Dynamic Soil Structure Interaction

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Chapter 5 :

Basis for Calculation of Dynamic Impedance Function

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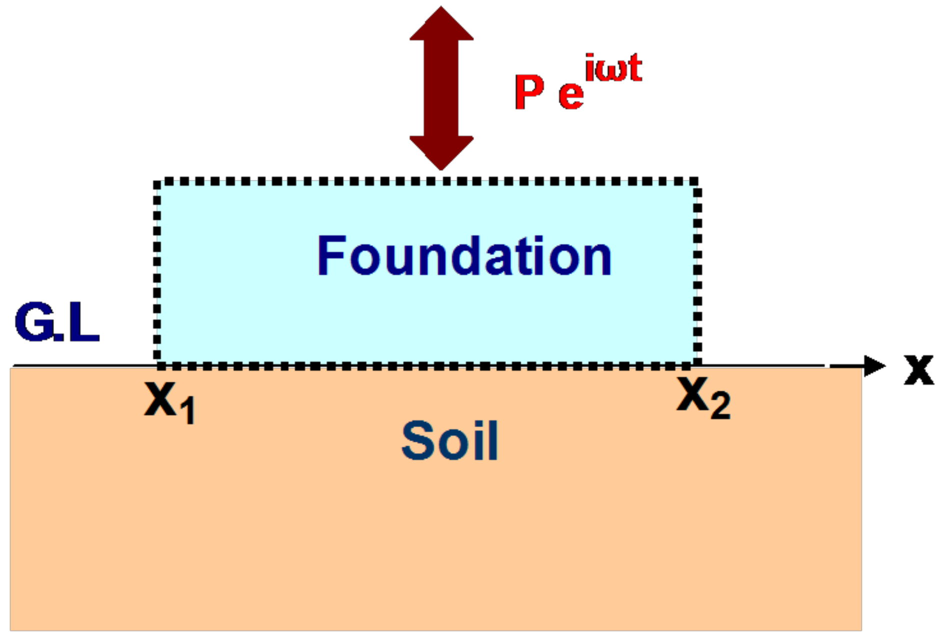
Vertical Dynamic Impedance Function for Surface Foundation

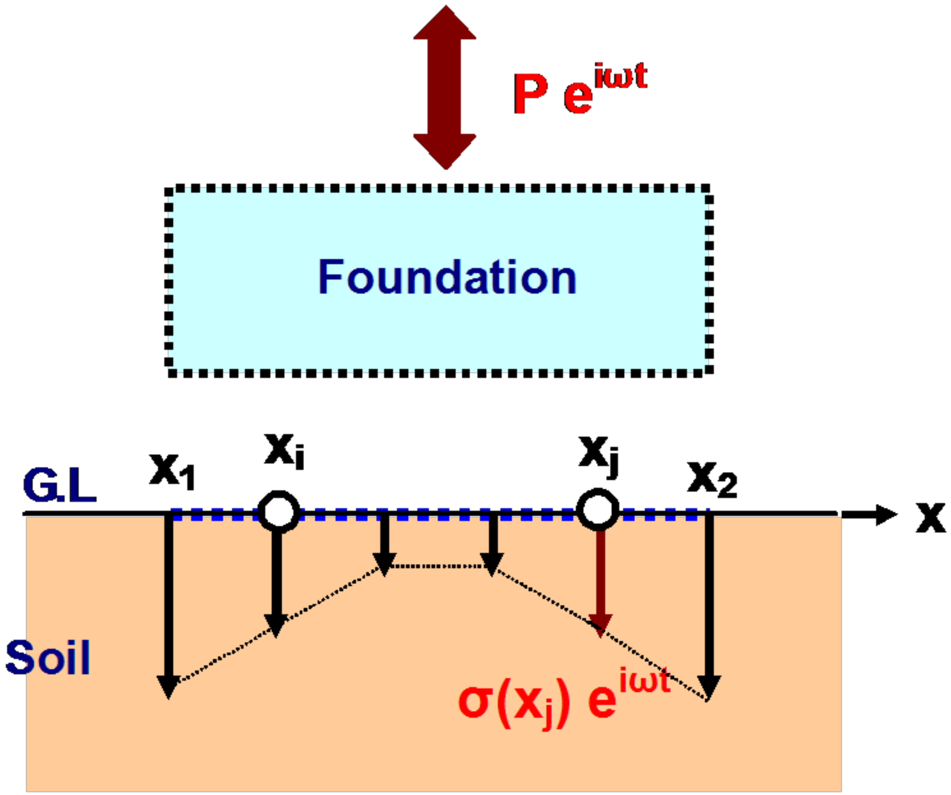
Let us consider how to calculate the dynamic impedance function.

For a simple explanation, two dimensional rigid foundation resting on the soil surface is employed, which is subject to a vertical load $P\,e^{i\omega t}$.



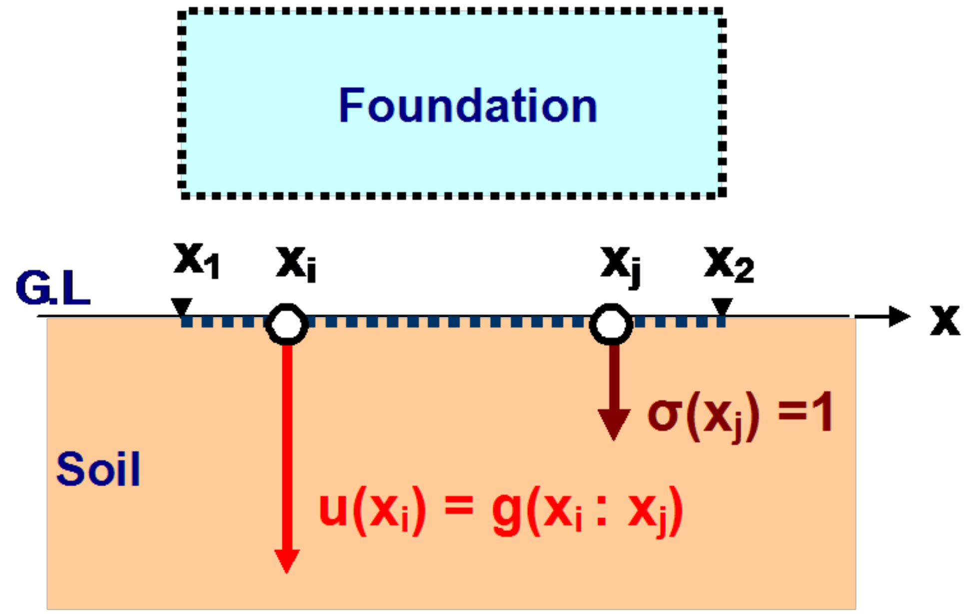
Surface Foundation subjected to a Dynamic Vertical Load

During a vertical excitation $Pe^{i\omega t}$, a vertical contact earth-pressure $\sigma(x_j)e^{i\omega t}$ is caused along the contact area($x_1<x_j<x_2$) between the foundation bottom and the soil surface.



Contact Earth-Pressure

Consider a displacement $u(x_i)$ at a point x_i due to a earth-pressure $\sigma(x_j)$ at a point x_j.



Green's Function

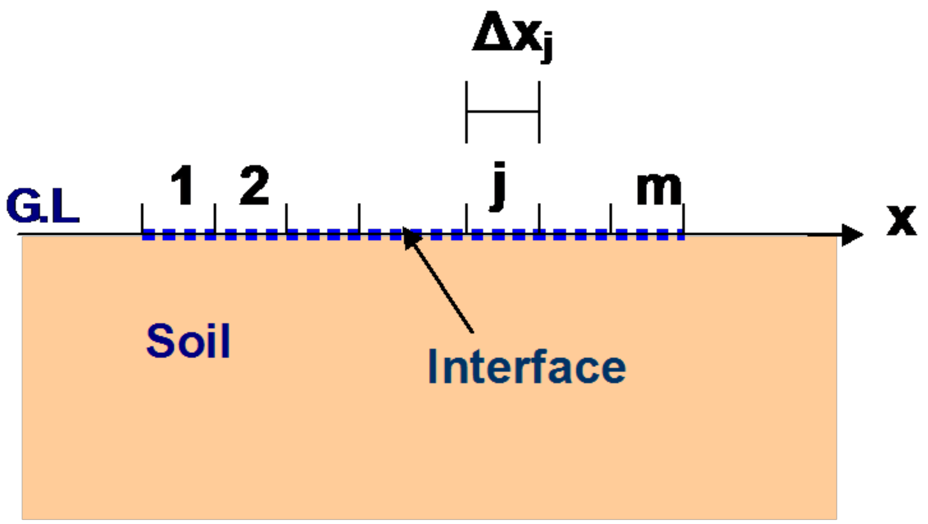
When $\sigma(x_j)=1$, a displacement $u(x_i)$ at a point x_i is called "Green's Function", and expressed by:

$$u(x_i) = g(x_i : x_j) \qquad (1)$$

The displacement at x_i due to whole contact earth pressure $\sigma(x_j)$, $x_1<x_j<x_2$ is given by:

$$u(x_i) = \int_{x_1}^{x_2} g(x_i : x_j)\sigma(x_j)\, dx_j \qquad (2)$$

Partition the interface between foundation and soil into m sub-regions.



Partition of the Interface

Then, Eq.(2) becomes,

$$u(x_i) \approx \sum_{j=1}^{m} g(x_i : x_j)\sigma(x_j)\Delta x_j \qquad (3)$$

Put $\sigma(x_j)\Delta x_j = f_j$, Eq.(3):

$$u(x_i) \approx \sum_{j=1}^{m} g(x_i : x_j)\sigma(x_j)\Delta x_j \qquad (3)$$

transformed to Eq.(4):

$$u(x_i) \approx \sum_{j=1}^{m} g(x_i : x_j)\, f_j \qquad (4)$$

Arrange Eq.(4) from $i=1$ to m, Eq.(4) is expressed in a matrix form as:

$$\{u\}=[G]\{ f \} \qquad (5)$$

$$\{\mathbf{u}\}=[\mathbf{G}]\{\,\mathbf{f}\,\} \tag{5}$$

where,

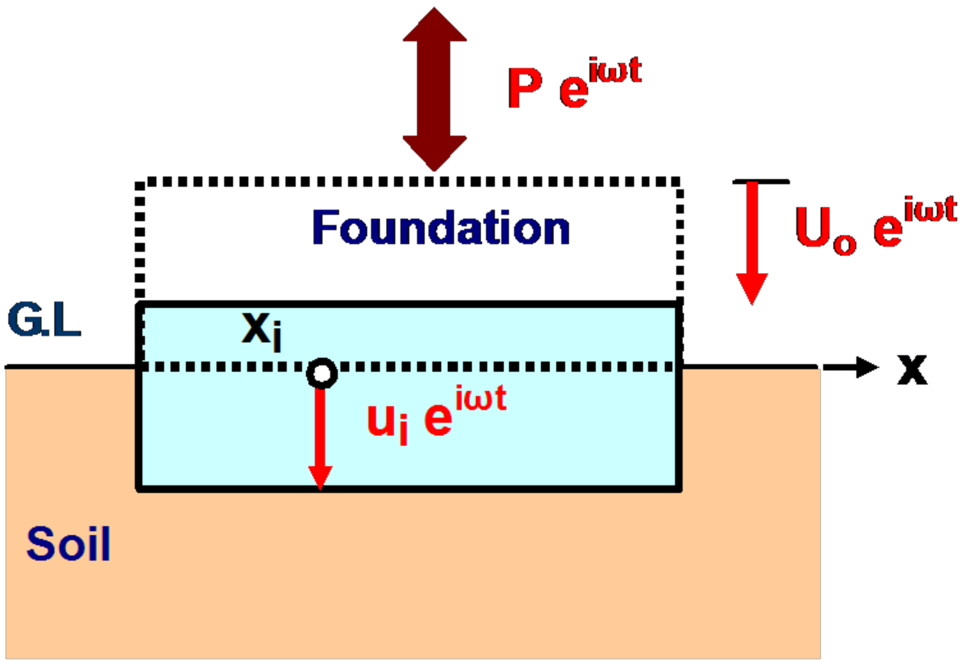
$$\{\mathbf{u}\}=\{u(x_1),\ u(x_2),\ \ldots\ldots,\ u(x_m)\}^T \tag{6-1}$$

$$\{\,\mathbf{f}\,\}=\{\,f_1,\ \ f_2,\ldots\ldots,\ \ f_m\}^T \tag{6-2}$$

$$[\mathrm{G}]=\begin{bmatrix} g(x_1:x_1) & g(x_1:x_2) & \cdot & \cdot & g(x_1:x_m) \\ g(x_2:x_1) & g(x_2:x_2) & \cdot & \cdot & g(x_2:x_m) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ g(x_m:x_1) & g(x_m:x_2) & \cdot & \cdot & g(x_m:x_m) \end{bmatrix} \tag{6-3}$$

Under rigid foundation condition, displacements {u} at the interface can be expressed by representative displacement U_0 of the foundation.

$$\{\mathbf{u}\}=\{1, 1, \ldots\ldots, 1\}^T \mathbf{U_0} \qquad (7)$$



Eq.(7) is rewritten as:

$$\{\mathbf{u}\}=\{R\} \mathbf{U_0} \qquad (8)$$

Where,

$$\{\mathbf{R}\}=\{1, 1, \ldots\ldots, 1\}^T \qquad (9)$$

$$\{\mathbf{u}\} = \{\mathbf{R}\}\, \mathbf{U_0} \qquad (8)$$

$$\{\mathbf{R}\} = \{1, 1, \ldots\ldots, 1\}^T \qquad (9)$$

{R} is called Restraint Vector, which relates the representative displacement U_0 of the foundation to the displacement vector {u} at the interface.

In multi-degrees of freedom cases, the restraint vector {R} becomes Restraint Matrix [R].

The applied force P coincides with combined contact earth pressure (sum of contact earth-pressure) .

$$\mathbf{F}(=\mathbf{P}) = f_1 + f_2 + \ldots\ldots + f_m = \{1, 1, \ldots\ldots, 1\}\{\mathbf{f}\}$$

$$=\{\mathbf{R}\}^T\{\mathbf{f}\} \qquad (10)$$

Eq.(5) :

$$\{\mathbf{u}\}=[\mathbf{G}]\{\mathbf{f}\} \qquad (5)$$

gives,

$$\{\mathbf{f}\}=[\mathbf{G}]^{-1}\{\mathbf{u}\} \qquad (11)$$

Substitute Eq.(11):

$$\{\mathbf{f}\}=[\mathbf{G}]^{-1}\{\mathbf{u}\} \qquad (11)$$

into Eq.(10):

$$\mathbf{F}(=\mathbf{P}) = \{\mathbf{R}\}^{T}\{\mathbf{f}\} \qquad (10)$$

then, we obtain Eq.(12):

$$\mathbf{F}(=\mathbf{P}) = \{\mathbf{R}\}^{T}\{\mathbf{f}\}=\{\mathbf{R}\}^{T}[\mathbf{G}]^{-1}\{\mathbf{u}\} \qquad (12)$$

Furthermore, the substitution of Eq.(8) :

$$\{\mathbf{u}\}=\{R\}\,\mathbf{U_0} \qquad (8)$$

into Eq.(12) gives Eq.(13):

$$\mathbf{F}(=\mathbf{P}) = \{\mathbf{R}\}^{\mathbf{T}}[\mathbf{G}]^{-1}\{\mathbf{R}\}\,\mathbf{U_o} \qquad (13)$$

Because Eq.(13):

$$\mathbf{F}(=\mathbf{P}) = \{\mathbf{R}\}^{T}[\mathbf{G}]^{-1}\{\mathbf{R}\}\,\mathbf{U}_{o} \qquad (13)$$

relates the applied force F(=P) to the representative displacement U_0 of the foundation,

the coefficient in a right hand side of Eq.(13) can be denoted by Dynamic Impedance Function K .

That is:

$$\mathbf{F}(=\mathbf{P}) = \mathbf{K}\,\mathbf{U}_{0} \qquad (15)$$

where,

$$\mathbf{K}=\{\mathbf{R}\}^{T}[\mathbf{G}]^{-1}\{\mathbf{R}\} \qquad (16)$$

E N D

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