Dynamic Soil Structure Interaction

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Chapter 3 :

Effect of SSI on

Dynamic Behaviors of Structure

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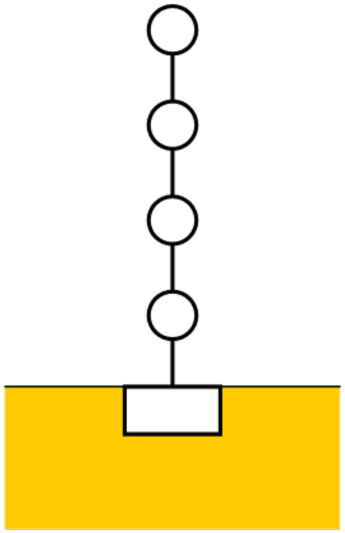
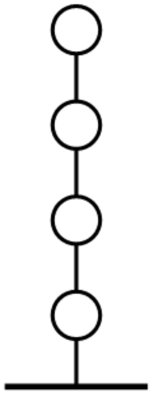
3. Effect of SSI on Dynamic Behaviors of Structure

The effect of Soil-Structure interaction (SSI) influences strongly the dynamic behaviors of the building.

If you will estimate dynamic behaviors of a structure such as the natural frequencies and the damping factors on the assumptions that the basement or the foundation footing is fixed without incorporating the SSI effects, the estimated values will be quite different from these of the actual structure.



SSI System



Fixed Base System

Here, we examine the effects of SSI on dynamic behaviors of a structure.

3.1 Numerical Condition of SSI System

Numerical conditions for the soil, superstructure and foundation described below are employed for examinations of SSI.

(1) Soil: Two layered strata shown in the figure:

G.L

Surface Stratum

$V_{S2}=100\mathrm{m/s}$ $\gamma_2=1.7\mathrm{tf/m^3}$

$\nu_2=0.45$ $h_2=0.05$

20m

Bearing Stratum

$V_{S1}=400\mathrm{m/s}$ $\gamma_1=1.9\mathrm{tf/m^3}$

$\nu_1=0.40$ $h_1=0.01$

(2) Superstructure and foundation

(a) Reinforced concrete structure

(b) Floor height: 3.0m

(c) Weight of each floor and foundation:1,080t and 2,700t

(d) Rigidity: Rigidities of each floor are identical, and its value is estimated from the first natural period T_1 as:

$T_1=0.025n$, n : number of floors

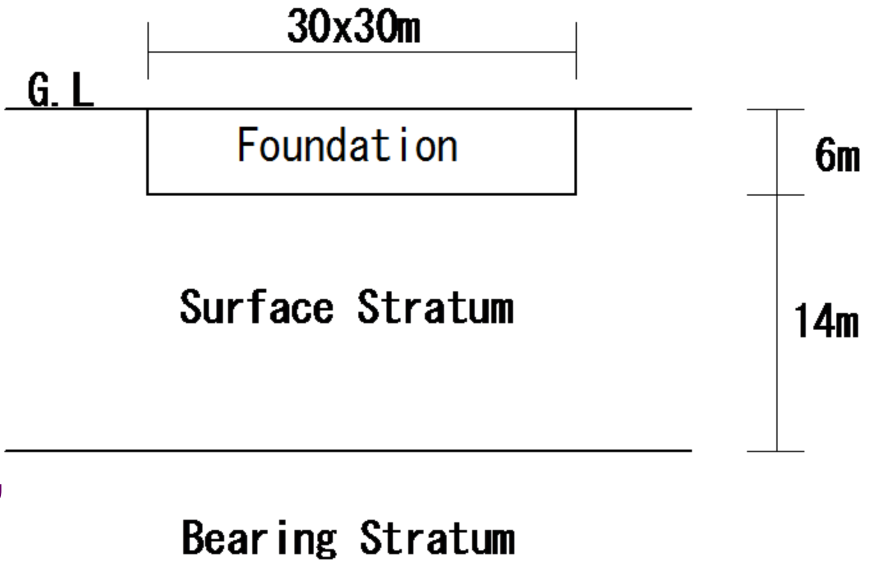
(e) Modeling: Superstructure is modeled by lumped masses model with shear rigidities on the rigid Basement.

(f) Basement: embedding depth = 6m.

(3) Foundations

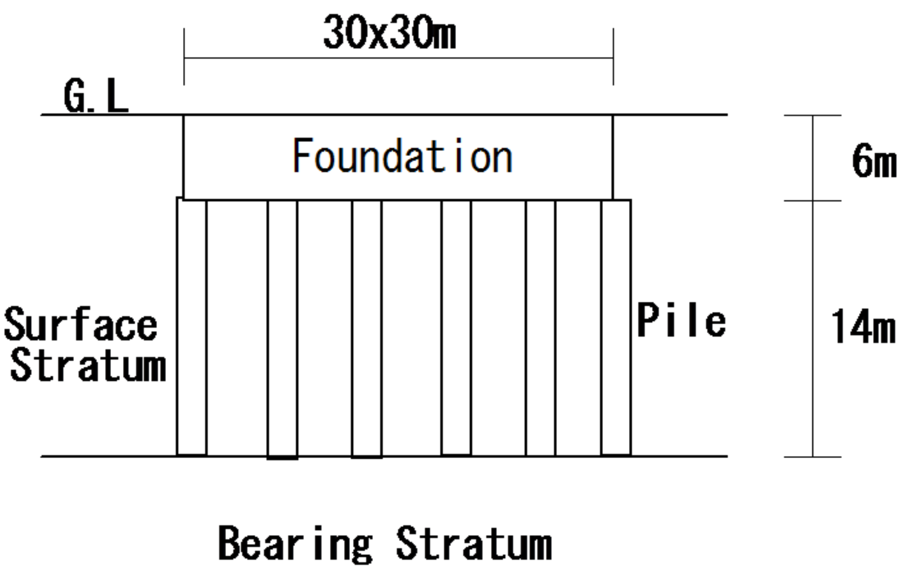
Two types of foundation are considered.

One is a spread foundation,



(a) spread Foundation

the other is a pile group foundation.



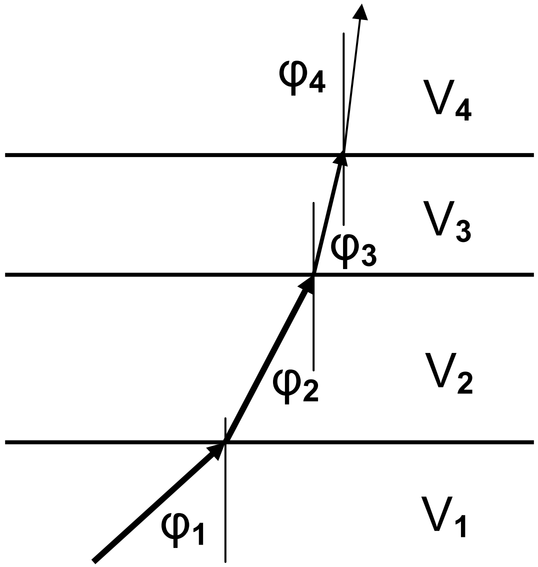
(b) Pile Group Foundation

(4) Earthquake Disturbance

An earthquake disturbance is assumed to result from an incident SH type wave propagating upward at a right angle to the ground surface.

At the time of an earthquake, four kinds of earthquake wave travel to the building. Among them, the S-wave comprises the principal shock of the earthquake motion and it must be incorporated into the seismic design of a structure.

As S-wave velocities of strata nearer to the ground surface becomes normally lower, the incident angle of the S-wave gets near a right angle to the ground surface.(Snell's Law)

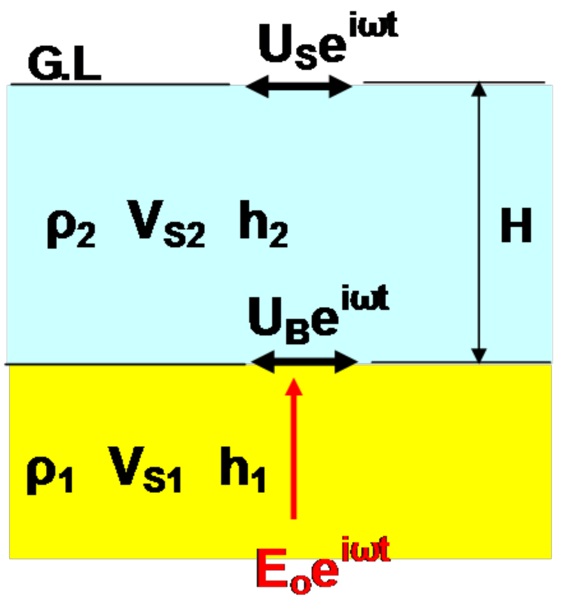


$$\frac{\sin\varphi_1}{V_1} = \frac{\sin\varphi_2}{V_2} = \frac{\sin\varphi_3}{V_3}$$

3.2 Response of Soil

Let take a look at soil response when a SH type earthquake wave is propagating upward at a right angle to the ground surface.

- **Response of Layered Strata**



: Incident SH Wave

First Natural Frequency f_1

$$f_1 = \frac{V_{S2}}{4H}$$

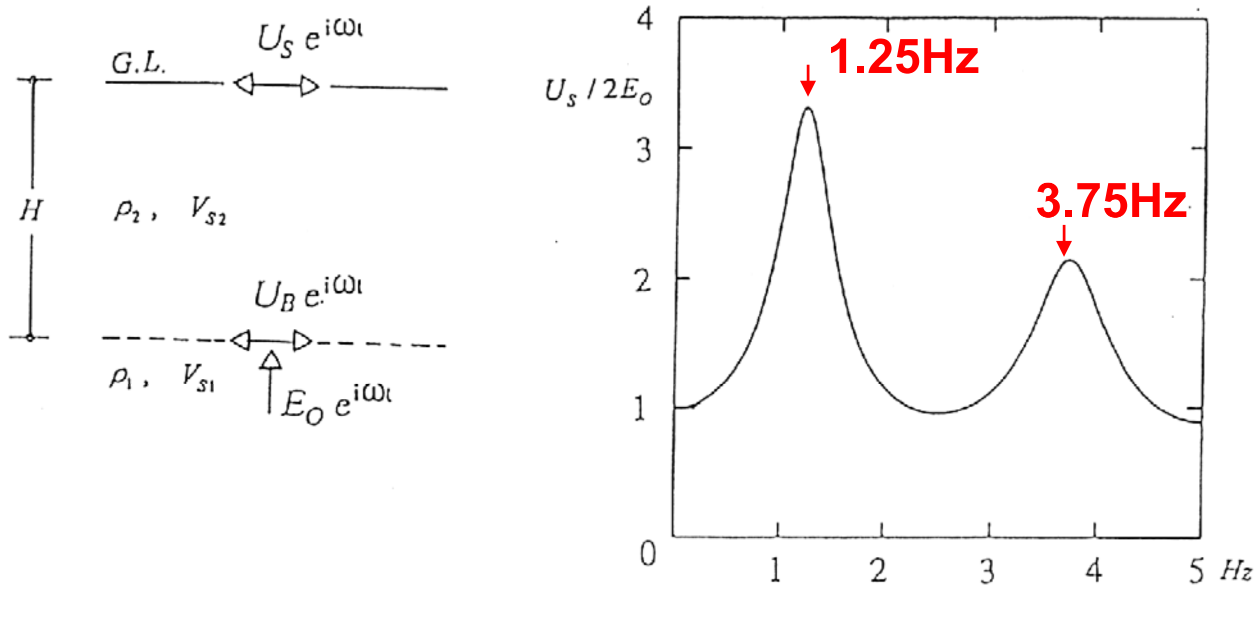
i-th Natural Frequency f_i

$$f_i = (2i-1)f_1$$

Impedance Ratio α

$$\alpha = \frac{\rho_2 V_{S2}}{\rho_1 V_{S1}}$$

The figure shows magnification factor(M.F) $U_S/2E_O$ of the ground surface response to the amplitude of the incident wave E_O. A denominator $2E_O$ of M.F denotes the amplitude of response at the surface of the bearing stratum when the surface stratum is removed.



(a) Incident S-Wave

(b) Magnification Factor

Fig. 3-3 Response of Soil

Two resonance peaks are observed. The lower peak corresponds to a first natural frequency 1.25Hz of the surface stratum, while the higher to a second 3.75Hz.

The i-th natural frequency f_i of the surface stratum is formularized by :

$$f_i = (2i - 1)V_{S2} /(4H) \qquad (3\text{-}1)$$

When the surface stratum consists of multi-layered strata, the natural frequency is similarly calculated using an equivalent shear wave velocity *V*se given by:

$$V_{Se} = \sum_{j=1}^{m} V_{Sj} d_j / H \qquad (3\text{-}2)$$

where *m* is number of layers over the bearing stratum, and V_{Sj} is shear wave velocity and d_j thickness of the j-th stratum. Substituting *V*se into in Eq.(3-1), the natural frequencies can be estimated approximately.

M.F at the natural frequencies depend on both a damping factor *h* of surface stratum and an impedance ratio α given by:

$$\alpha = \rho_2 V_{S2} / \rho_1 V_{S1} \qquad (3\text{-}3)$$

where ρ_i is mass density.

When the surface stratum has no damping, M.F at the first natural frequency is obtained as:

$$M.F = 1/\alpha \qquad (3\text{-}4)$$

In this soil condition, the impedance ratio is:

$$\alpha = \rho_2 V_{S2} / \rho_1 V_{S1} = 0.22 \qquad (3\text{-}5)$$

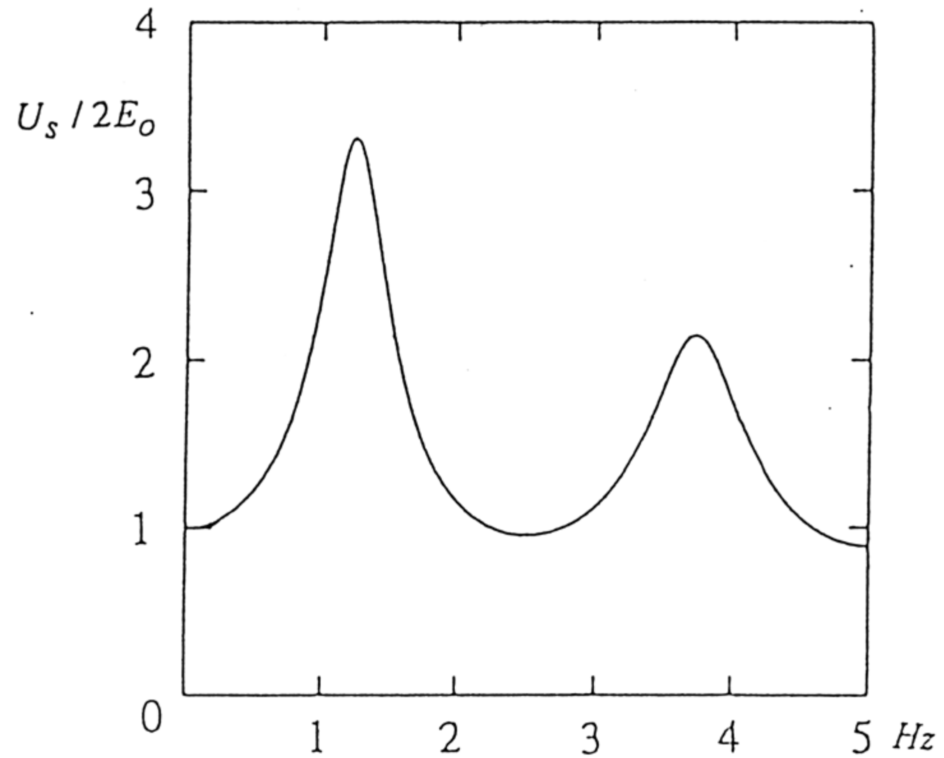
so, M.F is:

$$M.F = 1/\alpha = 1/0.22 = 4.5 \qquad (3\text{-}6)$$

As you can see, M.F= 4.5 obtained by Eq.(3-6) is larger than that of the resonance curve.

This discrepancy comes from the damping factor 0.05 of the surface stratum.

Note that M.F estimated from Eq.(3-6) gives the maximum M.F.



(b) Magnification Factor

When you want to know how larger the ground surface will be amplified in a given soil condition, you can estimate roughly it by Eq.(3-6).

3.3 Dynamic Impedance Function and Foundation Input Motion

Dynamic impedance functions (D.I.F) for the raft(spread) and the pile foundation are depicted in the figure.

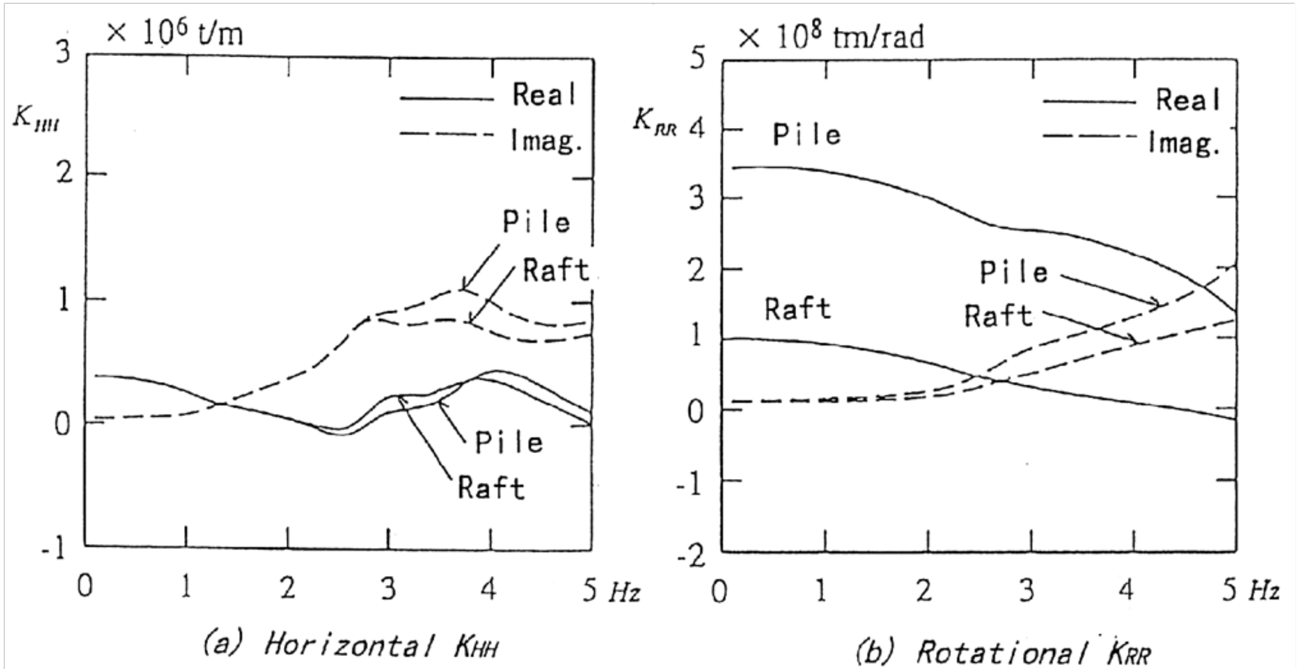


Fig. 3-4 Dynamic Impedance Function

D.I.F indicates dynamic soil springs, and it can be expressed by complex number. Real part of the function denotes dynamic stiffness, while imaginary part corresponds to damping capacities.

It was already explained in previous chapters that the damping came from both the radiation damping and the hysterics damping of soil medium.

In order to understand physical meaning of complex number of D.I.F, we consider a spring system shown in the figure comprising of a spring *K* and a viscous damping coefficient *C*.

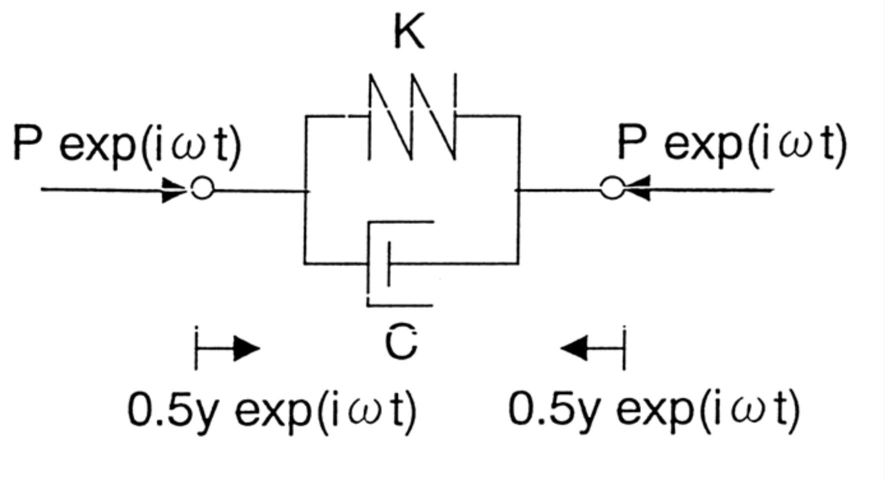
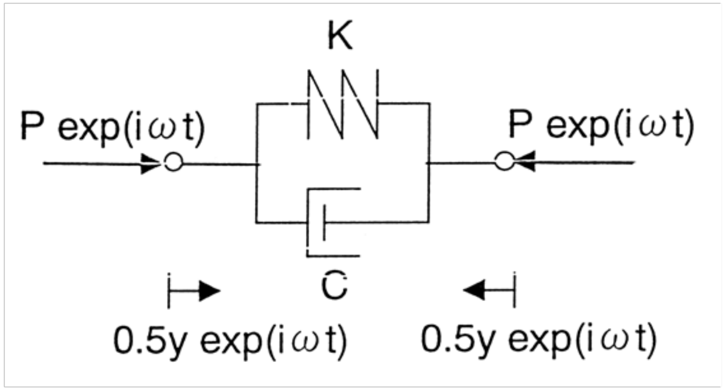


Fig. 3-5 Voigt Model

This spring-viscous damping system is called "Voigt Model".

When a harmonic external forces $Pe^{i\omega t}$ is applied to the both sides of the spring model and displacements $0.5ye^{i\omega t}$ are caused, an equation of the motion for this spring system can be expressed by:



$$Pe^{i\omega t} = Kye^{i\omega t} + C\,d(ye^{i\omega t})/dt = (K + i\omega C)ye^{i\omega t} \qquad (3\text{-}7)$$

Calculating the ratio of the left to the right hand side of Eq.(3-7), we obtain the relation as:

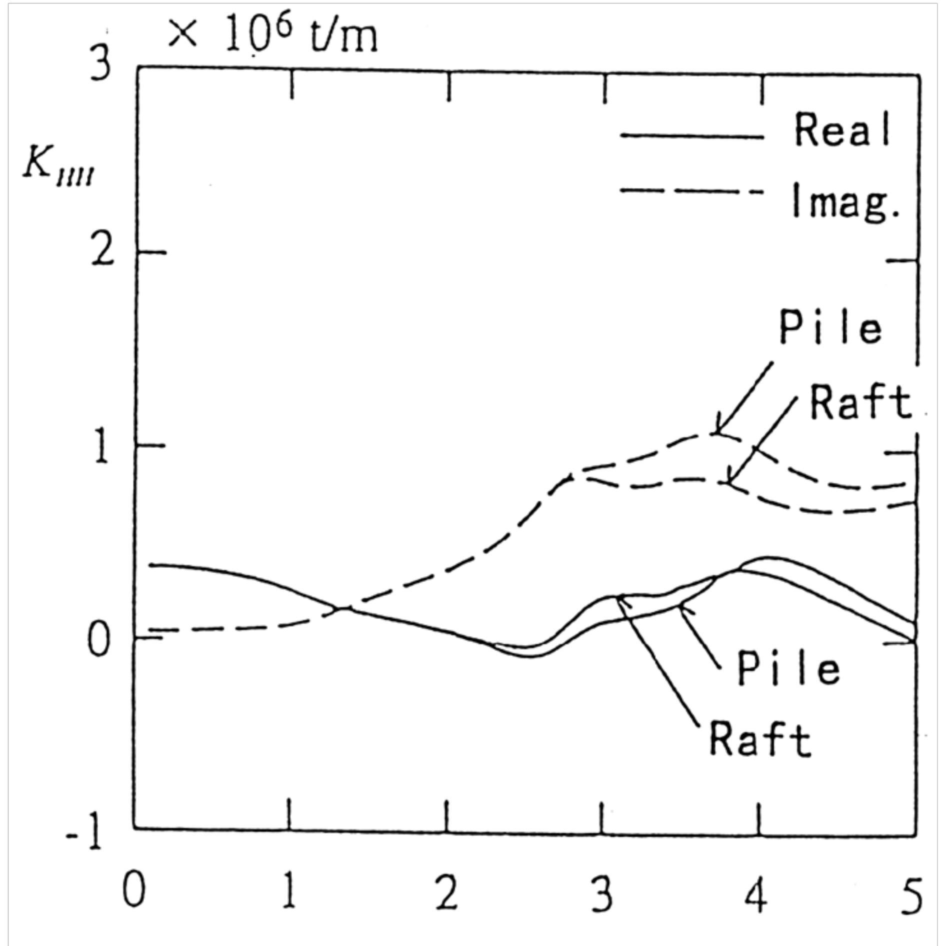
$$P/y = K + i\omega C \qquad (3\text{-}8)$$

The real part K of the dynamic spring constant for the Voigt Model is constant and independent of the excitation frequencies ω, while the imaginary part ωC increases in proportion to the excitation frequencies ω.

You can understand the physical meaning of the complex number, which expresses D.I.F.

However, D.I.F exhibits more inherent variation with changing of frequencies as shown in the figure.

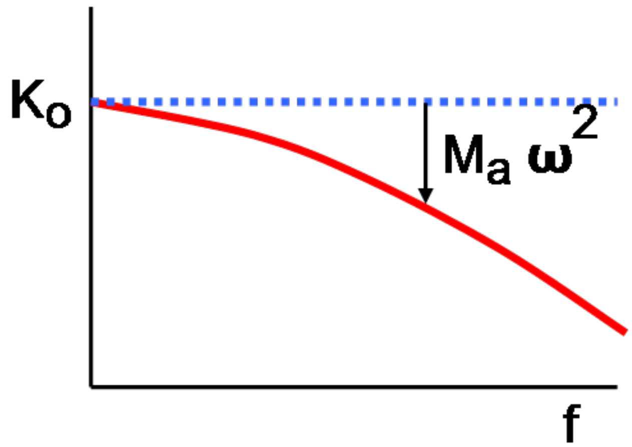
The real parts $K^{(R)}$ of D.I.F in the lower frequencies up to the first natural frequency 1.25Hz of the surface stratum decrease monotonously with increase of frequencies.



(a) Horizontal K_{HH}

This tendency can be approximated by:

$$K^{(R)} = K_o - \omega^2 M_a \qquad (3\text{-}9)$$

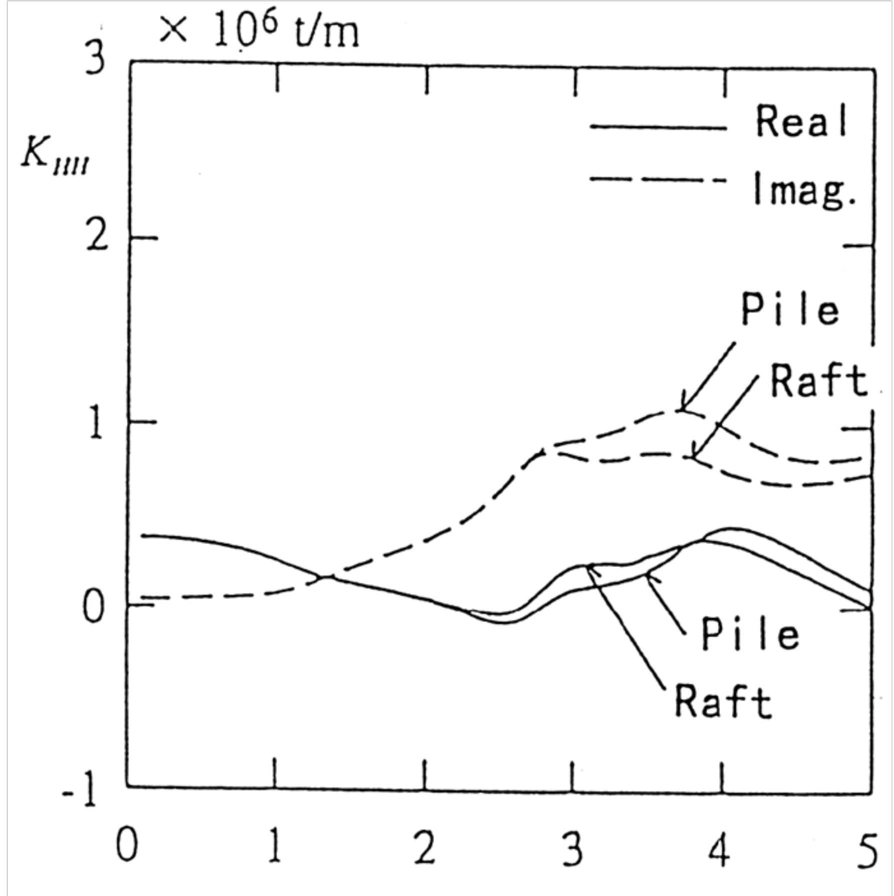


K_o is the real part at the frequency zero and denotes a static stiffness.

M_a corresponds to a mass of soil portion which is vibrating in same phase of the foundation vibration.

The imaginary parts of the horizontal D.I.F for both the raft and the pile foundation are nearly constant in lower frequencies up to 1.25Hz and they increase rapidly in higher frequencies.

This tendency of the imaginary parts results from the characteristic of the radiation waves.



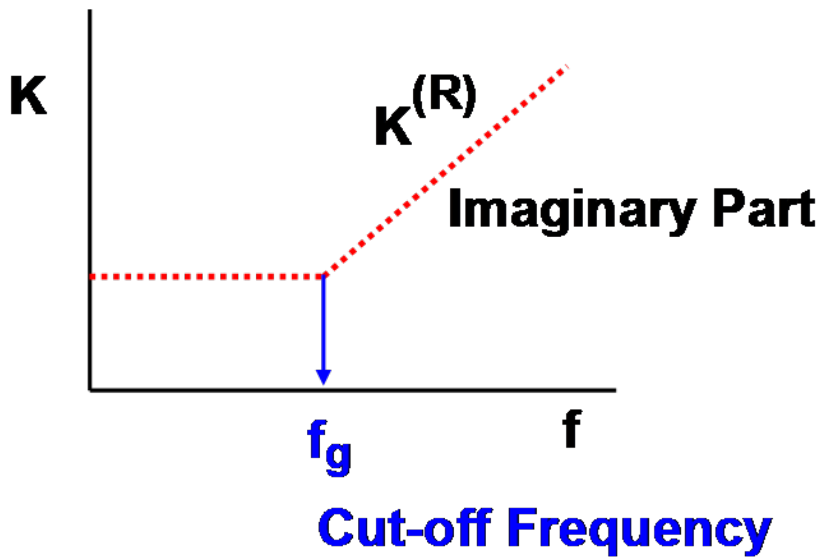
(a) Horizontal K_{HH}

The excitation forces on the foundation in higher frequencies are transmitted by the radiation wave and propagate outward.

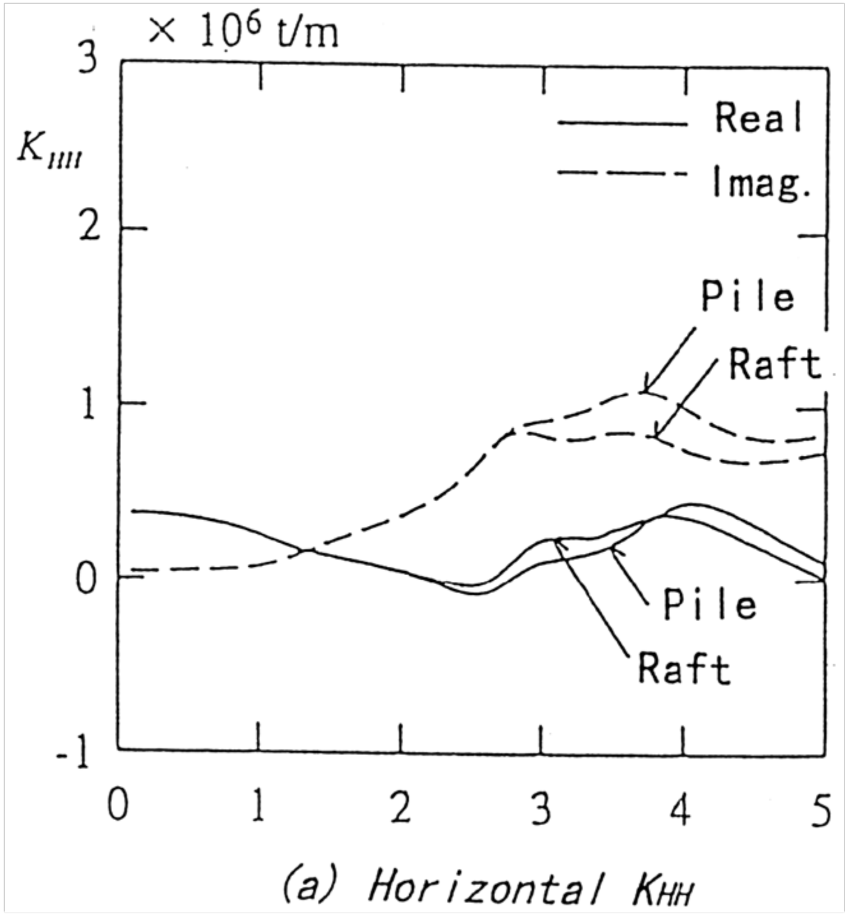
While, the forces in lower frequencies are hard to transmit outward in the form of the radiation wave and the forces transmit as same as the static loading on the soil.

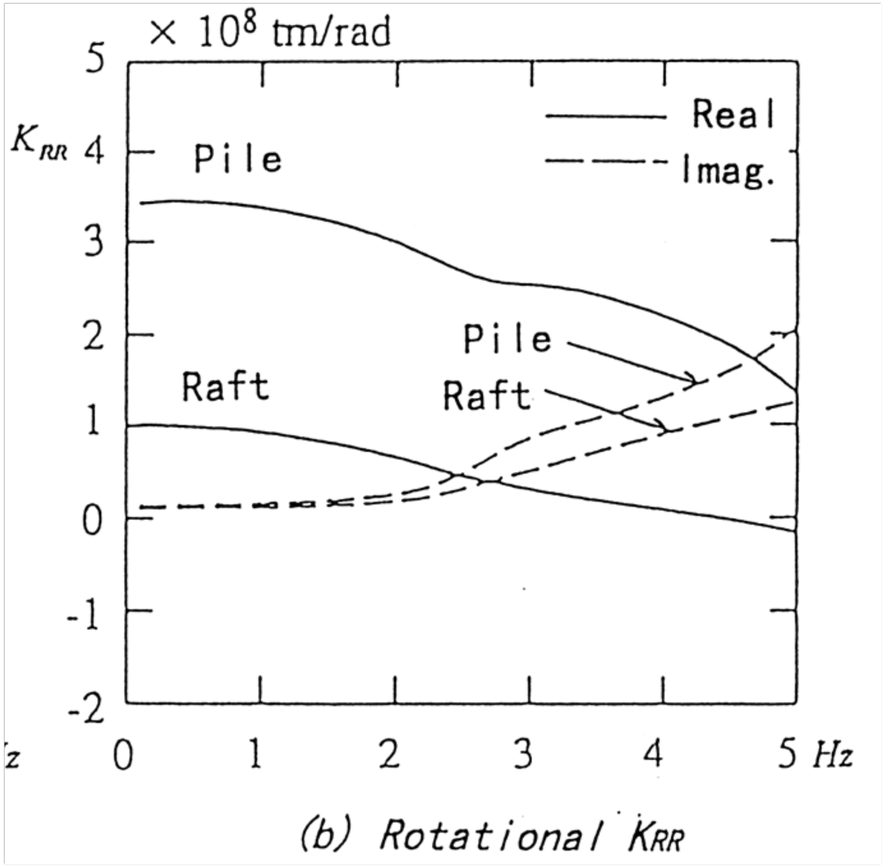
In order to express this tendency of the imaginary part, the first natural frequency f_g of the surface stratum is particularly termed "cut-off frequency".

The frequency characteristic of the imaginary part is depicted in the figure:



Both the real and the imaginary parts of the horizontal D.I.F are fluctuating in higher frequencies over 2.5Hz. The fluctuations are particular for the layered strata.





The fluctuations for the rotational D.I.F are also observed in far higher frequencies.

Comparing the raft foundation to the pile foundation, slight differences in the horizontal D.I.F are observed between both foundations. Distinct differences are recognized in the real parts of the rotational D.I.F.

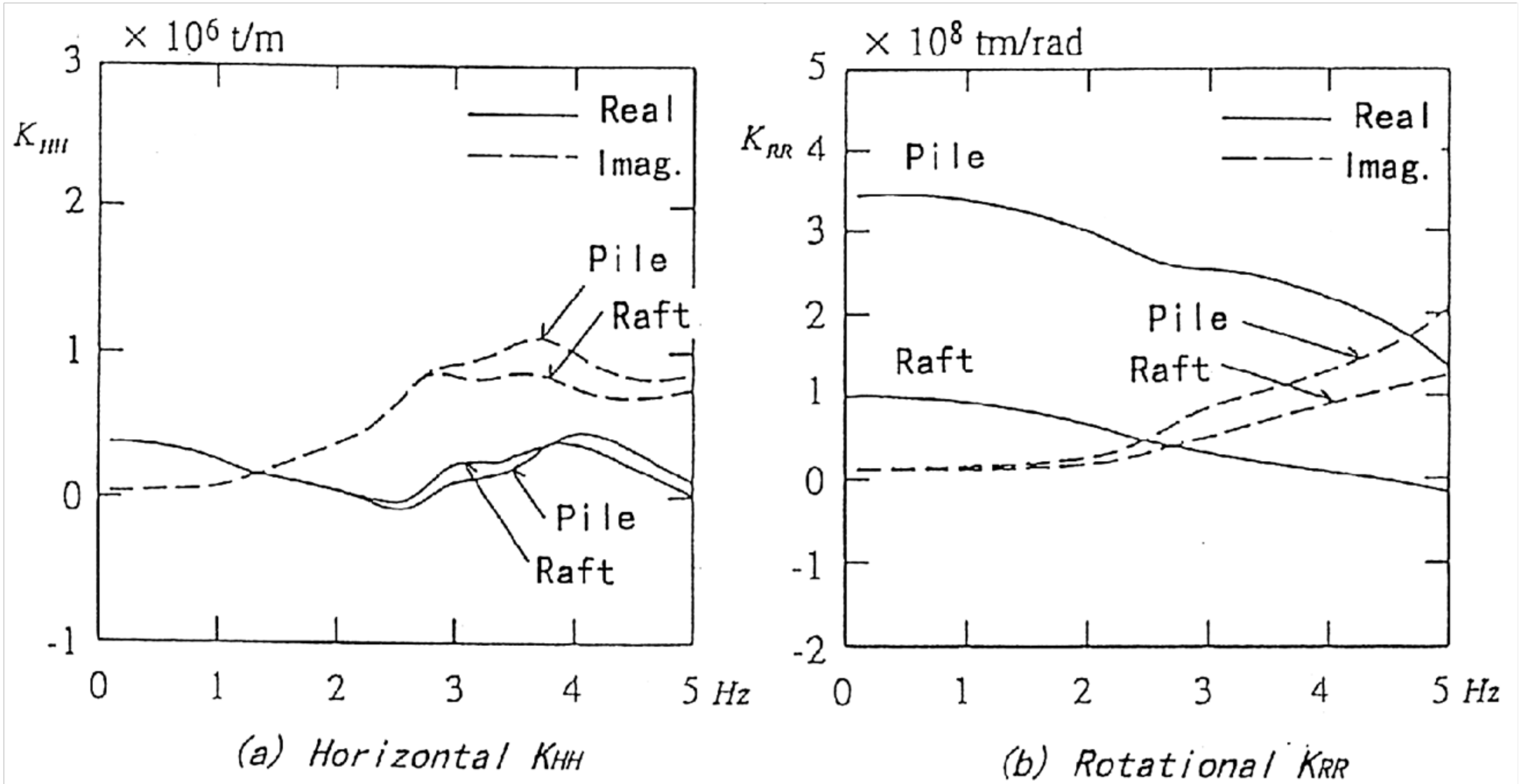


Fig. 3-4 Dynamic Impedance Function

The real parts of the pile foundation are much larger than that of the raft foundation.

The rotational stiffness of the pile group foundation is composed of:

(1) The rotational stiffness at each pile caps

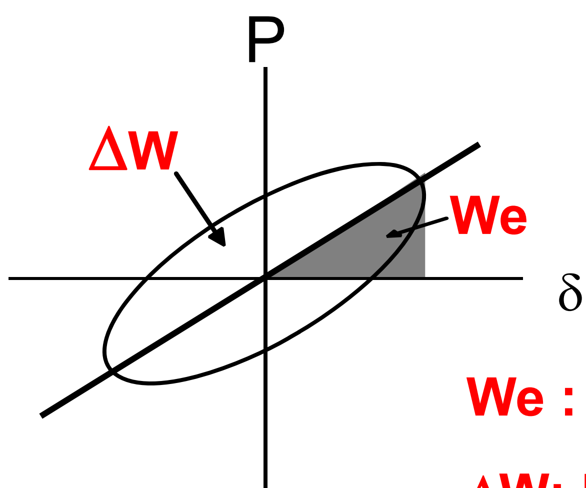
(2)The stiffness estimated by a product of the vertical stiffness at the each pile caps and the distance of the caps from a neutral axis of the foundation.

In a case of the pile group foundation, the latter (2) becomes significant. Consequently, the rotational stiffness of the pile foundation becomes larger than the raft foundation.

We can understand the differences in D.I.F between both the foundation types arise distinctly in the real parts of the rotational and the other components exhibit only trivial differences.

3.4 Damping Factor of Dynamic Impedance Function

A damping factor, which is evaluated from the hysterics loops of the structural components, is employed for a dynamic analysis of a superstructure. This damping factor is evaluated by a ratio of area of a hysterics loop to an elastic restoring energy.



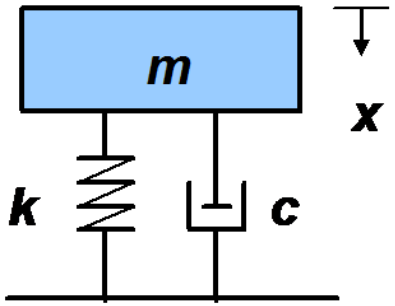
We : Elastic Potential Energy

ΔW: Loop Area

Damping Factor: $h=\Delta W/(2\pi We)$

As same as the damping factor aforementioned, we can estimate a damping factor of D.I.F. Let consider how to estimate it.

A equation of motion of a single degree of freedom system during the free vibration is expressed by:



$$m\ddot{x} + c\dot{x} + kx = 0 \qquad (3\text{-}10)$$

where, *m*=mass, *c*=coefficient of viscous damping and *k*= spring constant.

Introducing an undamped natural circular frequency $\omega_o(=\sqrt{k/m})$ and a damping factor $h(=c\omega_o/2k)$, Eq.(3-10) can be rewritten as:

$$\ddot{x} + 2h\omega_o\dot{x} + \omega_o^2 x = 0 \qquad (3\text{-}11)$$

Putting $x = X\exp(\lambda t)$ and substituting into Eq.(3-11), the following eigenvalue equation is obtained.

$$\lambda^2 + 2h\omega_o\lambda + \omega_o^2 = 0 \qquad (3\text{-}12)$$

Solving a quadratic equation of Eq.(3-12), the eigenvalue λ is obtained as:

$$\lambda = -h\omega_o + i\omega_o\sqrt{1-h^2} \qquad (3\text{-}13)$$

Express the real and the imaginary part of the eigenvalue λ by:

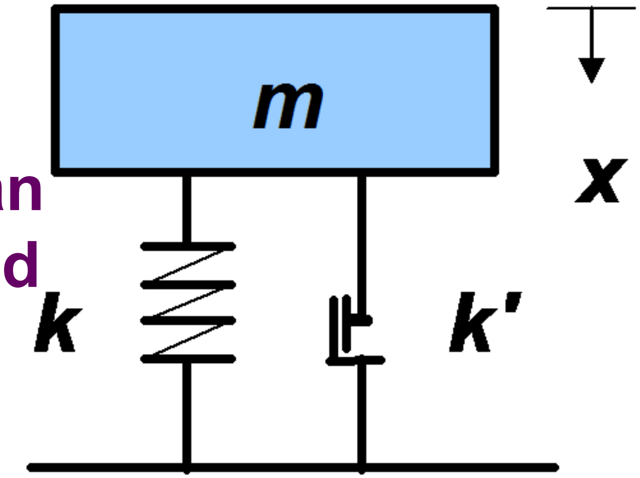
$$\mathrm{Re}.(\lambda) = -h\omega_o \qquad (3\text{-}14)$$

$$\mathrm{Im}.(\lambda) = \omega_o\sqrt{1-h^2} \qquad (3\text{-}15)$$

and calculate η which is defined by:

$$\eta = \frac{-\mathrm{Re}.(\lambda)}{\sqrt{(\mathrm{Re}.(\lambda))^2 + (\mathrm{Im}.(\lambda))^2}} = h \qquad (3\text{-}16)$$

In a case where a single degree of freedom system has a spring of complex number type such as D.I.F, an equation of free vibration is expressed by:



$$m\ddot{x} + (K + iK')x = 0 \qquad (3\text{-}17)$$

Where, *K*+i*K*' is the complex spring constant.

The eignvalue equation for Eq.(3-17) is:

$$m\lambda^2 + (K + iK') = m\lambda^2 + K^* \exp(i2\phi) = 0 \qquad (3\text{-}18)$$

where,

$$K^* = \sqrt{K^2 + K'^2} \qquad (3\text{-}19)$$

$$\phi = \frac{1}{2}\tan^{-1}\left(\frac{K'}{K}\right) \qquad (3\text{-}20)$$

Accordingly, the eignvalue is given by Eq.(3-21):

$$\lambda = i\omega_o^* \exp(i\phi) = \omega_o^*(-\sin\phi + i\cos\phi) \qquad (3\text{-}21)$$

where $\omega^*_o(=\sqrt{k^*/m})$. Substituting the real and the imaginary part of the eigenvalue of Eq.(3-21) into Eq.(3-16), we obtain:

$$\eta = \sin\phi = \sin\left(\frac{1}{2}\tan^{-1}\frac{K'}{K}\right) = h \qquad (3\text{-}22)$$

Eq.(3-22) equals damping factor of a single degree of freedom system with the complex spring constant.

When the damping factor is small, the damping factor can be approximately expressed by:

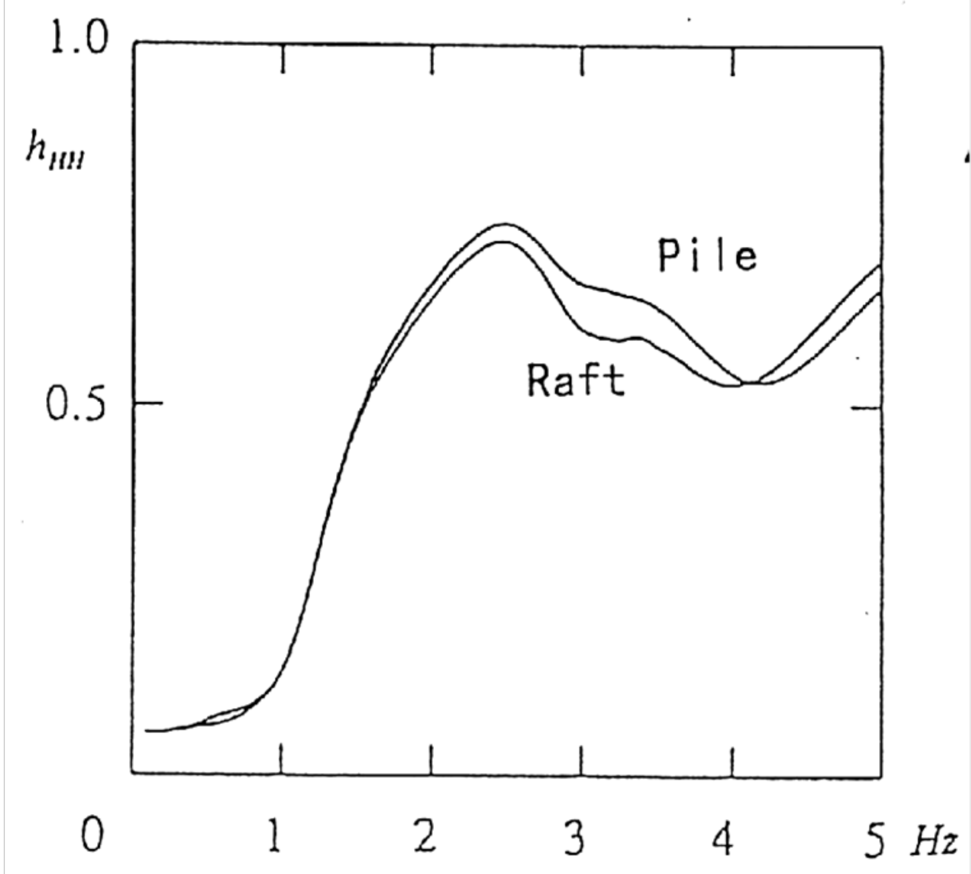
$$h = \frac{K'}{2K}$$

In a practical seismic design analysis, the rigidity is often given by:

$$K + iK' = K(1 + i2h)$$

The figure shows the damping factors h_{HH} of D.I.F.

Concerning the damping factors h_{HH} of the horizontal D.I.F, slight differences in the damping factors are only observed between the raft and the pile foundations.

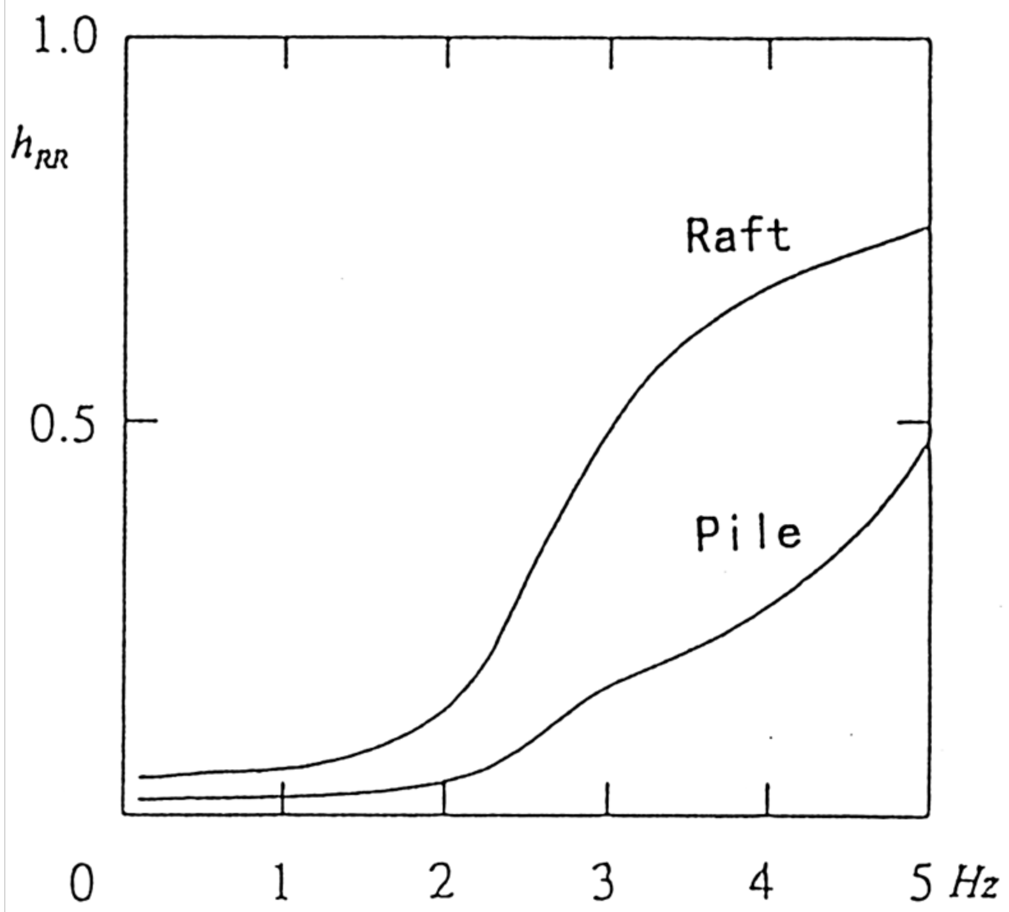


(a) Horizontal h_{HH}

h_{HH} are small and nearly constant in lower frequencies up to the cut-off frequency 1.25Hz, the values of h_{HH} are around the damping factor $h_2=0.05$ of the surface stratum.

h_{HH} becomes larger in higher frequencies over the cut-off frequency.

The damping factors h_{RR} of the rotational D.I.F for the raft foundation are larger than the pile foundation.



(b) Rotational h_{RR}

Comparing h_{RR} to h_{HH} , h_{HH} is larger than h_{RR} in all frequencies for the both foundation types.

3.5 Foundation Input Motion

A foundation input motion (F.I.M) is an effective input motion of earthquake disturbance, which is applied to D.I.F. The figure shows the amplitudes of F.I.M for the horizontal Δ and the rotational Θ components.

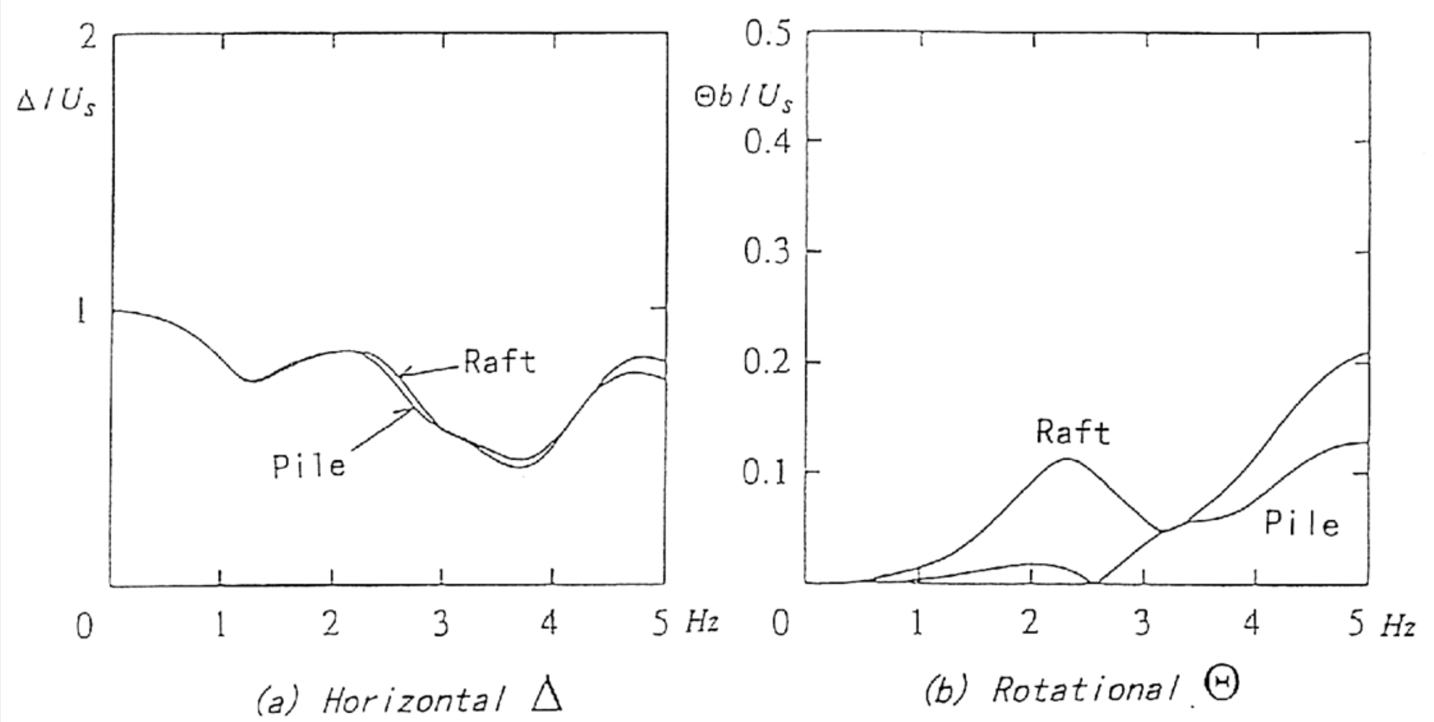
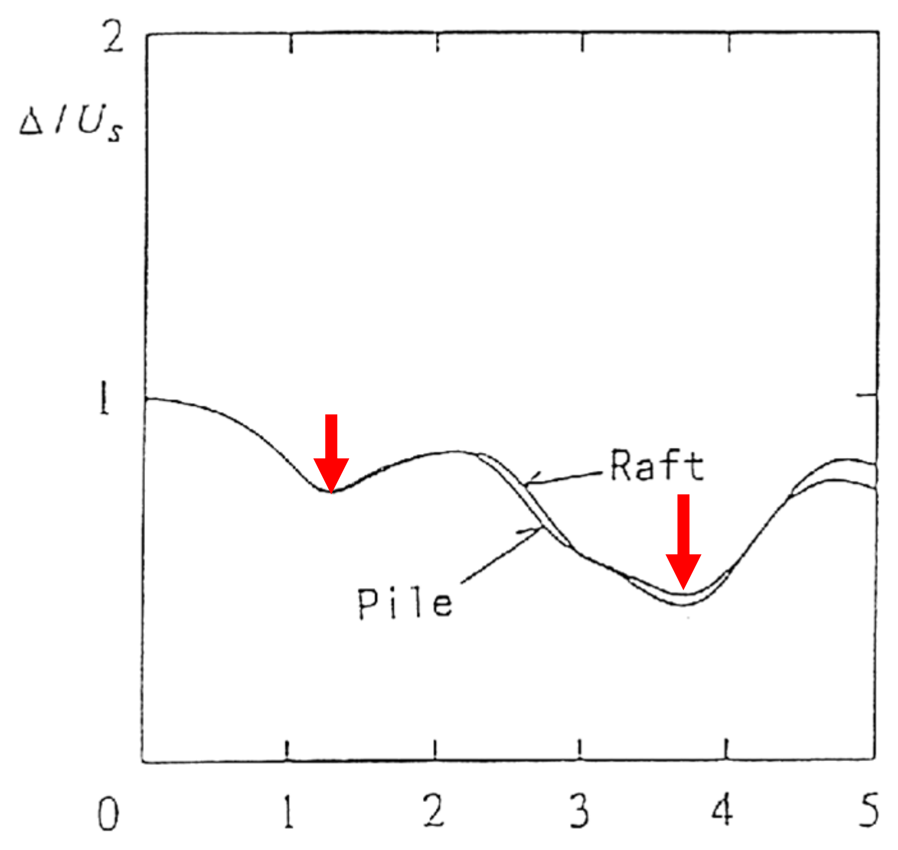


Fig. 3-7 Foundation Input Motion

F.I.M are depicted by the ratio to the ground surface response amplitudes U_s.

The values of Δ/U_s are less than 1.0 in all frequencies. This indicates that the horizontal F.I.M Δ is always less than the response U_s of the ground surface.

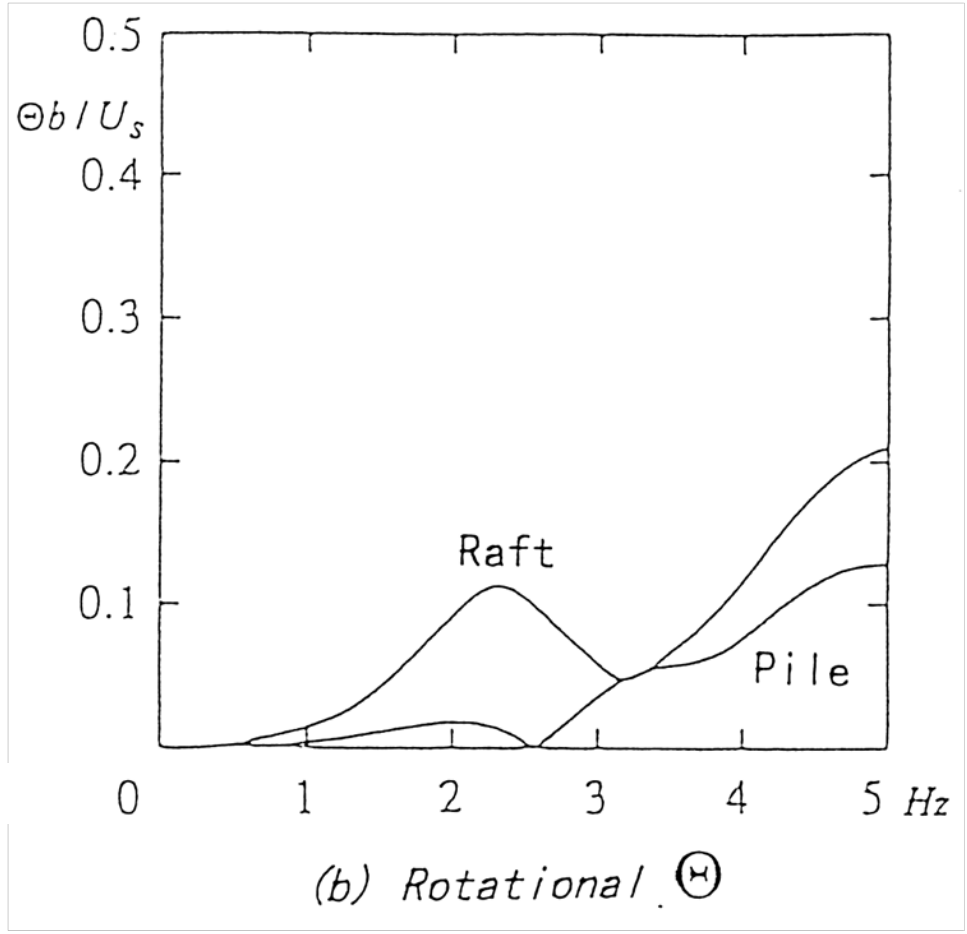
Two troughs are observed around the first and the second natural frequencies of the surface stratum.



(a) Horizontal Δ

Because the foundation footing or the basement possesses normally much higher rigidity than that of the surrounding soil and the growth of the response of the surface stratum is restrained, Δ forms the troughs at around the natural frequencies. The differences in the horizontal F.I.M are small between both foundation types.

The rotational F.I.M becomes larger with increase of frequency.



Because the real part of the rotational D.I.F for the pile foundation is much larger than that for the raft foundation, the rotation motion of the pile foundation is more restrained and Θ of the pile foundation becomes smaller than the raft foundation.

3.6 Response Characteristic

Resonance curves U_T at the roof floors are shown with increase of number of floor . Magnification factor (M.F) denotes $U_T/2E_o$, where E_o is the amplitude of the incident SH wave.

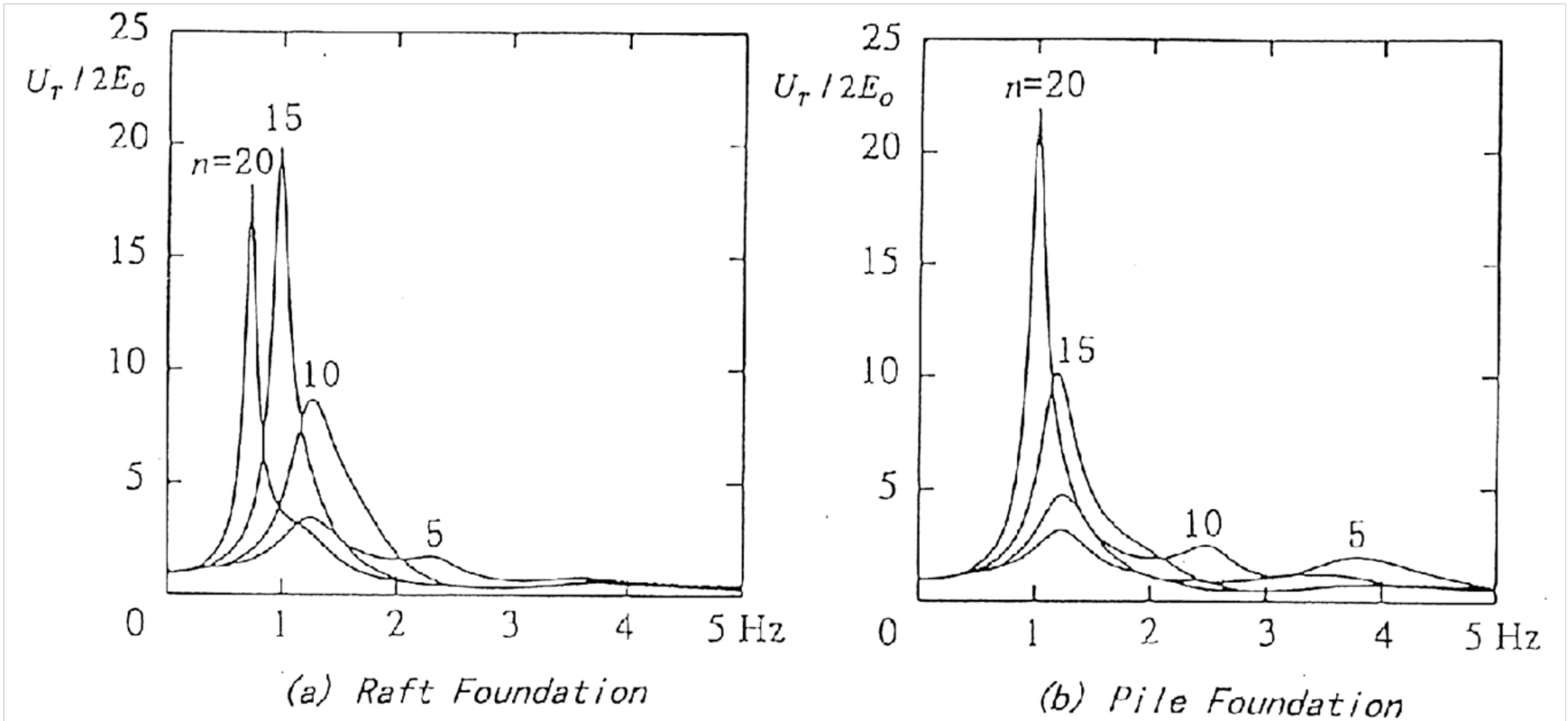
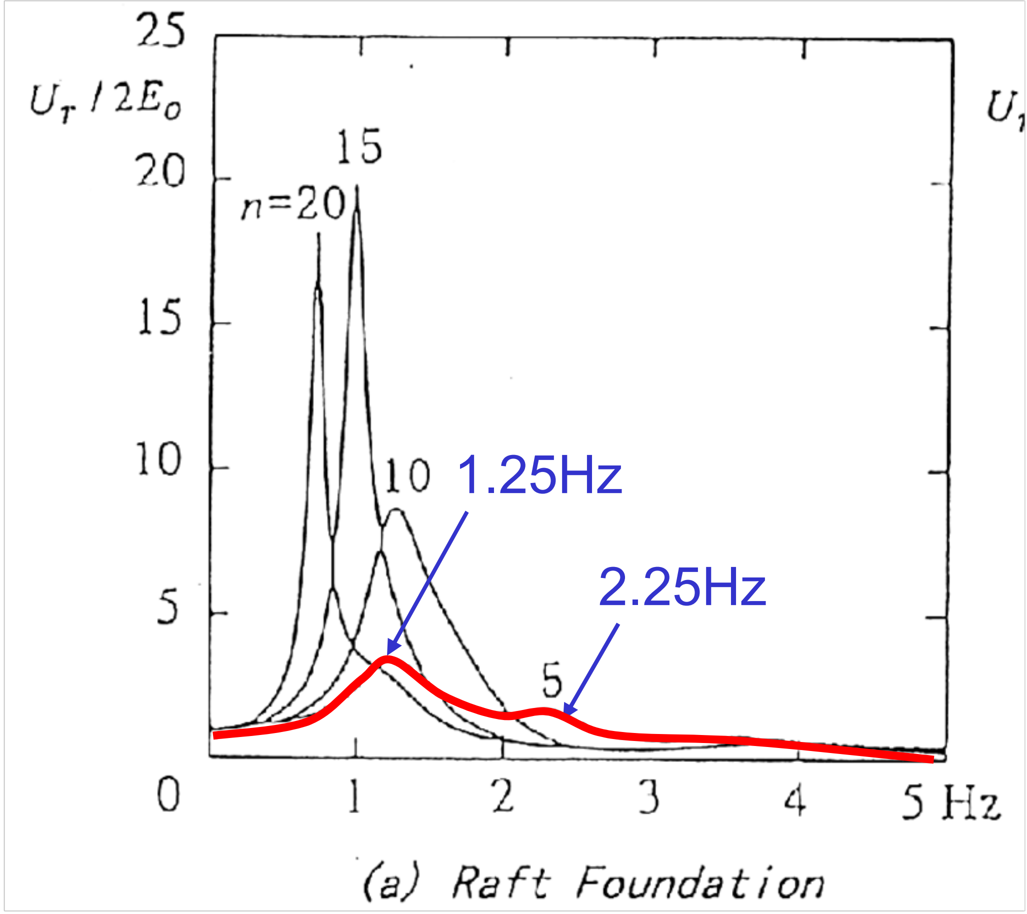


Fig. 3-8 Magnification Factor of Resonance Frequency at Roof Floor

M.F for the n=5 building on the raft foundation have two resonance peaks.

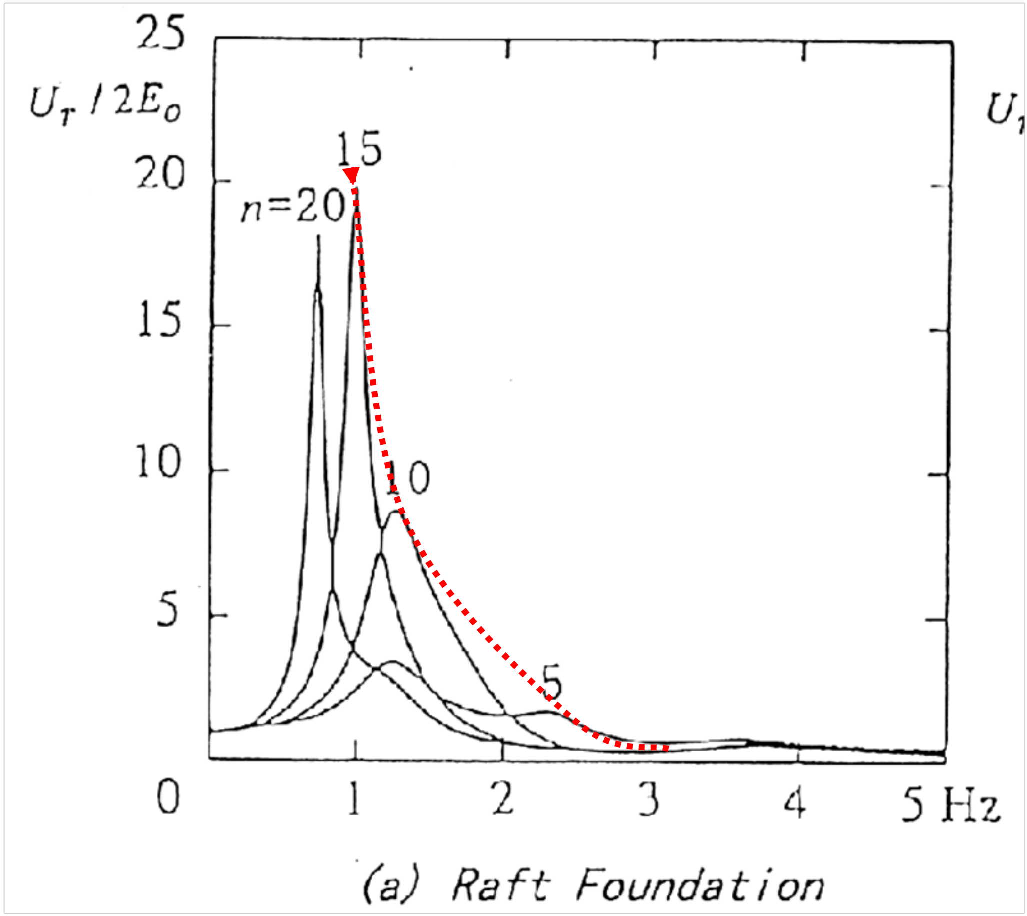


The lower resonance frequency is 1.25 Hz which corresponds to the first natural frequency of the surface stratum.

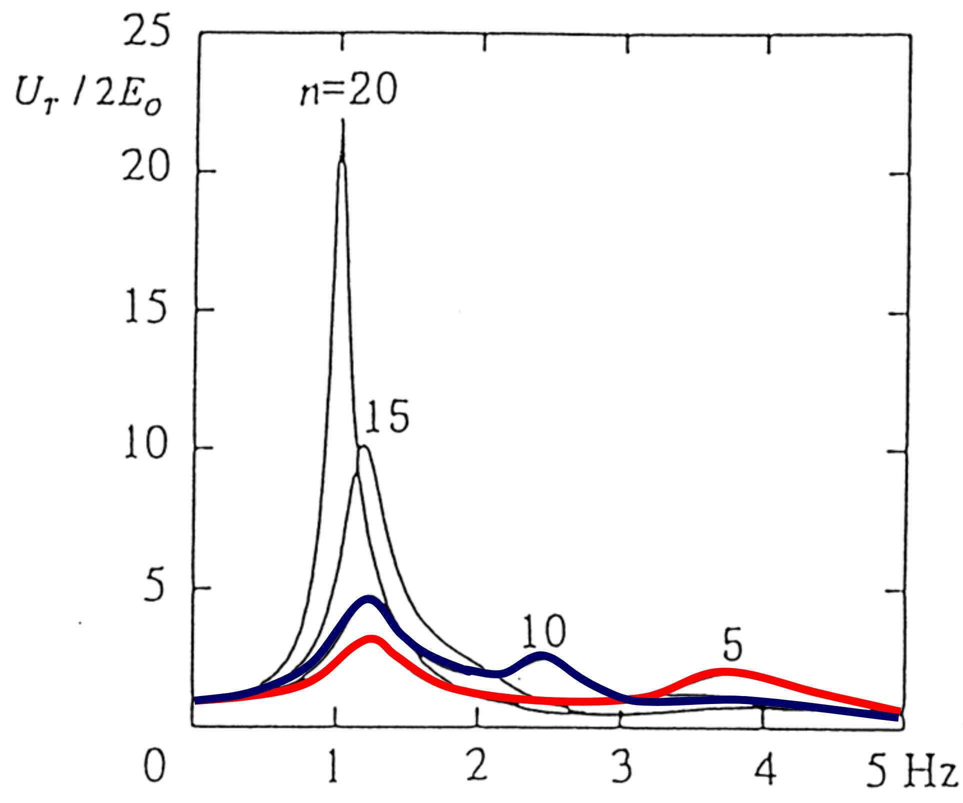
While the higher is 2.25Hz corresponding the natural frequency of the coupling system of a building, a foundation and ground.

With increase of the number of floors n, the natural frequency of the coupling system becomes lower and approaches to the first natural frequency of the surface stratum 1.25 Hz.

The natural frequency of the coupling system approaches to that of the surface stratum, two peaks converge on one peak with larger amplitude and the resonance phenomenon brings about between the coupling system and the surface stratum.



The tendencies of resonance curves described above are similarly recognized in the resonance curves of the building on the pile foundation .



(b) Pile Foundation

Because the rotational rigidity of the pile foundation is much larger than that of the raft foundation, the natural frequency of the coupling system for the pile foundation becomes higher than that for the raft foundation. This causes the differences in response characteristics of the building between on the pile and on the raft foundation.

The figure shows the relation between the number of floors *n* and the maximum M.F at the roof floor.

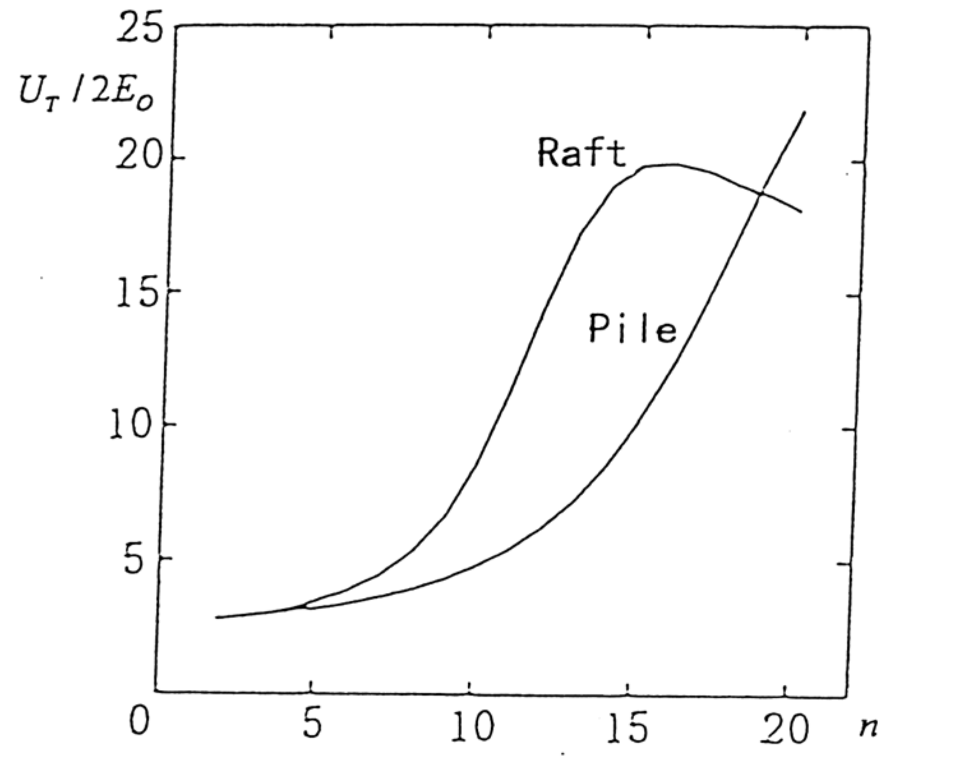


Fig. 3-9 Max. Magnification Factor at Roof Floor

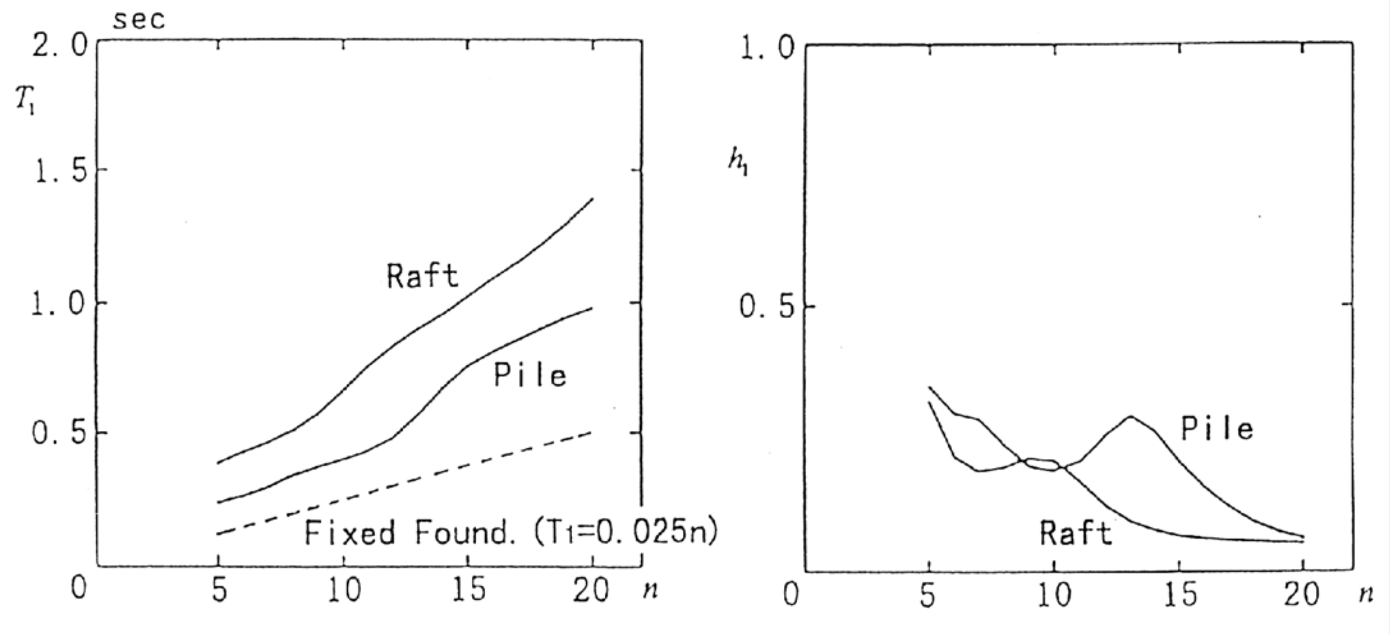
Of course, the building with larger *n* has the larger maximum M.F.

Comparing the raft to the pile foundation, M.F for the building on the raft foundation becomes larger.

You can point out the number of floor *n*=15 for the raft foundation at which M.F is the maximum. This indicates that the natural frequency of the coupling system for *n*=15 approaches the nearest to that of the surface stratum.

The first natural period T_1 or the the natural frequency $f_1(=1/T_1)$ and the damping factor h_1 at this period (is called the modal damping) are the most important quantities which affect the earthquake response of the building.

This period is not for the building on the fixed base, but for the coupling system.



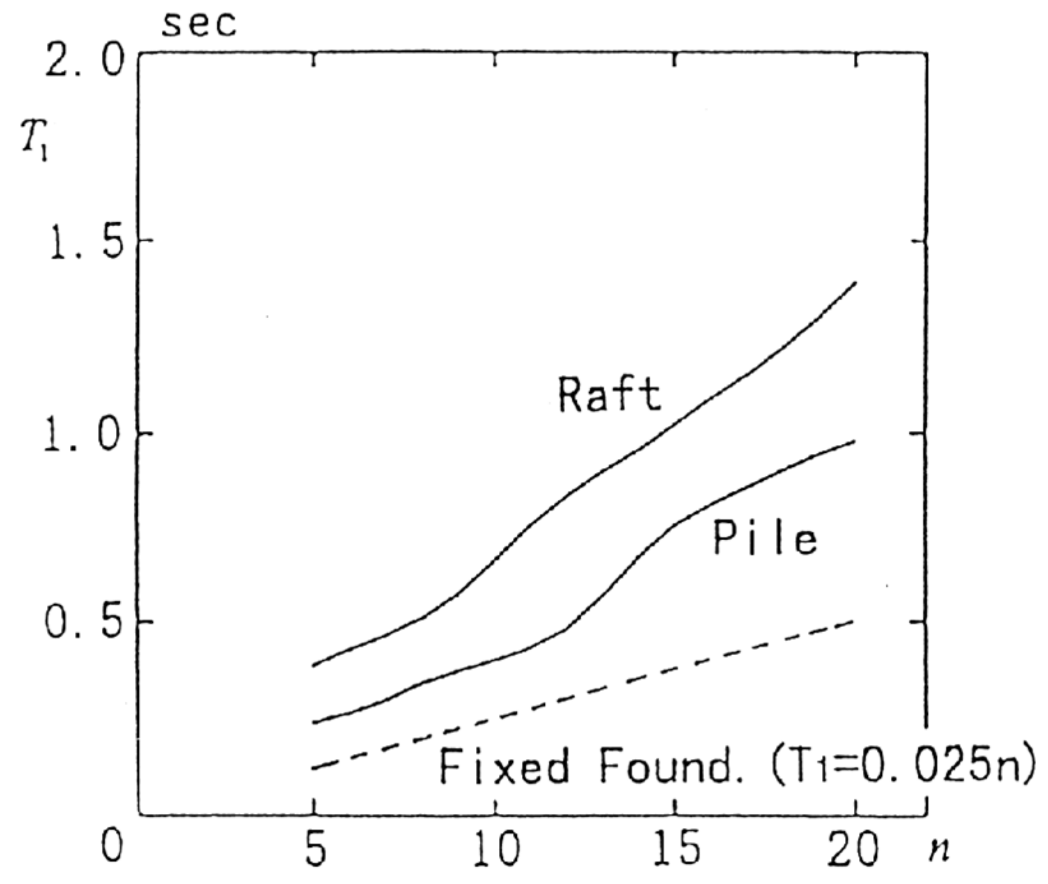
(a) First Natural Period T_1 (b) Modal Damping Factor h_1

Fig. 3-10 First Natural Period and Modal Damping Factor

The figure shows the relations between T_1, h_1 and n. The natural periods of the buildings under the fixed base condition are given by $T_1=0.025n$.

T_1 for the coupling system becomes longer in almost proportion to *n*.

T_1 for the raft foundation is longer than that for the pile foundation.

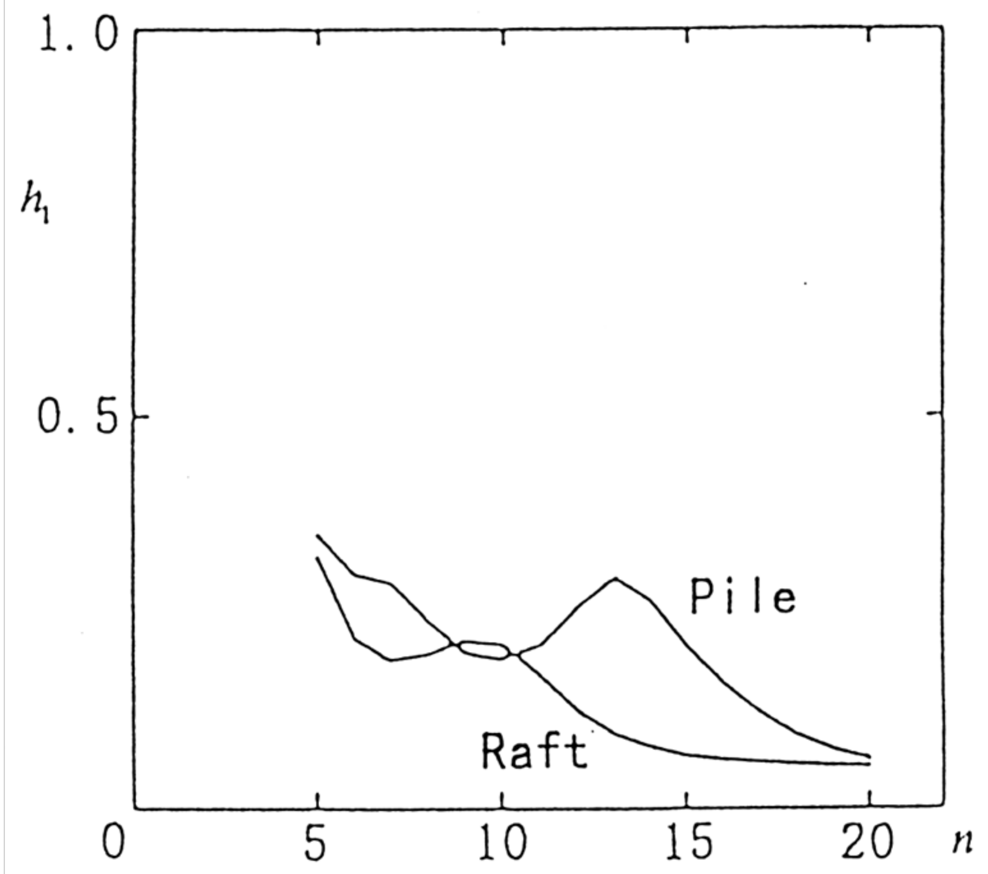


(a) First Natural Period T_1

Concerning the low or middle-rise buildings of *n*=5 to 10, T_1 are around 3 times for the raft foundation and around 2 times for the pile foundation as longer as T_1 under the fixed base condition.

The superstructures have 0.01 damping, but h_1 for the coupling system is always much bigger than 0.01.

With increase of n, the h_1 decreases gradually.



(b) Modal Damping Factor h_1

E N D

Chapter 3 :

Effect of SSI on

Dynamic Behaviors of Structure

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