite directions mutually. Consequently, the force in the X-direction is canceled and the cyclic force is generated in the Y-direction. The rotating speed is changed step by step and steady-state response at each step is measured. Finally the vibration generator test brings the resonance curve of the target structure.

The harmonic force generated by the equipment shown in Fig. 2 is given by the following equation:

$$F(t) = m_{\rm R} r \omega^2 e^{i\omega t} \tag{1}$$

where $m_{\rm R}$ is the total mass of the rotating weight, *r* is the radius of the rotating weights, and ω is the circular frequency (rotating speed). Harmonic force becomes larger as rotation speed increases. So, heavy weights are required for testing large structures with low natural frequencies.



Figure 2 Vibration generator test

(4) Free Vibration Test

Free vibration test is intuitive and relatively easy. The free vibration of the target structure is measured by providing initial movement as shown in Fig. 3. This test will be utilized for large-scale buildings if enough initial movement can be applied. The time history of free vibration provides the natural period and the damping ratio of the target structure.



Figure 3 Free vibration test

2. Practice in Dynamic Testing

(1) Testing Model and Measuring System

We use a small specimen which can be regarded as a single-degree-of-freedom system. The specimen consists of a heavy steel head, two thin steel plates, and a rigid steel base as shown in Fig. 4. Our objective is to determine the dynamic properties, i.e. the natural period and damping ratio, of this specimen.

Horizontal movements of the top and base of the model are important in most cases, so two accelerometers (acceleration sensors) are set up at the top and at the base of the specimen. The cables from the accelerometers are connected to the amplifier, which converts acceleration to electric signal. Outputs from the amplifier are drawn on a pen recorder as time histories.



Figure 4 Test specimen and measuring system

(2) Accelerometer Calibration

Before starting the dynamic testing, accelerometers must be calibrated to determine their sensitivity which is necessary to convert measured values to acceleration. Figure 5 indicates the procedure of the accelerometer calibration. First, an accelerometer is set on the flat place. The pen recorder draws a straight line representing zero of acceleration. When you turn its positive side down, the pen of the recorder shifts and indicates the acceleration of gravity. Then return the accelerometer to the neutral position, and turn its negative side down. Finally, return it to the neutral position again. We can know the sensitivity from the waveform as shown in Fig. 5.



Figure 5 Calibration of accelerometers using gravity

(3) Free Vibration Test

One of the simplest ways to determine the dynamic properties of a system is a free vibration test. A standing system starts vibrating freely when initial movement is applied. For instance, pull the top of the model and release it. Damped free vibration can be observed (Fig. 6). The natural period and the damping ratio can be read from the wave form of free vibration.



Figure 6 Damped free vibration

The equation of motion for a single-degree-of-freedom system without applied force is as follows:

$$\ddot{x}(t) + 2h\omega_0 \dot{x}(t) + \omega_0^2 x(t) = 0$$
⁽²⁾

where $\ddot{x}(t)$, $\dot{x}(t)$ and x(t) are acceleration, velocity and displacement of the mass, respectively. *h* is the damping ratio and ω_0 is the natural circular frequency.

The solution of Eq. 2 becomes:

$$x(t) = Ce^{-h\omega t} \sin(\sqrt{1 - h^2}\omega t + \varphi)$$
(3)

where the constants C and φ are determined from initial conditions.

The damped natural period is represented as time intervals, T'_m, T'_{m+1}, \cdots , of cyclic waves as shown in Fig. 6. The average of several time intervals should be used to ensure accuracy.

$$T_0' \simeq \frac{T_0}{\sqrt{1-h^2}} \simeq \frac{1}{n} \sum_{i=0}^{n-1} T_{m+i}' = \frac{1}{n} (t_{m+n} - t_m)$$
(4)

where T'_0 is the damped natural period and T_0 is the undamped natural period $(T_0 = 2\pi / \omega_0)$.

The damping ratio h can be estimated from a ratio of amplitude to one at the next cycle. The amplitude ratio of a_m to a_{m+1} is the function of the damping ratio.

$$\log_e \frac{a_m}{a_{m+1}} = \frac{2\pi h}{\sqrt{1 - h^2}}$$
(5)

The amplitude can be of displacement, velocity and acceleration. When damping ratio *h* is low, $\sqrt{1-h^2}$ is close to one. Consequently, *h* can be estimated as follows:

$$h \simeq \frac{\log_e \frac{a_m}{a_{m+1}}}{2\pi} \tag{6}$$

It is recommended to use the amplitude after several cycles (a_{m+n}) in order to reduce error:

$$h \simeq \frac{\log_e \frac{a_m}{a_{m+n}}}{2n\pi} \tag{7}$$

(4) Shaking Table Test

A simple test using a small shaking table will be performed to determine the dynamic properties of the specimen. The table is driven harmonically with the specified frequency. The frequency of the harmonic wave can be controlled step by step and accelerations at the base $(a_{\rm B})$ and at the top $(a_{\rm T})$ are captured on the recording paper at each step as shown in Fig. 7.



Figure 7 Acceleration record from the shaking table test

An exact frequency of the harmonic wave and peak-to-peak amplitudes $a_{\rm B}$ and $a_{\rm T}$ can be read from the waveform on the paper. It is recommended to use several cycles to get the frequency.



Figure 8 Reading a frequency

A resonance curve is obtained from the relationship between frequencies f of harmonic waves and amplitude ratios ($\rho = a_T / a_B$) of the top to the base. The frequency f_{max} at which the resonance curve becomes highest is approximately equal to the natural frequency of the system. A damping ratio is computed from the shape of the resonance curve as shown in Fig. 9.



Figure 9 Resonance curve

Damping ratio *h* is roughly determined from the peak value ρ_{max} of resonance curve using the following equation if *h* is low:

$$h \simeq \frac{1}{2\rho_{\max}} \tag{8}$$

Another method determined h from frequencies (f_1 and f_2 in Fig. 9) at which the response amplitude is reduced to the level $\rho_{\text{max}} / \sqrt{2}$, $1/\sqrt{2}$ times of its peak value. The damping ratio can be estimated using f_1 and f_2 as follows.

$$h \simeq \frac{f_2 - f_1}{f_2 + f_1} \tag{9}$$

Exercises

- (1) Find out the natural frequency and the damping ratio from the waveform of the free vibration test. Attach the waveform in the report as well.
- (2) Draw the resonance curve from the result of the steady-state shaking test. Find out the natural frequency and the damping ratios (using equation (8) and (9)) from the resonance curve.