Tsunamis and Storm Surges

Yoshinobu Tsuji
Earthquake Research Institute, University of Tokyo
Tel. 03-5842-5724, e-mail: tsuji@eri.u-tokyo.ac.jp

1． Basic Equations in Fluid Mechanics

1.1 Basic Equations in Fluid Mechanics

Ocean water and the air act mainly in the Ocean and in the Atmosphere. The word "fluid" is used both liquids such as the water in an ocean as well as for air. A liquid is defined as a body as changing its shape continuously time to time, while that a solid body (or a rigid body) hardly changes its shape.

There are two methods that can be used to describe the motion of fluids mathematically.

One method is called the "Eulerian method." Here, we first take a fixed three-dimensional coordinate system (x, y, z). Then, a velocity vector $\vec{u}(u, v, w)$ and a normal pressure p (or stress tensor in which case we also consider a tangential stress τ if the influence of viscosity is also considered) are expressed as subordinate variables of independent variables of (x, y, z) and time t.

The other method is called the "Lagrangian method". In this method, we consider the initial location (a,b,c) of a fluid particle at an initial time $t = 0$ to be independent variables. The location of the particle (x, y, z) and pressure p (in the form of subordinate valuables) are then considered to be

$x(a,b,c,t), y(a,b,c,t), z(a,b,c,t)$, and $p(a,b,c,t)$.

In reality, the Eulerian method is usually preferred for solving most of the problems in coastal engineering, ocean physics, and meteorology. It is well known that there are only two problems that can be solved better by using the Lagrangian method.

In this study, we derive the basic equations of fluid dynamics by using the Eulerian method.

[Equations of motion]

First, we derive the equations of motion of a perfect fluid, after neglecting its viscosity. A "Perfect fluid" is a fluid in which the stress (pressure p) acts only in a perpendicular direction to any surface in the fluid, and there is no tangential component.

Newton's Second Law of Motion gives the relationship between a force $\vec{F}$ and the induced acceleration $\vec{\alpha}$ for a solid particle; it is expressed given mathematically as follows:

$$\vec{F} = m\vec{\alpha} \qquad (1)$$

Let us transfer this equation of motion into the equations of motion for a fluid. First, we consider the x-component α_x of the acceleration vector $\vec{\alpha}$ of a fluid. We assume that a water particle is located at a point $P(x, y, z)$ at a time $t = t$, and that it has a velocity of $\vec{u}(u, v, w)$. In an infinitesimally small time δt after the initial time, this particle moves to the point $P'(x + u\delta t, y + v\delta t, z + w\delta t)$; hence, α_x is given by

$$\alpha_x = \lim_{\delta t \to 0} \frac{u(x + u\delta t, y + v\delta t, z + w\delta t, t + \delta t) - u(x, y, z, t)}{\delta t} \qquad (2)$$

We expand the part of $u(x + u\delta t, y + v\delta t, z + w\delta t, t + \delta t)$ in the form of Taylor's series as

$$u(x + u\delta t, y + v\delta t, z + w\delta t, t + \delta t) = u(x, y, z, t) + \frac{\partial u}{\partial x} \times u\delta t + \frac{\partial u}{\partial y} \times v\delta t + \frac{\partial u}{\partial z} \times w\delta t + \frac{\partial u}{\partial t} \times \delta t$$

By substituting this into equation (2), we obtain acceleration α_x in the Eulerian style as follows;

$$\alpha_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \qquad (3)$$

The operation $\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$ in this equation appears frequently in fluid mechanics, and we write it in brief $\frac{D}{Dt}$ (called "Lagrangian time differential" or simply "Lagrangian differential.") Physically, this operator is equivalent to "a change in any physical value" (velocity, pressure, temperature, density of salt (salinity), etc.) observed by a hypothetical dwarf "kobito or Songo-ku" riding on the particle. By using this operator, (3) can be expressed simply

$$\alpha_x = \frac{Du}{Dt} \qquad (3\text{-b})$$

Then we can naturally interpret that equation (3) implies that the change (with time) in the particle velocity represents the acceleration.

Next, we discuss the force $\vec{F}$ and the mass m given in equation (1). We consider a small rectangular parallelepiped body of a size $dx \times dy \times dz$ (See Fig. 1).

The mass of the brick m is given by

$$m = \rho dxdydz \qquad (4)$$

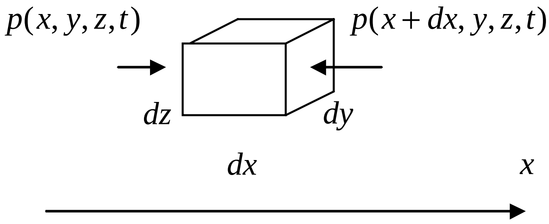


Fig. 1 Force balance in x direction

The x-component of the force $\vec{F}$, F_x can be estimated by considering the difference in pressure between the right and left surfaces, plus the body force coming from "horizontal gravity X" (if it exists). F_x is expressed as follows:

$$F_x = -\{p(x+dx) - p(x)\} \times dy \times dz + X \times \rho dxdydz$$

Since $p(x+dx) = p(x) + \partial p/\partial x \times dx$ approximately, Fx becomes

$$F_x = -\frac{\partial p}{\partial x} dxdydz + Xdxdydz \qquad (5)$$

By substituting (3-b), (4), and (5) into (1), and eliminating the common factor $dxdydz$, we derive the following equation, which is called "the equation of motion for a fliud" or "Euler's equation."

Identical equations can be derived for the y and z components as well and we have The follows:

$$\frac{Du}{Dt} = X - \frac{1}{\rho}\frac{\partial p}{\partial x} \qquad \frac{Dv}{Dt} = Y - \frac{1}{\rho}\frac{\partial p}{\partial y} \qquad \frac{Dw}{Dt} = Z - \frac{1}{\rho}\frac{\partial p}{\partial z} \qquad \text{(6-a, b , c)}$$

Since we are considering the Earth, the force field due to acceleration of gravity is $(0,0,-g)$; therefore, in such a case, the Euler's equations become;

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad \frac{Dv}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \qquad \frac{Dw}{Dt} = -g - \frac{1}{\rho}\frac{\partial p}{\partial z} \qquad \text{(6'-a, b, c)}$$

[Conservation of Mass]

In problems in the fluid mechanics, subordinate variables (=unknowns) are three components of the particle velocity (u, v, w) and pressure p; hence, we can not solve them by using only three equations (6-a, b, c). An additional equation is required, which is obtained from the condition of mass conservation, sometimes referred to "the equation of continuation" in fluid mechanics.

We can classify the various possible conditions into the following three categories:

Case A: For sea waves such as wind waves, tsunamis, and storm surges, we can consider the density of sea water ρ to be constant for the entire region. We refer to this case as "Case A" or "the case of constant water density."

Case B: For internal waves, that is a two-layer system in the sea region at a river mouth, we must consider the stratification (layer structure) of the density of the sea. Here, we cannot neglect the change in density ρ with depth, however, in many cases, we can neglect the influences of diffusion and mixing, which cause possible change in salinity, and the temperature of a particle. In such a case, we can neglect the influence of diffusion, and the water density ρ is assumed to be constant for every particle, even if it is not constant for position and time ($\rho = \rho(x, y, z, t)$). We refer to this case as "Case B" or "non-diffusive case". Here, a dwarf on a particle will not observe any change in density, and so

$$\frac{D\rho}{Dt} = 0 \qquad (7)$$

is satisfied.

Case C: For the vertical mixing of water in a stratified ocean, we must consider the influences of the diffusions of salinity S and water temperature T. Here, we must introduce the diffusion equations for salinity and temperature as;

$$\frac{DS}{Dt} = K_s \nabla^2 S \quad (8) \quad \text{and} \quad \frac{DT}{Dt} = K_T \nabla^2 T \quad (9),$$

Here, K_s and K_T are the diffusion constants for salinity, and temperature, respectively, and in general, $K_T > K_s$. Here, we have seven subordinate valuables (u, v, w, p, S, T, ρ), and the usable equations are the equations of motion (6-a, b, c), the mass conservation equation, two diffusion equations (8) and (9), and in addition, Knussen's density formula given as

$$\rho = \rho(p, S, T) \qquad (10)$$

Case C represents the most general case for ocean problems.

Next, we consider the mathematical formulation for mass conservation condition for Case A.

In Fig. 2, the volume of the brick is given by $dx \times dy \times dz$,

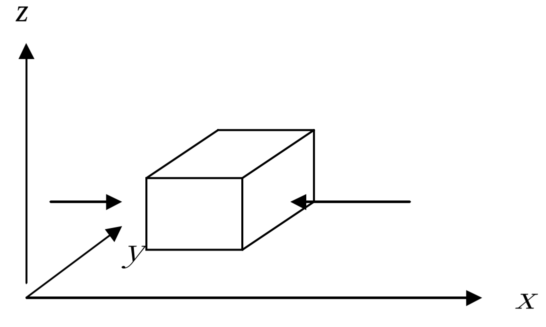


Fig. 2

If we assume that the density is ρ, and that the change in mass in a short time dt is dm, we have

$$\frac{dm}{dt}=\frac{\partial\rho}{\partial t}dxdydz \qquad (11)$$

If we consider the mass to be entering the left surface of the brick and leaving the brick through the right surface, the total mass balance can be expressed as follows: (We substitute $\rho u\equiv U(x)$, and we consider only the x-direction for simplification),

$$-\{U(x+dx)-U(x)\}dydz=-\frac{\partial U}{\partial x}dxdydz=-\frac{\partial(\rho u)}{\partial x}dxdydz \qquad (12\text{-a})$$

The same equations are satisfied for y,z directions as well. The sum of the three-mass balances should be equal to the increase in mass dm. Hence, the mass conservation condition is formulated as follows;

$$\frac{\partial\rho}{\partial t}=-\left(\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}\right) \qquad (13)$$

Eq. (13) is the general form of the mass conservation condition. It is possible to re-write this equation as

$$\frac{D\rho}{Dt}+\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=0 \qquad (13\text{-b})$$

[Mass conservation condition for non-diffusive case]

As mentioned above, for the cases A and B (non-diffusive cases), equation (7) is satisfied. Therefore, along with (13-b), we find that the following equation is the equation of mass conservation for these cases.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (14)$$

For case B, we have five unknowns (u,v,w,ρ,p), and five equations: the three equations of motion (6-a, b, c), the non-diffusion equation (13), and the mass conservation equation (14).

For case A, i.e., the case that the water density is absolutely constant, we have four unknowns (u,v,w,p) and four equations: the equations of motion (6-a, b, c), and the equation of mass conservation (14).

[Diffusion Equation]

Let us consider the concept of diffusion. Diffusion, such as diffusion of heat, diffusion of sugar in a coffee cup, can be defined as “such a motion that in the case heat (or density of sugar in a coffee cup) is inhomogeneous at the initial stage, makes uniform by making (heat or sugar) flux from densely distributed part to less distributed part". Flux of heat (or sugar) is in general proportional to the gradient of heat (or density of sugar).

Let us consider the one-dimensional case (Fig. 3). We assume that metal cubes are arranged one dimensionally (in a line), and that the initial temperature of the i-th cube is T_i. Let us discuss the change in temperature of each metal cube.

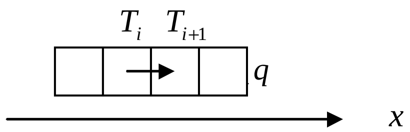


Fig. 3

If we assume that the temperatures of the i-th, and $i+1$-th cubes are T_i, T_{i+1}, then the heat flux q between these two cubes is given by

$$q = -KS\frac{T_{i+1} - T_i}{\Delta x} \quad (15) \quad \text{or} \quad q = -KS\frac{\partial T}{\partial x} \quad (15\text{-b})$$

where the negative "-" implies that heat flows from a hot cube to a cold one. S is the surface area of these cubes and K is the diffusion constant.

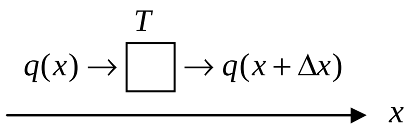


Fig. 4

Next we consider the temperature-change ΔT of a cube (Fig. 4). We can calculate this by estimating the balance between the incoming and outgoing heat, that is,

$$mc\frac{\partial T}{\partial t} = -\{q(x + \Delta x) - q(x)\} = -\frac{\partial q}{\partial x}\Delta x \quad (16)$$

where m is the mass of the cube and c is the specific heat. Since the mass of cube is given by $m = \rho V = \rho S \Delta x$,

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho c S}\frac{\partial q}{\partial x} \quad (17)$$

By differentiating (15-b) with respect to x, and substituting it into (17), q is eliminated, and we have

$$\frac{\partial T}{\partial t} = \frac{K}{\rho c}\frac{\partial^2 T}{\partial x^2} \quad (18)$$

If we apply $K/\rho c = K_T$, we finally obtain the equation of diffusion for a one dimensional case as follows;

$$\frac{\partial T}{\partial t} = K_T\frac{\partial^2 T}{\partial x^2} \quad (19)$$

For a three-dimensional case, the equation of diffusion is expressed as follows;

$$\frac{\partial T}{\partial t} = K_T \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) T \qquad (20)$$

Equation (20) is the diffusion equation for heat in a solid body, that is, for the case that the cube itself does not move. For the case that heat or sugar in fluid, the media (particle) itself moves, the left-hand side $\partial T/\partial t$ of these equations should be replaced by DT/Dt (Lagrangian differentiation).

$$\frac{DT}{Dt} = K_T \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) T \qquad (21)$$

In the case of an actual ocean or atmosphere, the diffusion coefficient K is not always constant across the entire region under consideration, moreover, and moreover it takes different values in the horizontal and vertical directions in the discussions of geophysical phenomena. For such cases, we introduce horizontal and vertical diffusion coefficients K_H, K_V which are different to each other. For such cases, the equation of diffusion takes the following form:

$$\frac{DT}{Dt} = \{ \frac{\partial}{\partial x}(K_H \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(K_H \frac{\partial T}{\partial y}) + \frac{\partial}{\partial z}(K_V \frac{\partial T}{\partial z}) \} \qquad (22)$$

Equation (22) is generally used while discussing oceanic or atmospheric problems.

1.2 Kinematic Conditions at Boundaries

We next consider the fluid conditions at boundaries, such as the sea bottom or surface of a sea. We assume that a sea surface can be expressed in the form $z = f(x, y)$ at a time $t = t$. Since such a function is a multi-valued function of z, it is better to express it in the style of an implicit function $F(x, y, z) = C$, in general. This function is only a snapshot image at a time $t = t$.

For more general cases, we can express the boundary function to be in the form of

$$F(x, y, z, t) = C \qquad (23)$$

We assume that a particle is situated at a point (x, y, z) on the boundary at a time $t = t$. If we substitute these values (x, y, z, t), in (23), (23) must be satisfied because the particle is situated on the boundary. An infinitesimally small time dt after the initial time, the particle moves with flow (u, v, w) to the point P' $(x + udt, y + vdt, z + wdt)$, where the particle is still on the boundary, unless the particle is "separated" from the boundary in some manner. Thus, the next condition will be satisfied.

$$F(x+udt, y+vdt, z+wdt, t+dt) = C \tag{24}$$

We expand the left-hand side of (24) in the form of a Taylor series, and we have

$$F(x,y,z,t) + \frac{\partial F}{\partial x}udt + \frac{\partial F}{\partial t}vdt + \frac{\partial F}{\partial t}wdt + \frac{\partial F}{\partial t}dt = C \tag{24-b}$$

By comparing (23) with (24-b), we have

$$\frac{\partial F}{\partial t} + u\frac{\partial F}{\partial x} + v\frac{\partial F}{\partial y} + w\frac{\partial F}{\partial z} = 0, \qquad \text{that is,} \qquad \frac{DF}{Dt} = 0 \tag{25}$$

This equation is called the "kinematic boundary condition," and it should be satisfied both for a rigid boundary (such as a sea bed) and a free surface (such as a sea surface).

If the sea surface is expressed in the form $z = \zeta(x,y,t)$, we can re-write it as $z - \zeta(x,y,t) = 0$. Then, (23) becomes $F = z - \zeta(x,y,t), C = 0$. We substitute this into (25), and thereby obtain,

$$-\frac{\partial \zeta}{\partial t} - u\frac{\partial \zeta}{\partial x} - v\frac{\partial \zeta}{\partial y} + w = 0,$$

that is,

$$w = \frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} \tag{26}$$

This is the kinematic boundary condition for sea surface waves. The operator $\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}$ can be written as a two-dimensional Lagrangian operator, i.e., $D_2/D\,t$, then (26) is then written as follows

$$w = \frac{D_2\zeta}{Dt} \tag{27}$$

If the sea bed is expressed as $z = b(x,y)$, we use b instead of ζ.

$$w = u\frac{\partial b}{\partial x} + v\frac{\partial b}{\partial y} \tag{28}$$

is then the sea bed kinematic condition. For a two-dimensional case (no change in the y direction), (28) becomes

$$w = u\frac{db}{dx} \quad (29) \qquad \text{or} \qquad \frac{w}{u} = \frac{db}{dx} \quad (29\text{-b})$$

1.3 Conservation of Vorticity

Definition of Vorticity

We introduce the definition of a "vortex vector" $\vec{\omega}$.

If we assume that the velocity field is given by $\vec{u}(u,v,w)$, we define a vorticity vector by operating the rotation operator ($\nabla \times$) on the velocity vector $\vec{u}$.

A "rotation operator" is expressed by the outer product (★外積) of the nabla operator $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$, and thus, we have

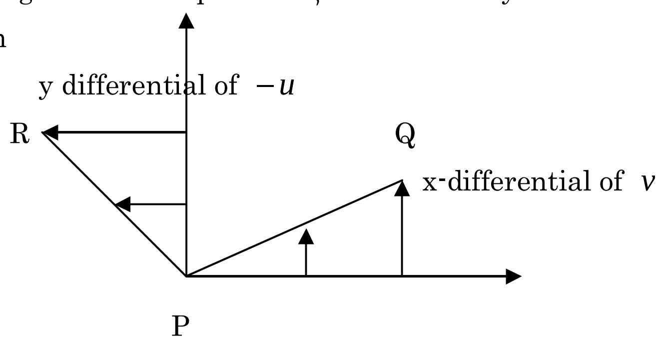
$$\vec{\omega}(\xi,\eta,\zeta) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \left(w_y - v_z, \quad u_z - w_x, \quad v_x - u_y \right) \qquad (30)$$

Here, $\vec{i}, \vec{j}, \vec{k}$ are the unit vectors in the $x, y,$ and z directions, respectively. Each component of the vorticity vector is written as

$\xi = w_y - v_z$, $\eta = u_z - w_x$, $\zeta = v_x - u_y$ (31-a, b, c).

Let us consider the meaning of the z-component ζ of the vorticity vector.

ζ can be regarded as the sum of the y differential of $-u$ and the x-differential of v,



which is equal to twice the average of the change speed with the directions OX and OY around the origin O. If we assume that an ant is placed at O, we see that the water particle Q on the x-axis moves around O with an angular speed v_x, while the water particle R on the y-axis moves around O with an angular speed $-u_y$. Thus, ζ-value means a local rotation around O (for the ant), and the value of ζ denotes the local rotation (= the speed of the change in the local direction of the next-door particles) at the point O. Hence, it is possible to use ζ to express "vorticity".

Conservation of Vorticity

Let us assume that the water density is constant.

First differentiating the y-component of the Euler's equation (6-b) by x and、 the

x-component（6-a）by y, and then subtract the results. In this manner, the pressure term p on the right-hand side is eliminated and after a rather long and complicated calculation, we arrive at the following.
(Note: We use the condition of continuity (14) in this calculation)

$$\frac{D\zeta}{Dt} = u_z\xi + v_z\eta + w_z\zeta \qquad (32\text{-c})$$

Similarly, the following equations can be derived for the x and y- component of the vorticity ξ, η

$$\frac{D\xi}{Dt} = u_x\xi + v_x\eta + w_x\zeta .... (32\text{-a})$$

$$\frac{D\eta}{Dt} = u_y\xi + v_y\eta + w_y\zeta \qquad (32\text{-b}).$$

Let us consider the meaning of the equations（32-a, b, c）. If there is no vorticity at an initial time $t = 0$, that is, if $\xi = \eta = \zeta = 0$ at t = 0, then the right-hand side of equations(32-a,b,c) becomes zero. The left-hand side expresses the change in the local rotation for the ant on the particle. Thus, equations (32-a, b, c) imply that the vorticity remains zero value even with an increase in time. In other words, "If there is no vortex initially in a perfect fluid having constant density, there will be no vortex even after time elapses."

Moreover, we can also prove that "If vorticity exists at an initial time, it will never vanish." There two assertions are called "The Lagrangian theory of vortex conservation."

We can say that, even if when we deal with problems on wind waves or tsunamis, no vorticity exists at any time if the waves are generated initially from still sea.

We refer to such a motion of liquids with no vorticity as "non-vortex motion."

1.4 Existence of the Velocity Potential Function ϕ

In the case of non-vortex motion, that is, when ξ = η = ζ = 0 has to be satisfied, we can introduce a velocity potential scalar function ϕ for the fluid motion and the components of the velocity vector $\vec{u}(u,v,w)$, .are given by the gradient of ϕ.

$$u = -\frac{\partial \phi}{\partial x}, v = -\frac{\partial \phi}{\partial y}, w = -\frac{\partial \phi}{\partial z} \qquad (33)$$

Note: In the present discussion, we set "—" at the top of the right hand side. This is because we intend to use the analogy of the relation between contour lines and rain drops (a stream over the surface of a hill). However, in many other text books, "— " is not introduced. This does not lead to any essential problem.

The two lemmas can be phrased as follows:

Lemma A: The flud is non-vortex nature.

Lemma B: A velocity potential function exists.

Here, we can easily prove that B $\Rightarrow$ A as follows:

We substitute (33) into the z -component of the vorticity ζ ; then we have

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -\frac{\partial^2 \phi}{\partial y \partial x} + \frac{\partial^2 \phi}{\partial x \partial y} = 0 \ldots\ldots \qquad (34)$$

Similarly, it can be shown that both ξ and η are zero; hence, and the flow is proved to be a non-vortex motion.

However, the proof of A $\Rightarrow B$ requires a slightly longer discussion, which is beyond the scope of this book. (refer a textbook of " vector analysis").

In this study, let us simply agree on this.

Note: When density is not constant (i.e., the case of a stratified fluid), the vorticity conservation law is not satisfied, and hence, we cannot introduce the velocity potential function ϕ for such cases.

1.5 Bernoulli's Theory

We proved that it is possible to represent the flow distribution $\vec{u}(u, v, w)$ by using a scalar function $\phi(x, y, z, t)$ for a non-vorticity fluid with constant density. By using the velocity function ϕ, the equation of continuity (14) can be re-written in the following form:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \ldots\ldots \qquad (35)$$

The operator $\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ is called "Laplacian Operator", and is sometimes written simply as "∇^2" or "Δ." The formula that satisfies the above equation is called "harmonic function" and takes the following general form:

(1) For an x, y, and z coordinate system

A product of sinusoidal and exponential functions

$$\phi = e^{kc} e^{ly} e^{mz} : k^2 + l^2 + m^2 = 0$$

(2) For a cylindrical coordinate system (r, θ, z)

A product of sinusoidal and Bessel's functions

$$\phi = \cos n\theta \times J_n(kr)$$

(3) For a spherical coordinate system

A product of sinusoidal and Legendre's Functions.

Next, we re-write the equations of motion (Euler's equations) by using the velocity potential function ϕ. We re-write (u, v, w) in (6-a) by (33), and we obtain

$$-\phi_{xt} + \phi_x\phi_{xx} + \phi_y\phi_{xy} + \phi_z\phi_{xz} = -\frac{1}{\rho} p_x .....(36)$$

Note the following relations

$$\phi_x\phi_{xx} = \frac{1}{2}\left(\phi_x^2\right)_x, = (\frac{1}{2}u^2)_x, \quad \phi_y\phi_{xy} = \frac{1}{2}\left(\phi_y^2\right)_x = (\frac{1}{2}v^2)_x,$$

and

$$\phi_z\phi_{xz} = \frac{1}{2}\left(\phi_z^2\right)_x = \left(\frac{1}{2}w^2\right)_x$$

Since, all the terms in equation (36) are those that were differentiated with respect to x, so, we can integrate it by x, yielding

$$-\phi_t + \frac{1}{2}\left(u^2 + v^2 + w^2\right) + \frac{p}{\rho} = F_1(y, z, t) \qquad \text{(37-a)}$$

where $F_1(y, z, t)$ in the right-hand side is an arbitrary function of y, z, t.

Similarly, begin from equations (6-b) and (6-c); we finally obtain (37-b) and (37-c).

$$-\phi_t + \frac{1}{2}\left(u^2 + v^2 + w^2\right) + \frac{p}{\rho} = F_2(x, z, t) \qquad \text{(37-b)}$$

$$-\phi_t + \frac{1}{2}\left(u^2 + v^2 + w^2\right) + \frac{p}{\rho} = -gz + F_3(x, y, t) \qquad \text{(37-c)}$$

Note that (37-a), (37-b), and (37-c) are independent of each other even though they have similar forms, and they must be satisfied independently. To satisfy all these three equations, we introduce an arbitrary function $F(t)$ and write it in the form of the following equation.

$$-\phi_t + \frac{1}{2}\left(u^2 + v^2 + w^2\right) + gz + \frac{p}{\rho} = F(t) \qquad (38)$$

By doing this, equations (37-a,b,c) are all satisfied simultaneously.

Equation (38) is called "Bernoulli's Law."

[Bernoulli's theory as taught to high school students]

In high school, "Bernoulli's Law".is usually taught as part of the Physics course. It is taught that "In the case of steady flow, at two points 1 and 2 on one streamline, the next value is constant”

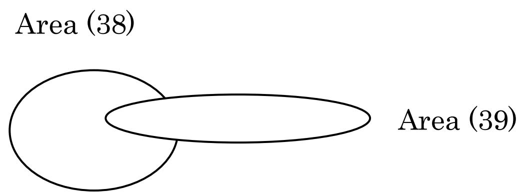
$$p_1 + \rho g h_1 + \frac{1}{2}\rho V_1^{\ 2} = p_2 + \rho g h_2 + \frac{1}{2}\rho V_2^2 \qquad (39)$$

where p is pressure, h is height, and V is velocity. We can derive (39) by assuming that the energy of a unit volume is constant. (39) and (38) are very similar to each other, and (39) is also called “Bernoulli’s formula.”

However, note that the assumptions of A and B are different, and we should apply them to different problems.

(38) Not steady, not always on a stream line, non-vorticity

(39) Steady, two points on a stream line, vorticity might exist



Please determine which equation(s) is(are) applicable for solving the following problems:

1. Ocean waves
2. Blood flow in a human body
3. Coffee rotating in a tea cup
4. Flow in a river
5. Brake oil in a car
6. Flow in a water supply pipe
7. Thin water layer on a pavement on a rainy day