Dynamic Soil Structure Interaction

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Chapter 6

Substructure Method for SSI Analysis

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Substructure Method for SSI Analysis

The substructure method is one of powerful numerical procedures for SSI analysis. We study the basic theoretical concept of the substructure method.

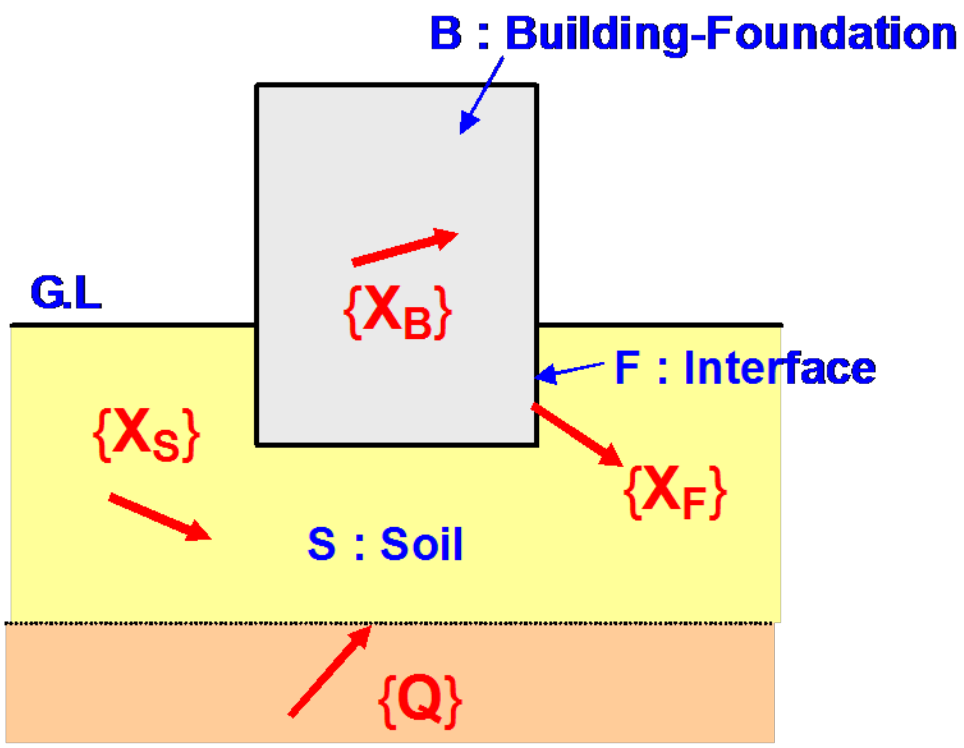


Fig.1   Building-Foundation and Soil System

Consider the Building-Foundation and Soil Interaction system shown in Fig.1.

The interaction system consists of B region (Building-Foundation), S region (Soil) and F region (Interface between B and S region).

The F region can be regarded as the contact surface between the B and the S region.

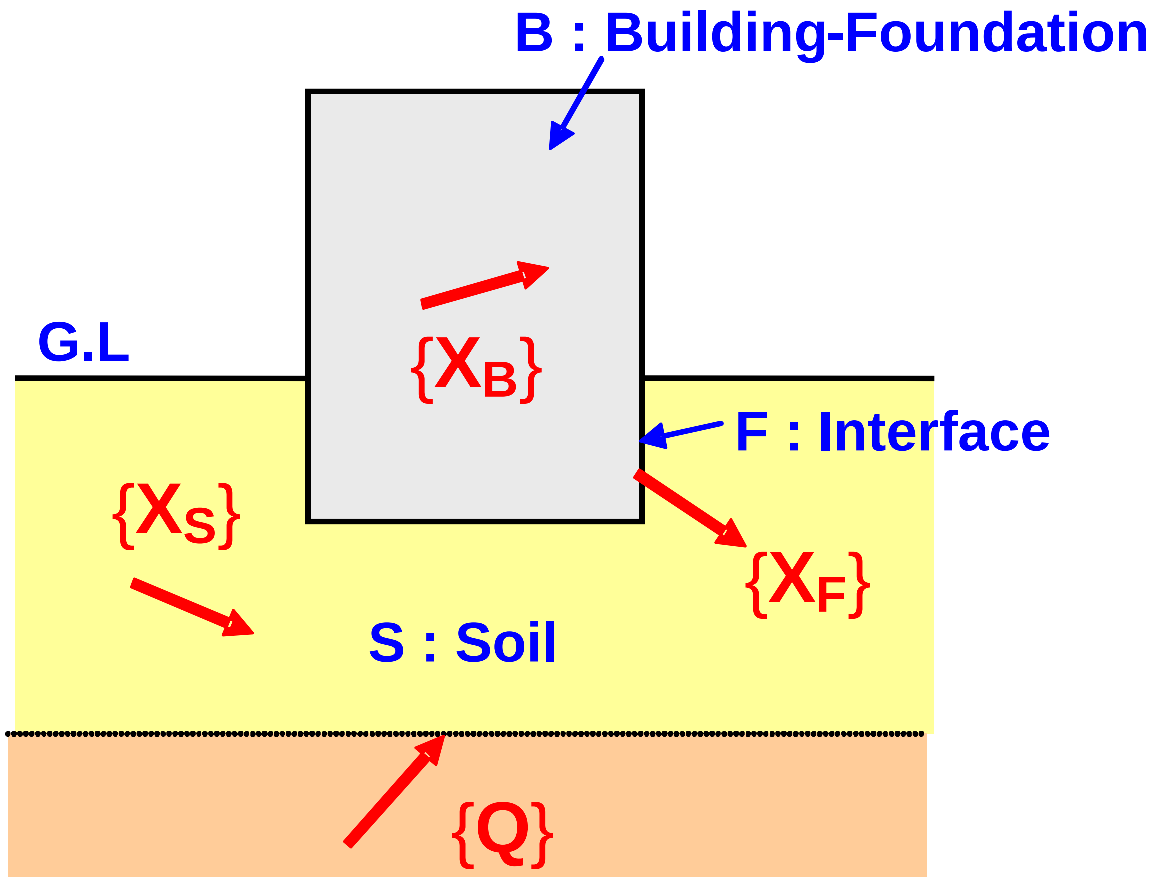
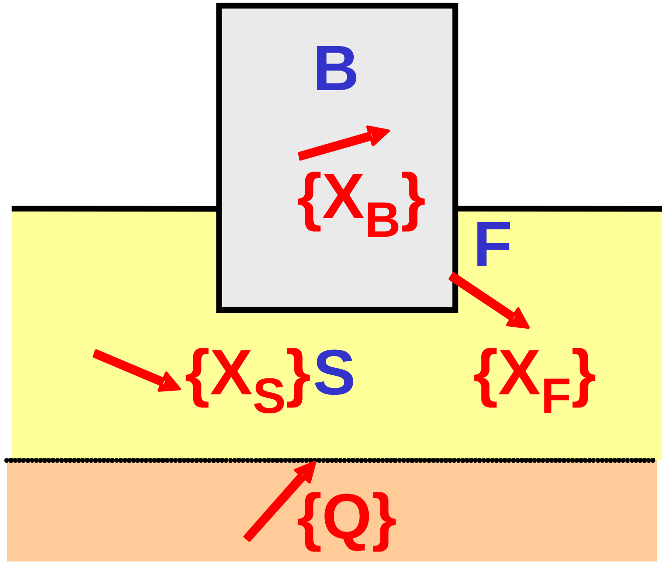


Fig.1 Building-Foundation and Soil System

Based on the FEM modeling technique, the equation of the motion for the Interaction System shown in Fig.1 is represented by:

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] & [0] \\ [K_{FB}] & [K_{FF}] & [K_{FS}] \\ [0] & [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \\ \{X_S\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{Q\} \end{Bmatrix} \tag{1}$$



where,

$[K_{IJ}]$: dynamic stiffness matrix

$\{X_I\}$: absolute displacement vector

$\{Q\}$: earthquake disturbance vector

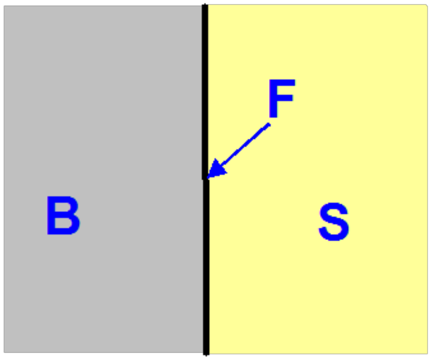
The indexes I and J indicate the B region, the F region and S region.

That is, the index B refers to the Building-Foundation, F to the interface and S to the soil.

The stiffness matrix in Eq.(1) includes [0] matrix,

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] & [0] \\ [K_{FB}] & [K_{FF}] & [K_{FS}] \\ [0] & [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \\ \{X_S\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{Q\} \end{Bmatrix} \qquad (1)$$

which means that the B region does not touch directly the S region but touches the S region through the F region.



Dynamic stiffness matrix $[K_{IJ}]$, which includes the mass, the damping and stiffness, is expressed by:

$$[K_{IJ}] = [k_{IJ}] - \omega^2 [m_{IJ}]\delta_{IJ} + i\omega^2 [c_{IJ}] \qquad (2)$$

where, $[k_{IJ}]$, $[m_{IJ}]$ and $[c_{IJ}]$ are stiffness, mass and damping matrix. And $\delta_{IJ}=1$ when I is equals to J, while $\delta_{IJ}=0$ when I is not equal to J.

Because the index F indicates the interface between B and S region,

the dynamic stiffness matrix $[K_{FF}]$ consists of the dynamic stiffness $[K_{FFB}]$ for B region and $[K_{FFS}]$ for S region as shown in Fig.2.

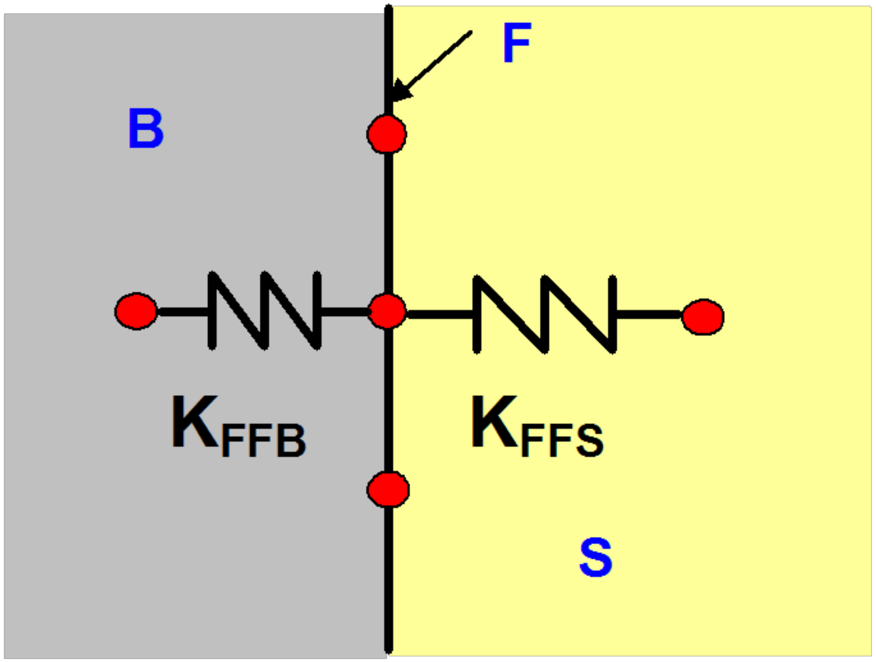


Fig.2 Decomposition of $[K_{FF}]$

That is:

$$[K_{FF}] = [K_{FFB}] + [K_{FFS}] \tag{3}$$

In Eq.(1) :

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] & [0] \\ [K_{FB}] & [K_{FF}] & [K_{FS}] \\ [0] & [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \\ \{X_S\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{Q\} \end{Bmatrix} \qquad (1)$$

The second equation in Eq.(1) (expressed by Eq.(1-2)) is:

$$[K_{FB}]\{X_B\} + [K_{FF}]\{X_F\} + [K_{FS}]\{X_S\} = \{0\} \qquad (4)$$

Substituting Eq.(3) : $[K_{FF}] = [K_{FFB}] + [K_{FFS}]$ (3)

into Eq.(4) gives:

$$[K_{FB}]\{X_B\} + ([K_{FFB}] + [K_{FFS}])\{X_F\} + [K_{FS}]\{X_S\} = \{0\}$$

$$\therefore \quad ([K_{FB}]\{X_B\} + [K_{FFB}]\{X_F\}) + ([K_{FFS}]\{X_F\} + [K_{FS}]\{X_S\}) = \{0\} \qquad (5)$$

Expressing the first parenthesis in Eq.(5):

$$\therefore \quad ([K_{FB}]\{X_B\} + [K_{FFB}]\{X_F\}) + ([K_{FFS}]\{X_F\} + [K_{FS}]\{X_S) = \{0\} \qquad (5)$$

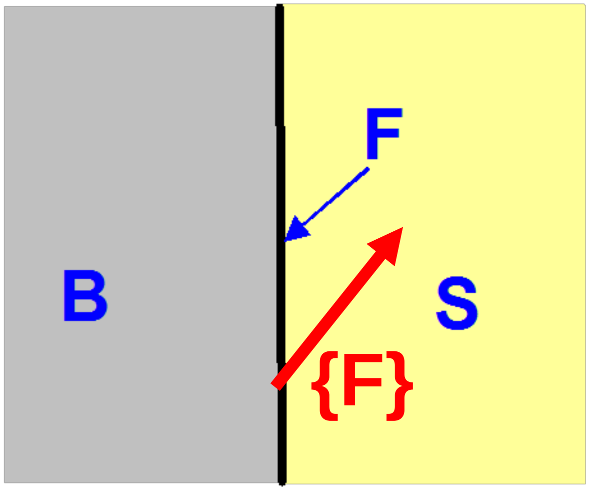
by:

$$[K_{FB}]\{X_B\} + [K_{FFB}]\{X_F\} = -\{F\} \qquad (6)$$

Then, the second parenthesis must be:

$$[K_{FFS}]\{X_F\} + [K_{FS}]\{X_S\} = \{F\} \qquad (7)$$

where, {F} denotes "Interaction Force Vector" between B and S region, and is applied on the interface F.



Joining Eq.(1-1):

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] & [0] \\ [K_{FB}] & [K_{FF}] & [K_{FS}] \\ [0] & [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \\ \{X_S\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{Q\} \end{Bmatrix} \qquad (1)$$

with Eq.(6) :

$$[K_{FB}]\{X_B\} + [K_{FFB}]\{X_F\} = -\{F\} \qquad (6)$$

gives:

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] \\ [K_{FB}] & [K_{FFB}] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ -\{F\} \end{Bmatrix} \qquad (8)$$

Also, joining Eq.(7) :

$$[K_{FFS}]\{X_F\} + [K_{FS}]\{X_S\} = \{F\} \qquad (7)$$

with Eq.(1-3) :

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] & [0] \\ [K_{FB}] & [K_{FF}] & [K_{FS}] \\ [0] & [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \\ \{X_S\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{Q\} \end{Bmatrix} \qquad (1)$$

leads to:

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_F\} \\ \{X_S\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{Q\} \end{Bmatrix} \qquad (9)$$

All entries in the dynamic stiffness matrix of Eq.(8) :

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] \\ [K_{FB}] & [K_{FFB}] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ -\{F\} \end{Bmatrix} \tag{8}$$

can be determined by the material constants of B region,

while that of Eq.(9):

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_F\} \\ \{X_S\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{Q\} \end{Bmatrix} \tag{9}$$

by S region.

You can see that Eq.(1):

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] & [0] \\ [K_{FB}] & [K_{FF}] & [K_{FS}] \\ [0] & [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \\ \{X_S\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{Q\} \end{Bmatrix} \qquad (1)$$

is separated into Eq.(8) :

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] \\ [K_{FB}] & [K_{FFB}] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ -\{F\} \end{Bmatrix} \qquad (8)$$

and Eq.(9):

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_F\} \\ \{X_S\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{Q\} \end{Bmatrix} \qquad (9)$$

Eq.(8) and Eq.(9) are jointed by the interaction force vector {F} on the interface F.

The mechanics for Eq.(8) :

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] \\ [K_{FB}] & [K_{FFB}] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ -\{F\} \end{Bmatrix} \tag{8}$$

can be expressed by Fig.3:

which governs the B region including the F region.

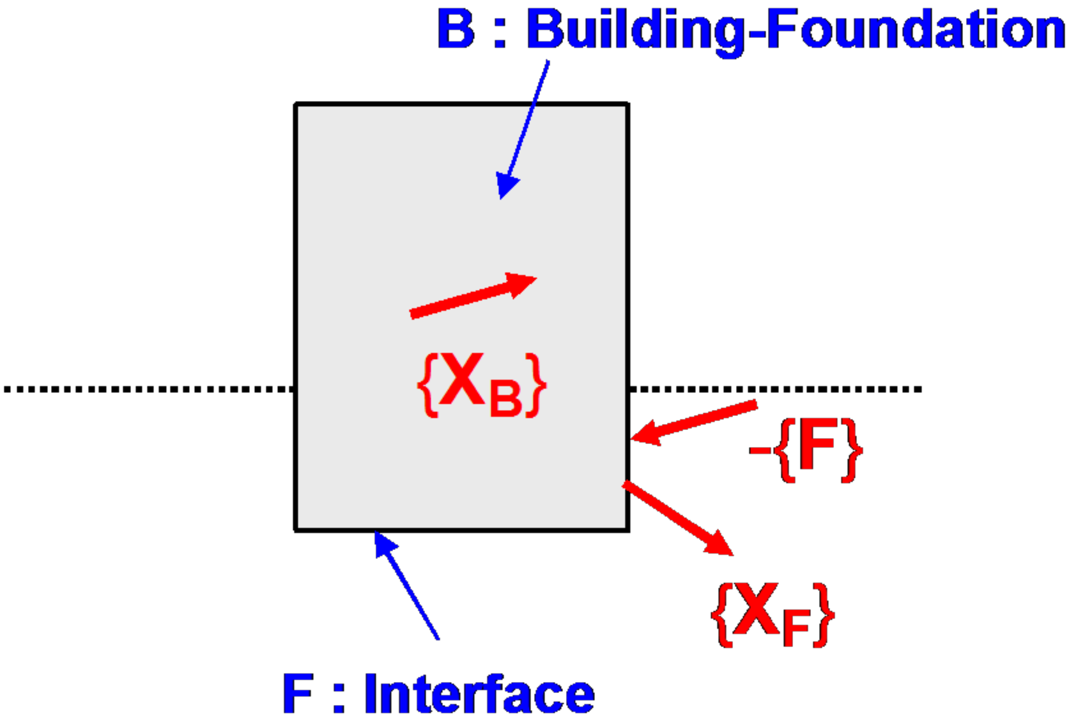


Fig.3 Mechanics of Eq.(8)

And also, the mechanics of Eq.(9) :

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_F\} \\ \{X_S\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{Q\} \end{Bmatrix} \qquad (9)$$

is illustrated by Fig.4:

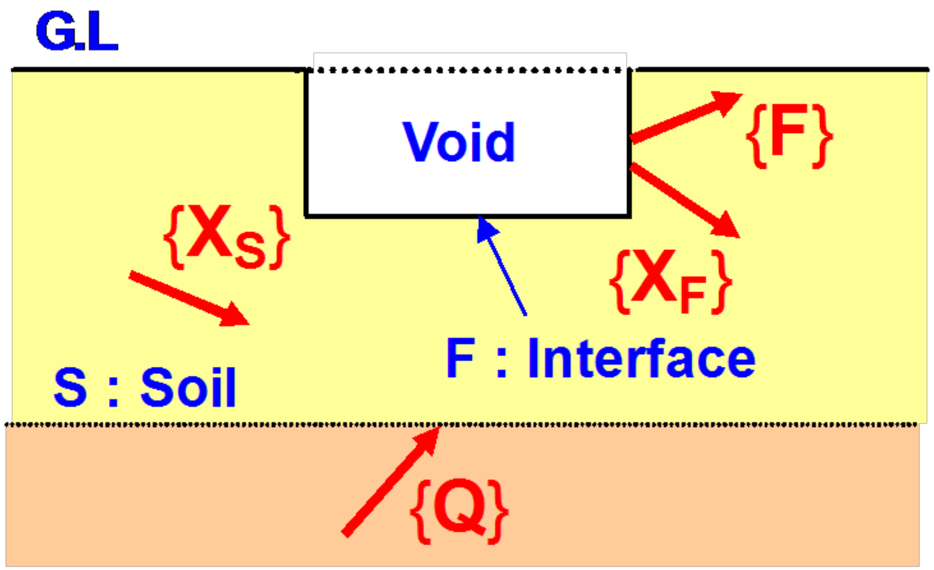


Fig.4 Mechanics of Eq.(9)

The equation governs the S region including the F region with a void.

The void indicates the soil part which is excavated.

In Eq.(9):

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_F\} \\ \{X_S\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{Q\} \end{Bmatrix} \qquad (9)$$

the external force term vector can be decomposed into:

$$\begin{Bmatrix} \{F\} \\ \{Q\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{0\} \end{Bmatrix} + \begin{Bmatrix} \{0\} \\ \{Q\} \end{Bmatrix} \qquad (10)$$

The corresponding displacement vector are defined by:

$$\begin{Bmatrix} \{X_F\} \\ \{X_S\} \end{Bmatrix} = \begin{Bmatrix} \{X_{Ff}\} \\ \{X_{Sf}\} \end{Bmatrix} + \begin{Bmatrix} \{X_{Fq}\} \\ \{X_{Sq}\} \end{Bmatrix} \qquad (11)$$

The first vector $\{\{X_{Ff}\}\ \{\{X_{Sf}\}\}^T$ in the right hand side is due to the interaction force vector $\{\{F\}\ \{0\}\}^T$,

while the second $\{\{X_{Fq}\}\ \{\{X_{Sq}\}\}^T$ to the earthquake disturbance vector $\{\{0\}\ \{Q\}\}^T$.

That is, the first and the second terms of Eq.(11):

$$\begin{Bmatrix} \{X_F\} \\ \{X_S\} \end{Bmatrix} = \begin{Bmatrix} \{X_{Ff}\} \\ \{X_{Sf}\} \end{Bmatrix} + \begin{Bmatrix} \{X_{Fq}\} \\ \{X_{Sq}\} \end{Bmatrix} \tag{11}$$

are the solutions of the equations below:

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_{Ff}\} \\ \{X_{Sf}\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{0\} \end{Bmatrix} \tag{12}$$

and

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_{Fq}\} \\ \{X_{Sq}\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{Q\} \end{Bmatrix} \tag{13}$$

The mechanics of Eq.(12) :

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_{Ff}\} \\ \{X_{Sf}\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{0\} \end{Bmatrix} \tag{12}$$

can be explained by Fig.5:

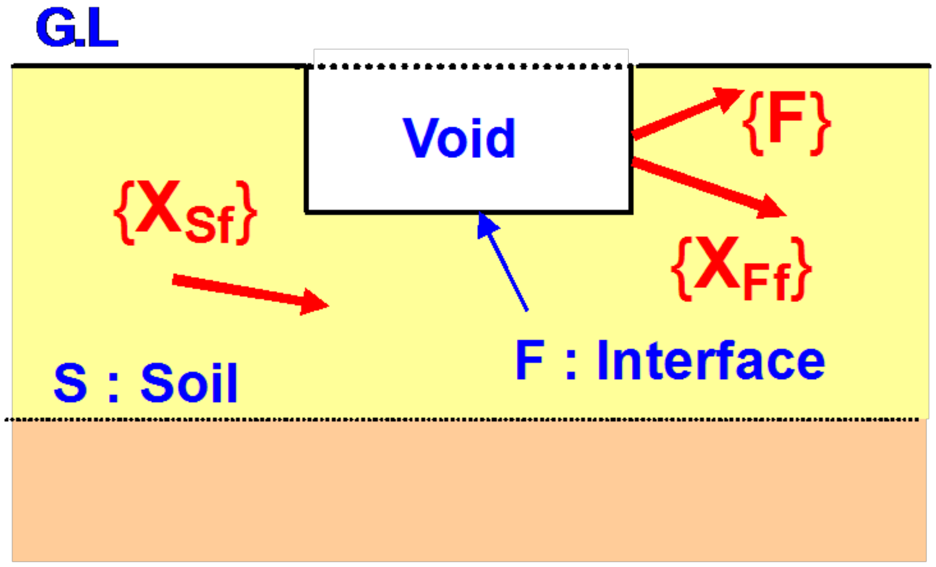


Fig.5 Mechanics of Eq.(12)

And also, the mechanics of Eq.(13):

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_{Fq}\} \\ \{X_{Sq}\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{Q\} \end{Bmatrix} \tag{13}$$

can be explained by Fig.6.

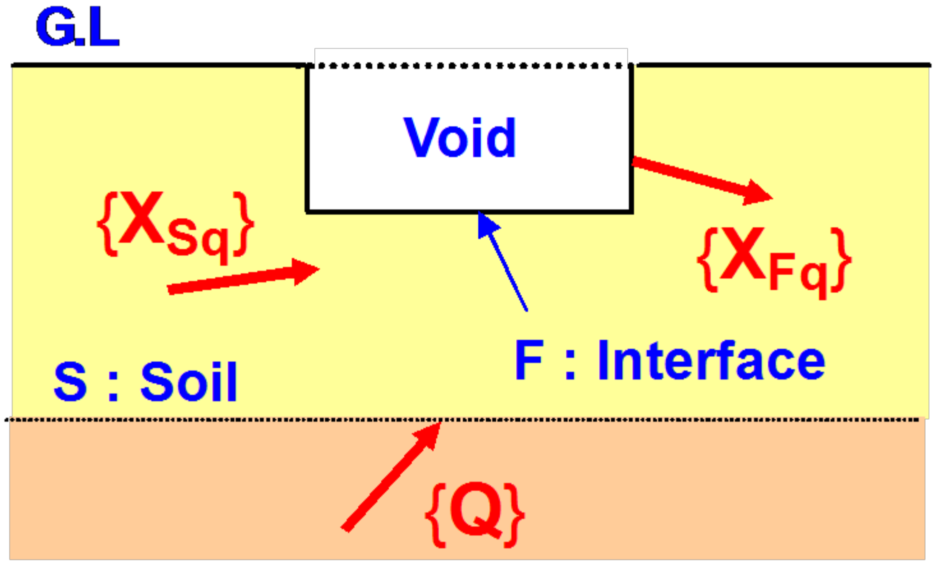


Fig.6 Mechanics of Eq.(13)

In Eq.(12):

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_{Ff}\} \\ \{X_{Sf}\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{0\} \end{Bmatrix} \tag{12}$$

Eq.(12-2) is:

$$[K_{SF}]\{X_{Ff}\} + [K_{SS}]\{X_{Sf}\} = \{0\} \tag{12-2}$$

and the displacement vector $\{X_{Sf}\}$ is obtained as:

$$\{X_{Sf}\} = -[K_{SS}]^{-1}[K_{SF}]\{X_{Ff}\} \tag{14}$$

Substituting Eq.(14) into Eq.(12-1) leads to:

$$\begin{aligned} &[K_{FFS}]\{X_{Ff}\} + [K_{FS}]\{X_{Sf}\} \\ &= [K_{FFS}]\{X_{Ff}\} - [K_{FS}][K_{SS}]^{-1}[K_{SF}]\{X_{Ff}\} \\ &= ([K_{FFS}] - [K_{FS}][K_{SS}]^{-1}[K_{SF}])\{X_{Ff}\} = \{F\} \end{aligned} \tag{15}$$

Because $\{X_{Ff}\}$ is the displacement vector and $\{F\}$ is the force vector in Eq.(15):

$$([K_{FFS}]-[K_{FS}][K_{SS}]^{-1}[K_{SF}])\{X_{Ff}\}=\{F\} \qquad (15)$$

the term in the parenthesis in the left hand side refers to the stiffness matrix.

Putting:

$$[K_F]=[K_{FFS}]-[K_{FS}][K_{SS}]^{-1}[K_{SF}] \qquad (16)$$

Then, Eq.(15) can be expressed by:

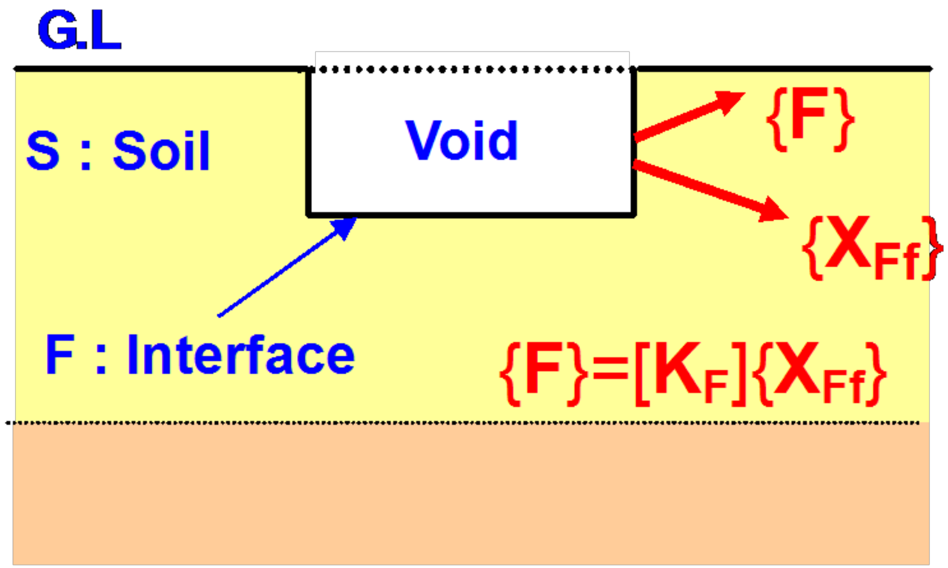
$$[K_F]\{X_{Ff}\}=\{F\} \qquad (17)$$

The matrix $[K_F]$ in Eq.(17):

$$[K_F]\{X_{Ff}\} = \{F\} \qquad (17)$$

indicates the dynamic stiffness matrix of the soil with the void which is evaluated at the interface F.

The mechanics of Eq.(17) is illustrated by the figure:



The matrix $[K_F]$ is called "Dynamic Impedance matrix of soil at the interface F".

Because Eq.(11-1) in Eq.(11):

$$\begin{Bmatrix} \{X_F\} \\ \{X_S\} \end{Bmatrix} = \begin{Bmatrix} \{X_{Ff}\} \\ \{X_{Sf}\} \end{Bmatrix} + \begin{Bmatrix} \{X_{Fq}\} \\ \{X_{Sq}\} \end{Bmatrix} \tag{11}$$

is:

$$\{X_F\} = \{X_{Ff}\} + \{X_{Fq}\} \tag{18}$$

the interaction force vector {F} of Eq.(17) :

$$[K_F]\{X_{Ff}\} = \{F\} \tag{17}$$

can be expressed by:

$$\{F\} = [K_F](\{X_F\} - \{X_{Fq}\}) \tag{19}$$

Substituting Eq.(19):

$$\{F\} = [K_F](\{X_F\} - \{X_{Fq}\}) \qquad (19)$$

into Eq.(8) :

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] \\ [K_{FB}] & [K_{FFB}] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ -\{F\} \end{Bmatrix} \qquad (8)$$

leads to:

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] \\ [K_{FB}] & [K_{FFB}] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ -[K_F](\{X_F\} - \{X_{Fq}\}) \end{Bmatrix} \qquad (20)$$

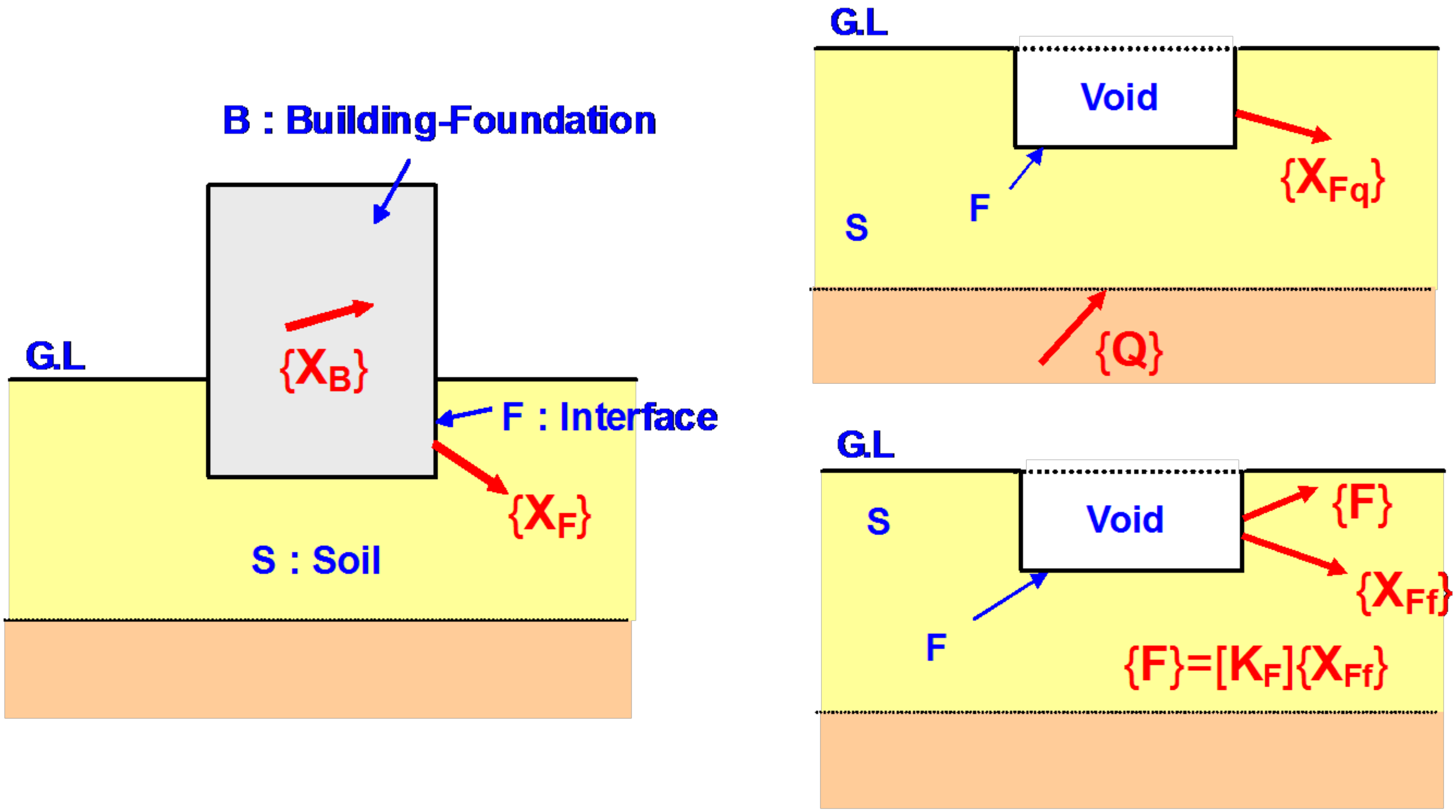
And transforming to :

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] \\ [K_{FB}] & [K_{FFB}] + [K_F] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ [K_F]\{X_{Fq}\} \end{Bmatrix} \qquad (21)$$

The term $[K_F]\{X_{Fq}\}$ is called "Driving Force Vector".

There are a few textbooks on the soil structure interaction: in which the explanation of SSI begins with Eq.(21):

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] \\ [K_{FB}] & [K_{FFB}]+[K_F] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ [K_F]\{X_{Fq}\}) \end{Bmatrix} \tag{21}$$



Getting back to Eq.(13):

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_{Fq}\} \\ \{X_{Sq}\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{Q\} \end{Bmatrix} \tag{13}$$

The dynamics of Eq.(13) can be explained by Fig.7:

When the soil has the void, the stresses along the F must disappear.

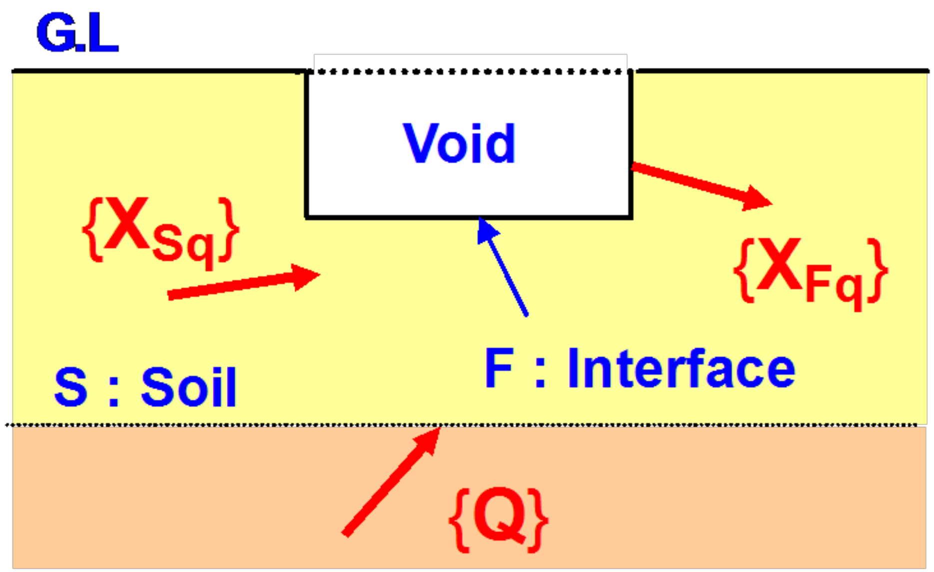


Fig.7 Soil with the Void

The displacement vectors $\{\{X_{Fq}\}\ \{X_{Sq}\}\}^T$ are the earthquake response displacements of the soil with the void,

and it is slightly difficult to compute them.

Consider the soil before the excavation as shown in Fig.8:

where the void is occupied with the soil.

The soil without the void becomes a layered soil media without the geometrical irregularity.

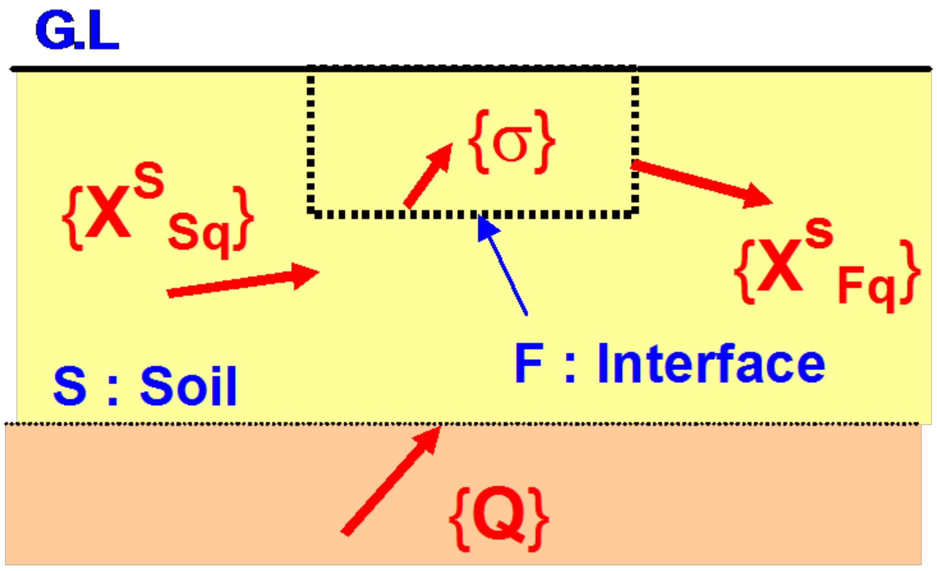


Fig.8 Soil before Excavation

When the earthquake disturbance is the incident SH wave, the earthquake response analysis of this type of the soil can be done easily by the one dimensional shear wave propagation theory (a computer code "SHAKE").

After the earthquake response analysis of the soil shown in Fig.8 without a void, the displacement vector $\{\{X^s{}_{Fq}\}\{X^s{}_{Sq}\}\}^T$, and the stress vector $\{\sigma\}$ on the F can be obtained.

Appling the stress vector $\{\sigma\}$ in the reverse direction on the interface F as shown in Fig.9:

the displacement vector $\{\{X^{\sigma}_{Fq}\}\ \{X^{\sigma}_{Sq}\}\}^T$ are caused.

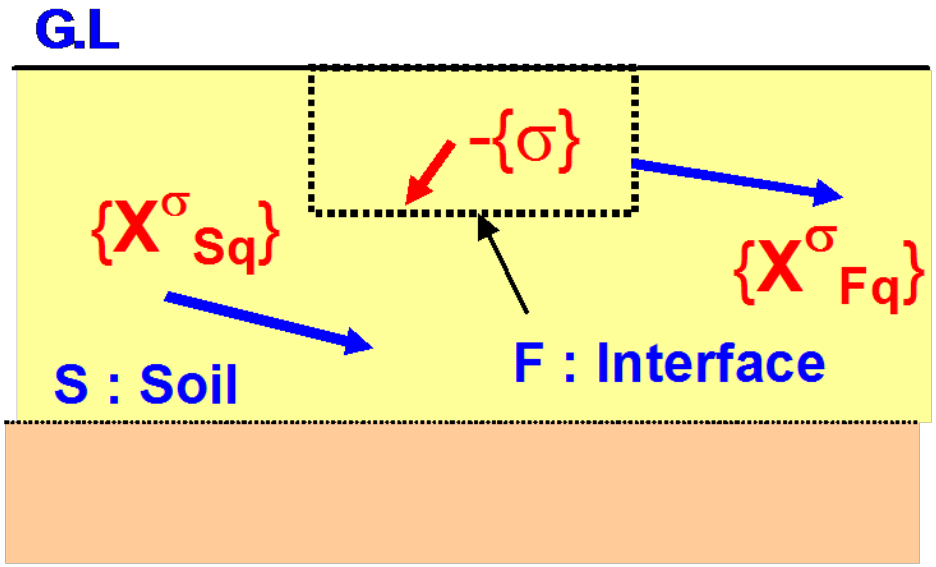


Fig.9 Counter stress $-\{\sigma\}$ application

The numerical condition of Fig.7:
can be decomposed into that of Fig.8 and Fig.9:

and sum of Fig.8 and Fig.9 satisfy the disappearance of the stress $\{\sigma\}$ along the F.

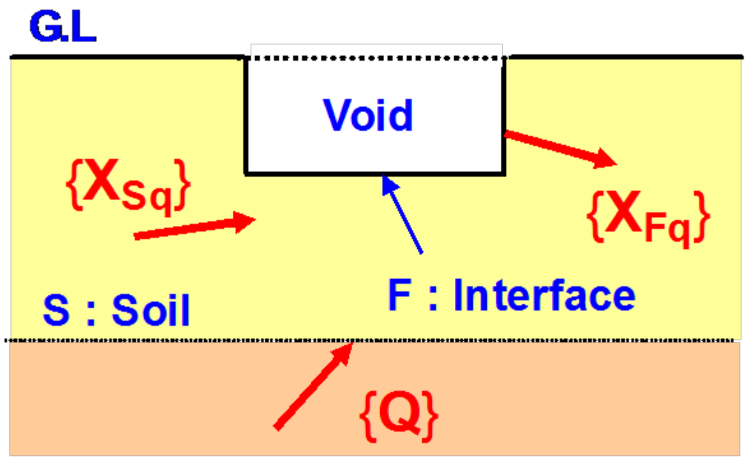


Fig.7　Soil with the Void

$$\{X_{Fq}\}=\{X^{S}_{Fq}\}+\{X^{\sigma}_{Fq}\}$$

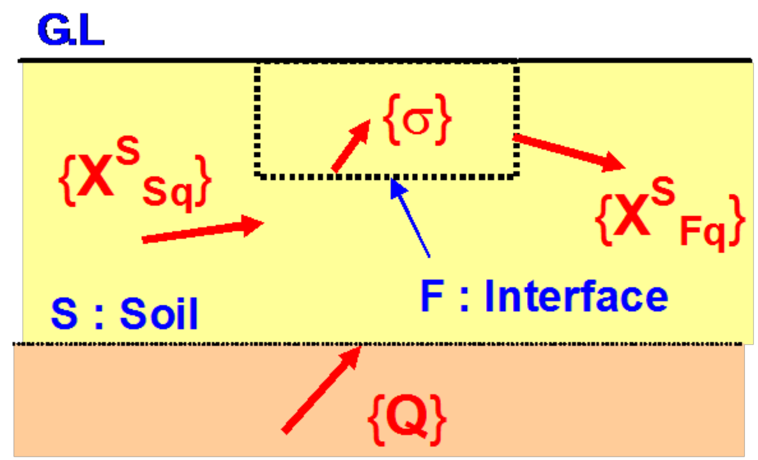


Fig.8　Soil before Excavation

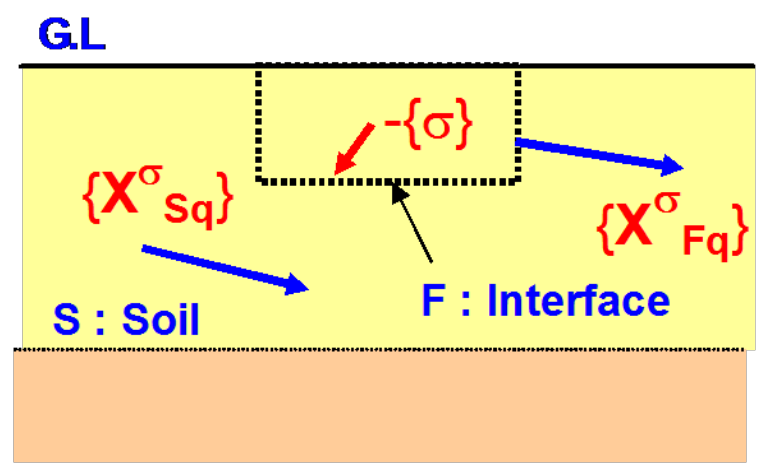


Fig.9　Counter stress $-\{\sigma\}$ application

The displacement vectors $\{\{X_{Fq}\}\ \{X_{Sq}\}\}^T$ are the earthquake response of the soil with the void,

$$\begin{Bmatrix} \{X_{Fq}\} \\ \{X_{Sq}\} \end{Bmatrix} = \begin{Bmatrix} \{X^S_{Fq}\} \\ \{X^S_{Sq}\} \end{Bmatrix} + \begin{Bmatrix} \{X^\sigma_{Fq}\} \\ \{X^\sigma_{Sq}\} \end{Bmatrix} \tag{22}$$

which can be expressed by the sum of $\{\{X^s{}_{Fq}\}\ \{X^s{}_{Sq}\}\}^T$ and $\{\{X^\sigma{}_{Fq}\}\ \{X^\sigma{}_{Sq}\}\}^T$.

In Eq.(22):

$$\begin{Bmatrix} \{X_{Fq}\} \\ \{X_{Sq}\} \end{Bmatrix} = \begin{Bmatrix} \{X^S_{Fq}\} \\ \{X^S_{Sq}\} \end{Bmatrix} + \begin{Bmatrix} \{X^\sigma_{Fq}\} \\ \{X^\sigma_{Sq}\} \end{Bmatrix} \quad (22)$$

The first term in the right hand side corresponds to the earthquake response of the soil without a void,

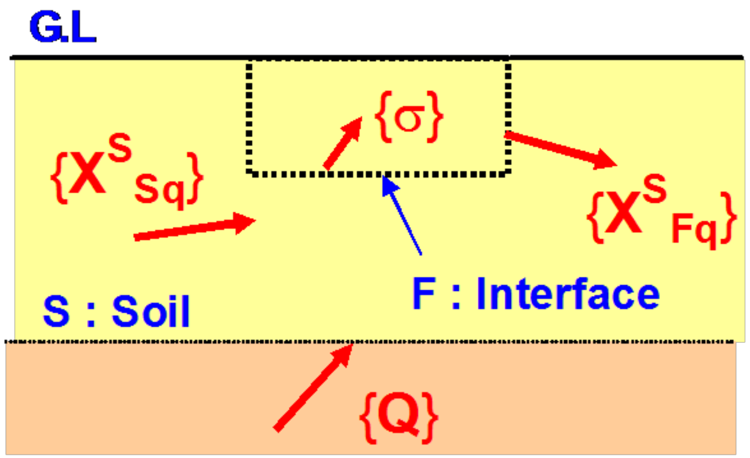


Fig.8 Soil before Excavation

while the second to the response of the soil without the void due to the counter stress -{σ} on the interface F.

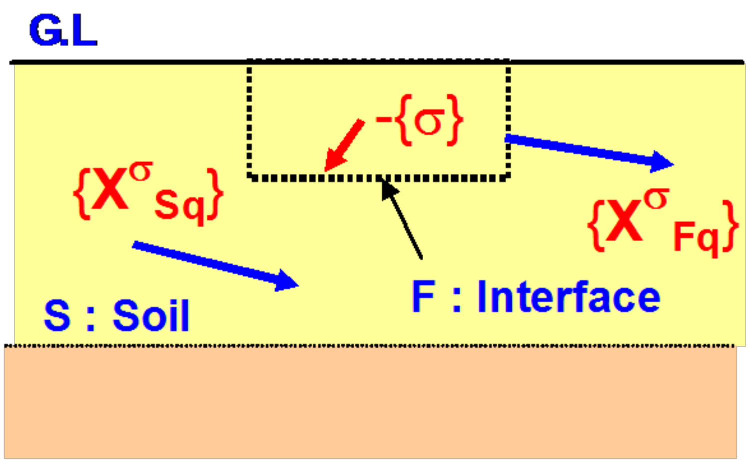


Fig.9 Counter stress -{σ} application

Expressing the dynamic stiffness of the soil within the void by $[K_V]$, which will be excavated as shown in Fig.10:

Then, the equation of motion for Fig.8 condition:

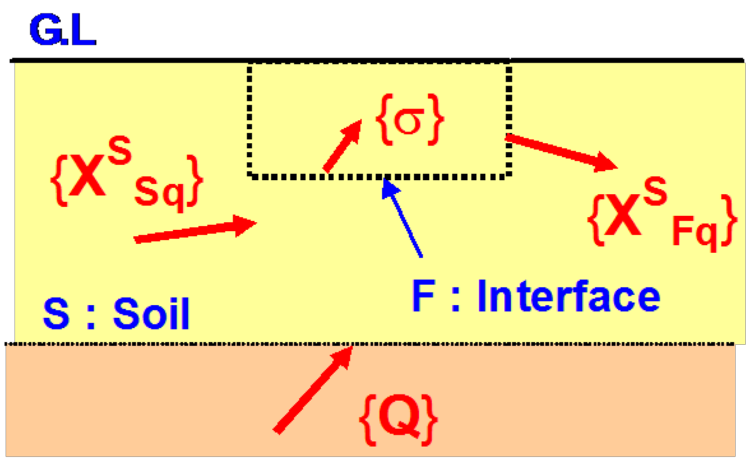


Fig.8 Soil before Excavation

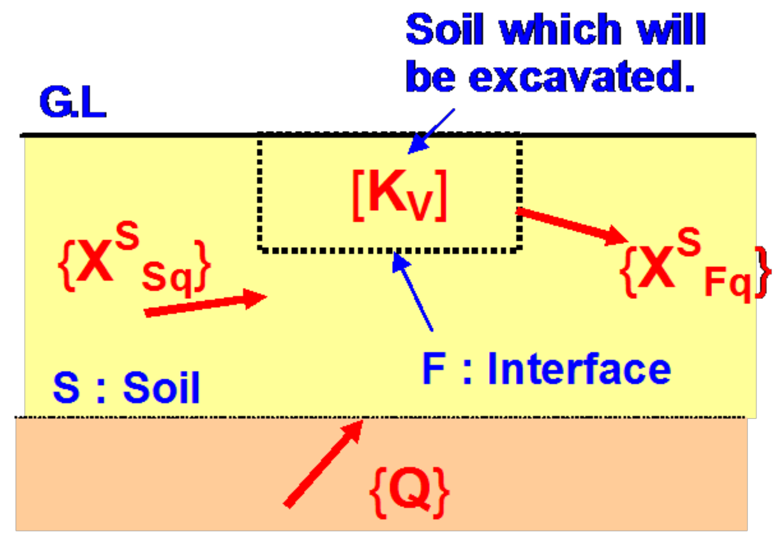


Fig.10 Soil before Excavation

can be expressed by:

$$\begin{bmatrix} [K_{FFS}]+[K_V] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X^S_{Fq}\} \\ \{X^S_{Sq}\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{Q\} \end{Bmatrix} \qquad (23)$$

In Eq.(23):

$$\begin{bmatrix} [K_{FFS}]+[K_V] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_{Fq}^S\} \\ \{X_{Sq}^S\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{Q\} \end{Bmatrix} \qquad (23)$$

The first equation becomes:

$$([K_{FFS}]+[K_V])\{X_{Fq}^S\}+[K_{FS}]\{X_{Sq}^S\}$$

$$=[K_V]\{X_{Fq}^S\}+([K_{FFS}]\{X_{Fq}^S\}+[K_{FS}]\{X_{Sq}^S\})=\{0\} \qquad (24)$$

As shown in Fig.11:

Because $[K_V]$ is the dynamic stiffness matrix of the soil part which will be excavated

and $\{X^S{}_{Fq}\}$ is the displacement vector of the soil without the void,

$[K_V]$ times $\{X^S{}_{Fq}\}$ coincides with the stress vector $\{\sigma\}$ on the F.

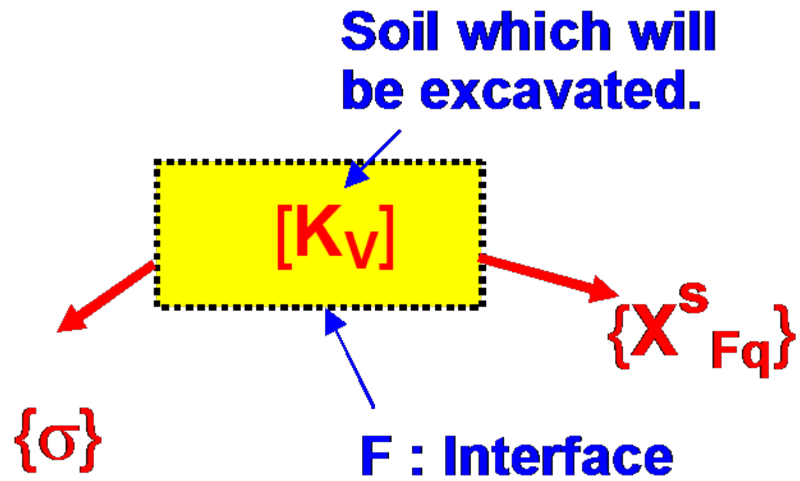


Fig.11 $\{\sigma\}=[K_V]\{X^S{}_{Fq}\}$

That is: $\{\sigma\} = [K_V]\{X^S_{Fq}\}$ (25)

Substituting Eq.(25) into Eq.(24):

$$[K_V]\{X^S_{Fq}\} + ([K_{FFS}]\{X^S_{Fq}\} + [K_{FS}]\{X^S_{Sq}\}) = \{0\} \quad (24)$$

Then,

$$[K_{FFS}]\{X^S_{Fq}\} + [K_{FS}]\{X^S_{Sq}\} = -\{\sigma\} \qquad (26)$$

Joining Eq.(26):

$$[K_{FFS}]\{X_{Fq}^{S}\}+[K_{FS}]\{X_{Sq}^{S}\}=-\{\sigma\} \tag{26}$$

with the second equation:

$$[K_{SF}]\{X_{Fq}^{S}\}+[K_{SS}]\{X_{Sq}^{S}\}=\{Q\} \tag{23-1}$$

in Eq.(23):

$$\begin{bmatrix} [K_{FFS}]+[K_{V}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_{Fq}^{S}\} \\ \{X_{Sq}^{S}\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{Q\} \end{Bmatrix} \tag{23}$$

leads to:

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_{Fq}^{S}\} \\ \{X_{Sq}^{S}\} \end{Bmatrix} = \begin{Bmatrix} -\{\sigma\} \\ \{Q\} \end{Bmatrix} \tag{27}$$

From Eq.(22):

$$\begin{Bmatrix} \{X_{Fq}\} \\ \{X_{Sq}\} \end{Bmatrix} = \begin{Bmatrix} \{X_{Fq}^{S}\} \\ \{X_{Sq}^{S}\} \end{Bmatrix} + \begin{Bmatrix} \{X_{Fq}^{\sigma}\} \\ \{X_{Sq}^{\sigma}\} \end{Bmatrix} \qquad (22)$$

the displacement vector $\{X^S_{Fq}\ X^S_{Sq}\}^T$ becomes:

$$\begin{Bmatrix} \{X_{Fq}^{S}\} \\ \{X_{Sq}^{S}\} \end{Bmatrix} = \begin{Bmatrix} \{X_{Fq}\} \\ \{X_{Sq}\} \end{Bmatrix} - \begin{Bmatrix} \{X_{Fq}^{\sigma}\} \\ \{X_{Sq}^{\sigma}\} \end{Bmatrix} \qquad (28)$$

where,

$\{X^S_{Fq}\ X^S_{Sq}\}^T$: earthquake response of the soil before the excavation

$\{X_{Fq}\ X_{Sq}\}^T$: earthquake response of the soil after the excavation

$\{X^s_{Fq}\ X^s_{Sq}\}^T$: response of the soil before the excavation due to the stress vector $\{\sigma\}$ on the F

Substituting Eq.(28):

$$\begin{Bmatrix} \{X_{Fq}^{S}\} \\ \{X_{Sq}^{S}\} \end{Bmatrix} = \begin{Bmatrix} \{X_{Fq}\} \\ \{X_{Sq}\} \end{Bmatrix} - \begin{Bmatrix} \{X_{Fq}^{\sigma}\} \\ \{X_{Sq}^{\sigma}\} \end{Bmatrix} \qquad (28)$$

into Eq.(27):

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_{Fq}^{S}\} \\ \{X_{Sq}^{S}\} \end{Bmatrix} = \begin{Bmatrix} -\{\sigma\} \\ \{Q\} \end{Bmatrix} \qquad (27)$$

We obtain Eq.(29):

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \left(\begin{Bmatrix} \{X_{Fq}\} \\ \{X_{Sq}\} \end{Bmatrix} - \begin{Bmatrix} \{X_{Fq}^{\sigma}\} \\ \{X_{Sq}^{\sigma}\} \end{Bmatrix} \right) = \begin{Bmatrix} \{0\} \\ \{Q\} \end{Bmatrix} - \begin{Bmatrix} \{\sigma\} \\ \{0\} \end{Bmatrix} \qquad (29)$$

Note: The right hand side of Eq.(27) is decomposed.

Eq.(29):

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \left(\begin{Bmatrix} \{X_{Fq}\} \\ \{X_{Sq}\} \end{Bmatrix} - \begin{Bmatrix} \{X_{Fq}^{\sigma}\} \\ \{X_{Sq}^{\sigma}\} \end{Bmatrix} \right) = \begin{Bmatrix} \{0\} \\ \{Q\} \end{Bmatrix} - \begin{Bmatrix} \{\sigma\} \\ \{0\} \end{Bmatrix} \qquad (29)$$

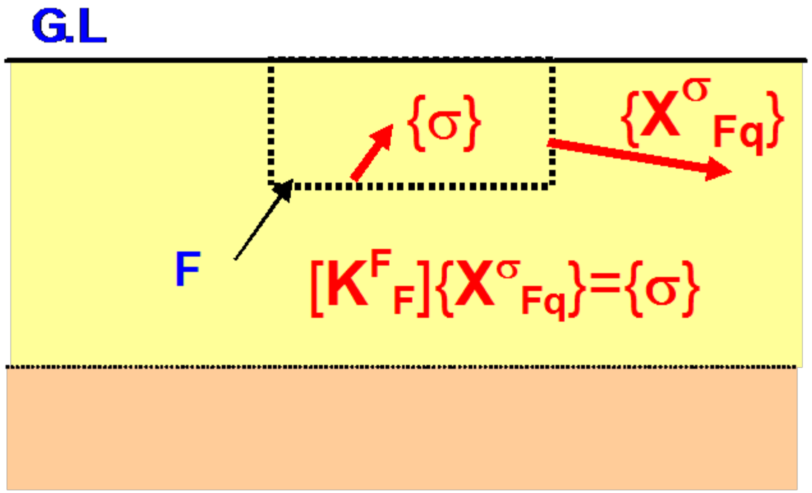
can be decomposed into:

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_{Fq}\} \\ \{X_{Sq}\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{Q\} \end{Bmatrix} \qquad (30)$$

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_{Fq}^{\sigma}\} \\ \{X_{Sq}^{\sigma}\} \end{Bmatrix} = \begin{Bmatrix} \{\sigma\} \\ \{0\} \end{Bmatrix} \qquad (31)$$

Of course, Eq.(30) is the same to Eq.(13).

The matrix $[K^F{}_F]$ indicates the dynamic impedance of the soil without the void as shown in the figure.



And,

$$\{X^\sigma_{Fq}\} = [K^F_F]^{-1}\{\sigma\} \qquad (35)$$

Using Eq.(22-1),

$$\{X_{Fq}\} = \{X^S_{Fq}\} + \{X^\sigma_{Fq}\} = \{X^S_{Fq}\} + [K^F_F]^{-1}\{\sigma\} \qquad (36)$$

Getting back to Eq.(21):

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] \\ [K_{FB}] & [K_{FFB}]+[K_F] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ [K_F]\{X_{Fq}\} \end{Bmatrix} \qquad (21)$$

In Eq.(31):

$$\begin{bmatrix} [K_{FFS}] & [K_{FS}] \\ [K_{SF}] & [K_{SS}] \end{bmatrix} \begin{Bmatrix} \{X_{Fq}^{\sigma}\} \\ \{X_{Sq}^{\sigma}\} \end{Bmatrix} = \begin{Bmatrix} \{\sigma\} \\ \{0\} \end{Bmatrix} \tag{31}$$

the second equation is:

$$[K_{SF}]\{X_{Fq}^{\sigma}\} + [K_{SS}]\{X_{Sq}^{\sigma}\} = \{0\}$$

that is:

$$\therefore \quad \{X_{Sq}^{\sigma}\} = -[K_{SS}]^{-1}[K_{SF}]\{X_{Fq}^{\sigma}\} \tag{32}$$

and the first equation is:

$$[K_{FFS}]\{X_{Fq}^{\sigma}\} + [K_{FS}]\{X_{Sq}^{\sigma}\}$$

$$= ([K_{FFS}] - [K_{FS}][K_{SS}]^{-1}[K_{SF}])\{X_{Fq}^{\sigma}\} = \{\sigma\} \tag{33}$$

As same as Eq.(17) :

$$[K_F]\{X_{Ff}\} = \{F\} \qquad (17)$$

which denotes the dynamic impedance of the soil with the void on the F,

Eq.(33):

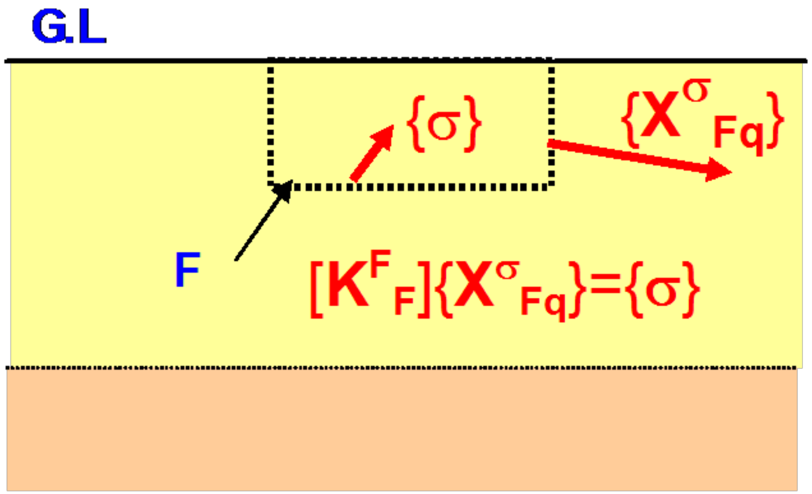
$$([K_{FFS}] - [K_{FS}][K_{SS}]^{-1}[K_{SF}])\{X^{\sigma}_{Fq}\} = \{\sigma\} \qquad (33)$$

can be represented by:

$$[K^F_F]\{X^{\sigma}_{Fq}\} = \{\sigma\} \qquad (34)$$

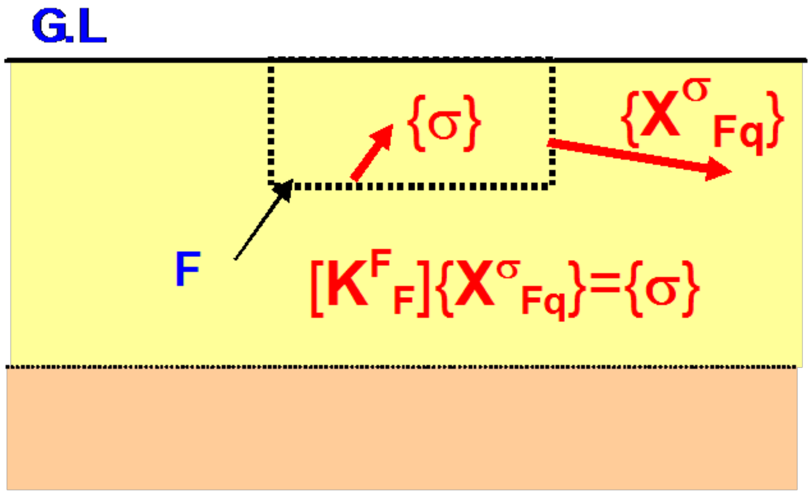
where,

$$[K^F_F] = [K_{FFS}] - [K_{FS}][K_{SS}]^{-1}[K_{SF}]$$



The mechanics of Eq.(34) is shown in the figure.

The matrix $[K^F_F]$ indicates the dynamic impedance of the soil without the void as shown in the figure.



And,

$$\{X^\sigma_{Fq}\} = [K^F_F]^{-1}\{\sigma\} \qquad (35)$$

Using the first equation in Eq.(22):

$$\begin{Bmatrix} \{X_{Fq}\} \\ \{X_{Sq}\} \end{Bmatrix} = \begin{Bmatrix} \{X^S_{Fq}\} \\ \{X^S_{Sq}\} \end{Bmatrix} + \begin{Bmatrix} \{X^\sigma_{Fq}\} \\ \{X^\sigma_{Sq}\} \end{Bmatrix} \qquad (22)$$

The displacement vector $\{X_{Fq}\}$ can be expressed by:

$$\{X_{Fq}\} = \{X^S_{Fq}\} + \{X^\sigma_{Fq}\} = \{X^S_{Fq}\} + [K^F_F]^{-1}\{\sigma\} \qquad (36)$$

Getting back to Eq.(21):

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] \\ [K_{FB}] & [K_{FFB}]+[K_F] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ [K_F]\{X_{Fq}\} \end{Bmatrix} \quad (21)$$

Putting Eq.(36):

$$\{X_{Fq}\} = \{X^S_{Fq}\} + \{X^\sigma_{Fq}\} = \{X^S_{Fq}\} + [K^F_F]^{-1}\{\sigma\} \quad (36)$$

into Eq.(21),

Then, Eq.(21) is transformed into:

$$\begin{bmatrix} [K_{BB}] & [K_{BF}] \\ [K_{FB}] & [K_{FFB}]+[K_F] \end{bmatrix} \begin{Bmatrix} \{X_B\} \\ \{X_F\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ [K_F](\{X^F_{Fq}\} + [K^F_F]^{-1}\{\sigma\}) \end{Bmatrix}$$

(37)

$[K_F]$ is the dynamic impedance function of the soil with the void,

The calculation of $[K_F]$ is little bit difficult.

However,

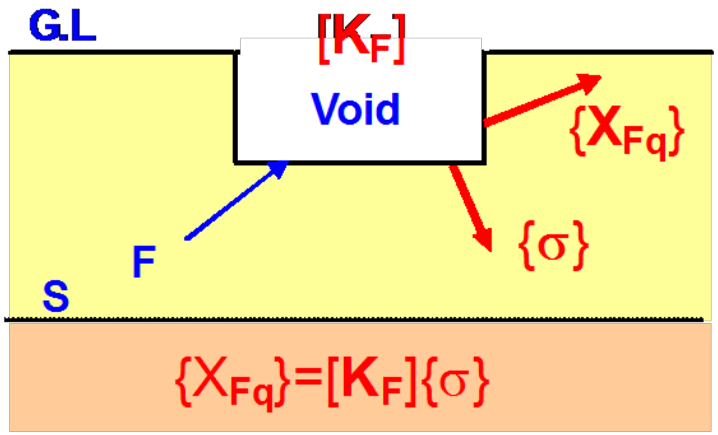
$[K_F]$ is $[K^F_F]$ minus $[K_V]$:

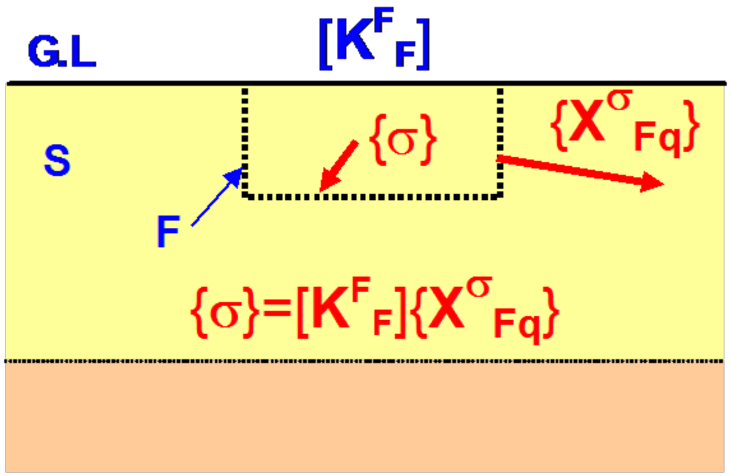
$$[K_F] = [K_F^F] - [K_V]$$

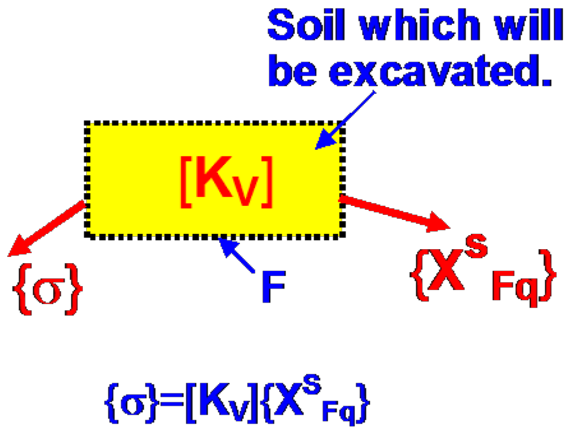
Where,

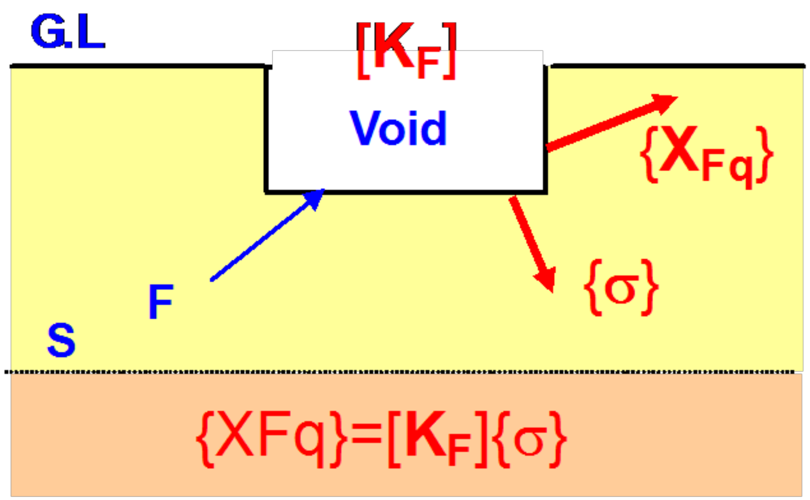
$[K^F_F]$ is the dynamic stiffness matrix of the soil without the void

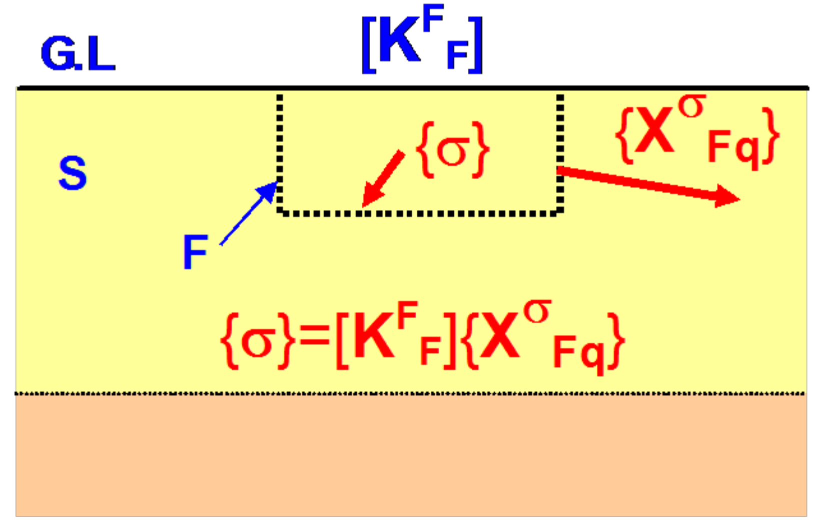
and $[K_V]$ is the dynamic stiffness of the soil part which will be excavated.



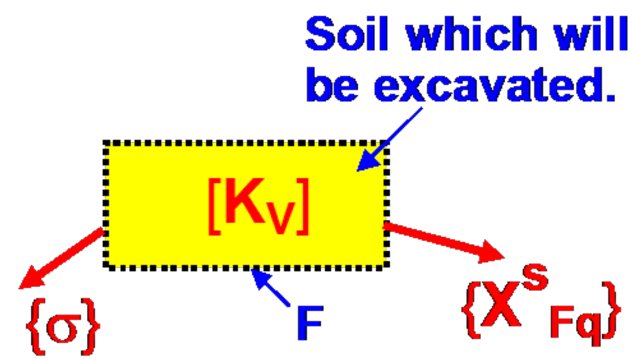








$$[K_F]=[K^F{}_F]-[K_V]$$



$$\{\sigma\}=[K_V]\{X^S{}_{Fq}\}$$

E N D

Substructure Method

For SSI Analysis

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