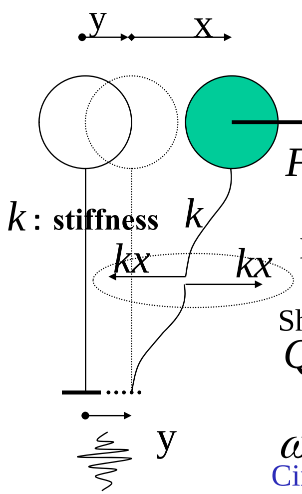
Lesson One

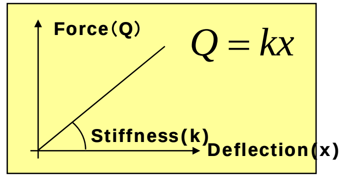
Basic Theory of High Damping Structures

Formulation of the equation of motion of a SDOF linear model without Damping



Inertia Force

$$F = -m(\ddot{x} + \ddot{y})$$



Equilibrium Condition $\quad F = Q$

Shear Force

$$Q = kx$$

Equation of Motion

$$\ddot{x} + \omega^2 x = -\ddot{y}$$

$$\omega = \sqrt{k/m}$$

Circular Frequency

$$T = 2\pi/\omega$$

Natural Period

Free vibration of a SDOF model without damping

- Equation of Motion

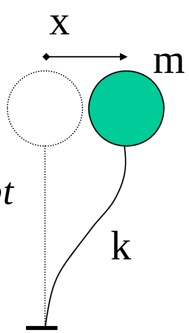
$$m\ddot{x} + kx = 0$$

$$\ddot{x} + \omega^2 x = 0 \qquad \omega^2 = k/m \qquad T = \frac{2\pi}{\omega}$$

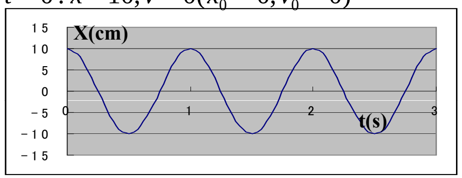
- Initial Conditions

$$x = x_0, \dot{x} = v_0 \quad at \;\; t=0$$

- Solution $\; x = x_0 \cos \omega t + (v_0/\omega) \sin \omega t$



$t = 0 : x = 10, v = 0 (x_0 = 0, v_0 = 0)$



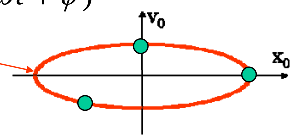
Energy conservation low in the free vibration of a SDOF model without damping

$$x = x_0 \cos \omega t + (v_0/\omega) \sin \omega t$$

$$= \sqrt{(v_0/\omega)^2 + x_0^{\,2}} \sin(\omega t + \phi) \qquad \phi = \tan^{-1}\left(\frac{x_0}{v_0/\omega}\right)$$

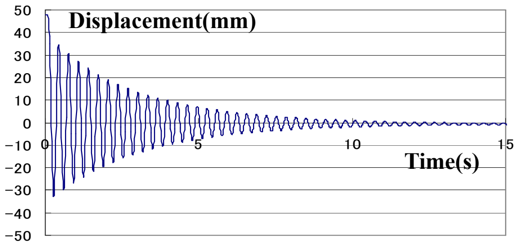
$$= \sqrt{\frac{2}{k}\left(\frac{1}{2} m v_0^{\,2} + \frac{1}{2} k x_0^{\,2}\right)} \sin(\omega t + \phi)$$

Kinetic Energy $\quad$ Potential Energy

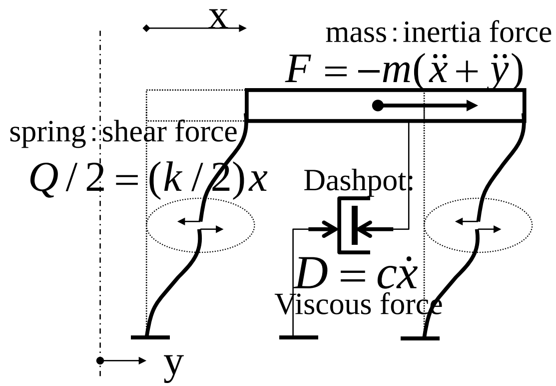


1)The total energy in free vibration system without damping is conserved
2)Amplitude of the free vibration would be always the same so long as the sum of the initial kinetic and the initial potential energy are the same, regardless of the initial displacement and initial velocity.

Free vibration of a SDOF model with damping



Formulation of equation of motion of a SDOF model with Damping



$$F=Q+D$$

$$h=c/2\sqrt{mk}$$

c :viscous coefficient

h Damping factor

$$\omega=\sqrt{k/m}$$

Equation of motion

$$m\ddot{x}+c\dot{x}+kx=-m\ddot{y}$$

$$\ddot{x}+2h\omega\dot{x}+\omega^2x=-\ddot{y}$$

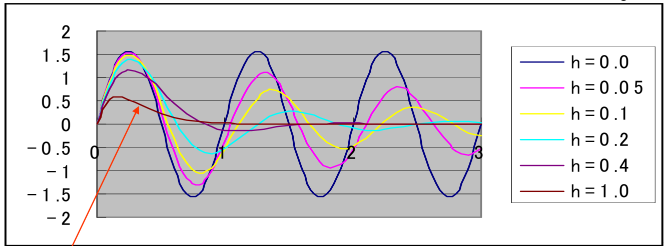
Free vibration of a SDOF model with viscous damping

Decaying function

$$x=\frac{v_0}{\sqrt{1-h^2}\,\omega}e^{-h\omega t}\sin\sqrt{1-h^2}\,\omega t$$

Harmonic function

In the case of initial conditions are $t=0: x=0, v_0=10$



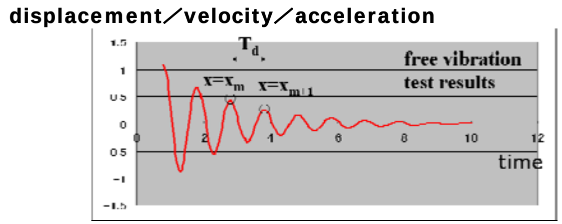
h≧1:Overcritical damping h=1:Critical damping

Calculation of the logarithm damping ratio δ and the damping factor h

$$x=\frac{v_0}{\sqrt{1-h^2}\,\omega}e^{-h\omega t}\sin\sqrt{1-h^2}\,\omega t \Rightarrow x_m/x_{m+1}=e^{h\omega T_d}$$

Amplitude ratio

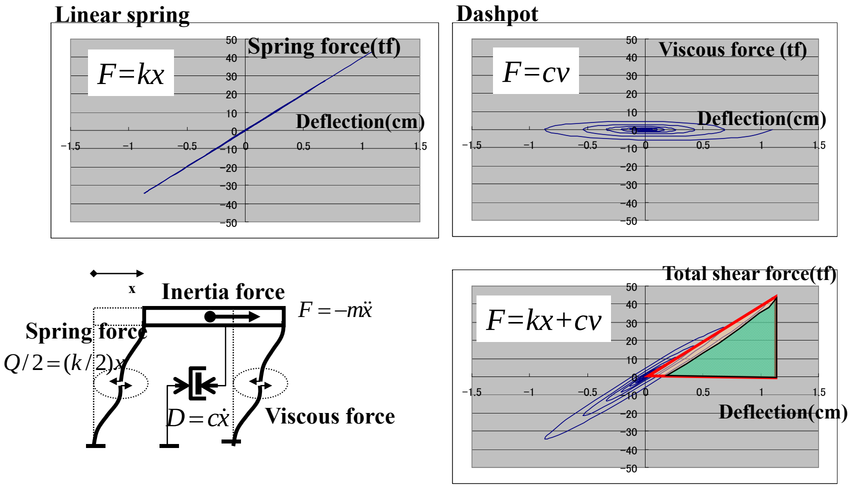
displacement／velocity／acceleration



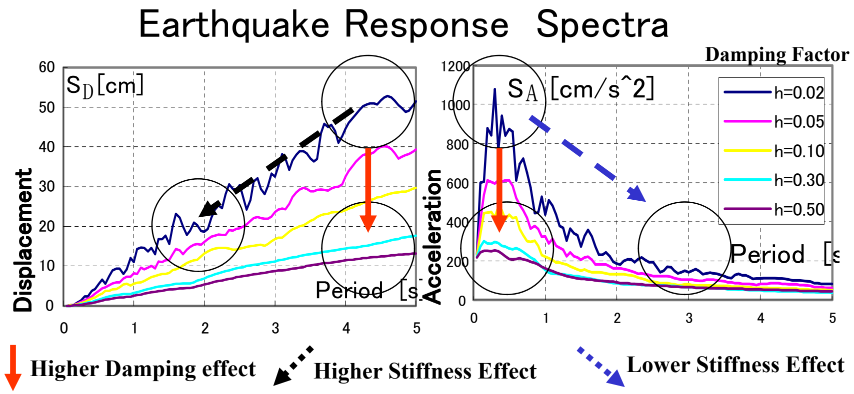
Logarithm damping ratio $\delta=\ln x_m/x_{m+1}=h\omega T_d$

Hence, $\delta\cong 2\pi h$

Energy loss during free vibration of a SDOF linear model with viscous damping

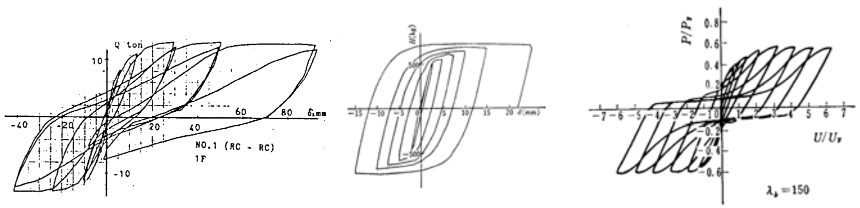


Role of damping in seismic response of a linear SDOF model



Damping Effect : 1)to minimize maximum response
2)to make less sensitive to randomness in structural model and seismic motion

Hysteretic property of different types of structures



a. RC Frame b. Steel Frame c. Braced Steel Frame

Building itself can dissipate input seismic energy, but it is done at the sacrifice of the damage to the main structural members.

Summary of Part 1

- **The cause of structural damping is the energy loss that corresponds to the area of load-deflection relation of structural members.**
- **High damping really works to decrease the seismic response of building structures.**
- **Normal structural members do exhibit damping properties, but it is done at the sacrifice of damage to those structural members.**
- **If we use additional damping device to absorb seismic input energy, we can design building structures without excessive damage to such main structural members as columns, beams walls, etc.**
- **High damping works to make the structural system less sensitive to randomness included in the ground motion, which can improve the accuracy of predicted seismic response.**

END of Lesson One