International Institute of Seismology
and Earthquake Engineering (IISEE)
Seismology Course Lecture Notes

TRAINING COURSE IN
SEISMOLOGY AND EARTHQUAKE ENGINEERING II

Theory and Practice

For Calibration of Moving Coil Type Seismometer.

2nd Edition

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1. Introduction

The equation of motion of the pendulum mass for moving coil type seismometer (Fig. 1) is

$$\ddot{x}(t) + 2\omega_0 h\dot{x}(t) + \omega_0^{\,2}\; x(t) = -\,\ddot{y}(t), \tag{1}$$

where $x(t)$ denotes the displacement of the pendulum mass relative to the flame of seismometer, $\ddot{y}(t)$ the input ground acceleration, ω_0 the natural angular frequency ($\omega_0 = 2\pi f_0 = 2\pi / T_0$), f_0 the natural frequency, T_0 the natural period, h the damping constant which is the sum of the mechanical one h_m and electro-magnetic one h_e which is controlled by the shunt resistance R_s (*e.g.,* Yokoi (1996)). The discussion here is always based on this equation of motion.

If we can apply a controlled input ground displacement $y(t)$ or acceleration $\ddot{y}(t)$, direct calibration can be performed. Controlled $y(t)$, e.g., can be provided by a shaking table.

Calibration can be applied also by making use of the electro-magnetic force induced by the input current that is given to the transducer coil itself or to the calibration coil if exists. Willmore impedancebridge is an effective tool to separate the output current from input one in case of single coil seismometer (*e.g.*, Willmore, 1959).

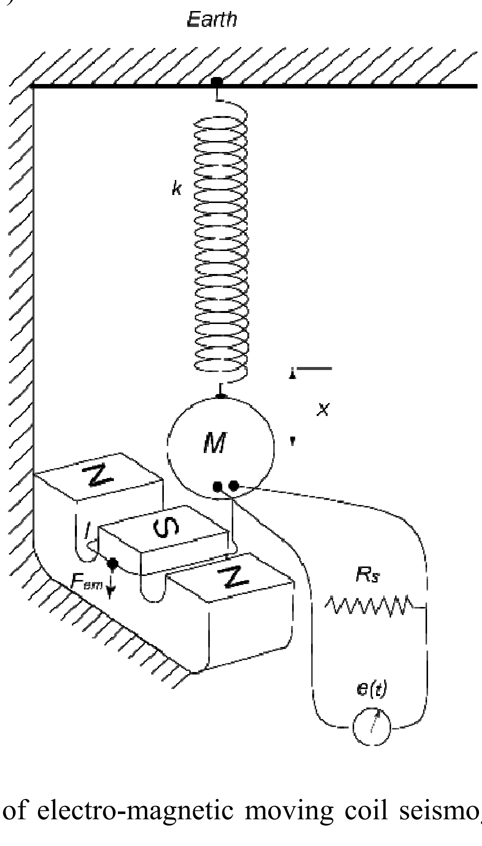


Fig.1 Simplified sketch of electro-magnetic moving coil seismograph. When a coil attached to the mass moves through the magnetic field, the voltage across the coil terminal is proportional to the relative velocity between mass and magnet. Redrawn based on Aki and Richards (1980).

2. Theory and Method

2.1. Calibration on Shaking Table

The output voltage of the seismometer directly connected to the parallel shunt resistance R_s as shown in Fig.2 is given as follows.

$$e(t) = \frac{GR_s}{R_0 + R_s} \cdot \dot{x}(t) \ , \tag{2}$$

where G is the electro-dynamical constant so called the sensitivity, R_0 the coil resistance of the seismometer. A shaking table gives a sinusoidal input displacement

$y(t) = y_m exp(i\omega t)$

with arbitrary angular frequency $\omega = 2\pi f$, where f is the frequency. The mass displacement and the output voltage may also have sinusoidal form.

$x(t) = x_m exp(i\omega t),$

$e(t) = e_m exp\ (i\omega t).$

Then, Eq.(1) can be solved in the form

$$\frac{-x_m}{y_m} = \frac{1}{1 - 2ih(\omega_0/\omega) - (\omega_0/\omega)^2} \ . \tag{3}$$

The ratio of the output voltage e_m to the input ground velocity $i\omega y_m$ at ω is derived from Eq. (2) and (3).

$$\begin{aligned} \frac{e_m}{i\omega y_m} &= \frac{-e_m}{i\omega x_m} \cdot \frac{-i\omega x_m}{i\omega y_m} \\ &= \frac{GR_s}{R_0 + R_s} \cdot \frac{1}{1 - 2ih(\omega_0/\omega) - (\omega_0/\omega)^2} \ . \end{aligned} \tag{4}$$

Consider the relation $\omega_0 = 2\pi f_0 = 2\pi / T_0$, $\omega = 2\pi f = 2\pi / T$,then,

$$-i\{1 + (R_0 + R_s)\} \cdot \frac{e_m T}{2\pi y_m} = \frac{G}{1 - 2ih(T/T_0) - (T/T_0)^2} \ . \tag{5}$$

Define the function $F_1(T)$ by the absolute value of this function.

$$F_1(T) = \{1 + (R_0 / R_s)\} \cdot \frac{e_m T}{2\pi y_m} = \frac{G}{\sqrt{\{1 - (T/T_0)^2\}^2 + 4h^2 (T/T_0)^2}} \ . \tag{6.1}$$

The high frequency asymptote of $F_1(T)$ gives G itself (Fig.3).

The argument of the both members of Eq.(5) gives the phase lag of the output voltage from the input ground displacement $Arg(e_m/y_m)$. The function $P_1(T)$is defined as follows.

$$P_1(T) = Arg(e_m / y_m) - \frac{\pi}{2} = -\tan^{-1}\left[-2h(T/T_0)/\{1 - (T/T_0)^2\}\right] + 2N\pi \tag{6.2}$$

At the natural period $T=T_0$, $P_1(T)$ takes the value $\pi/2$.

Fig.2 Circuit for the shaking table test.

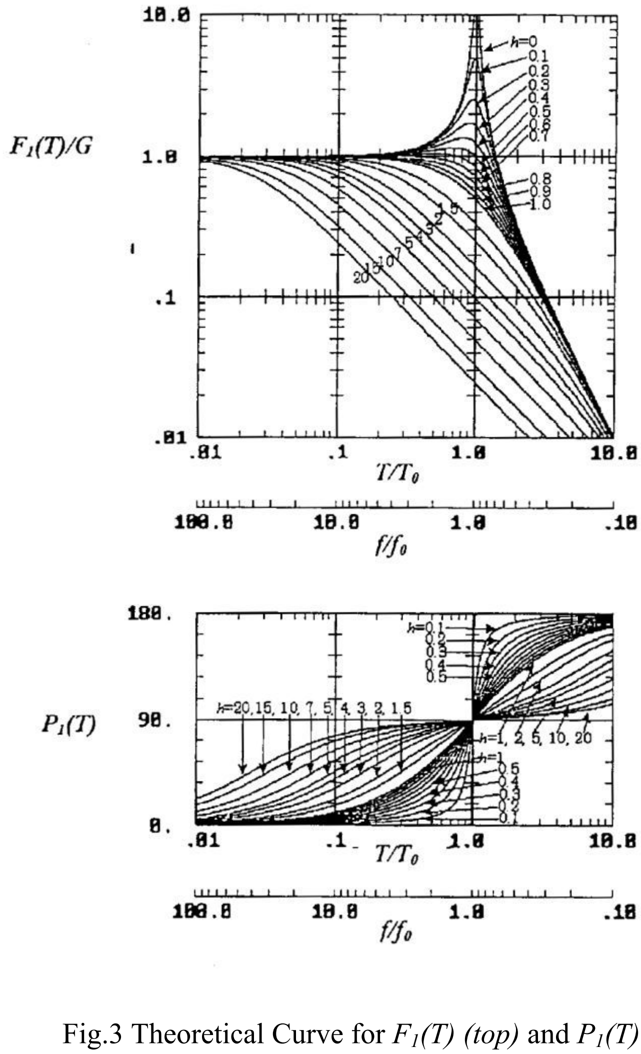


Fig.3 Theoretical Curve for $F_1(T)$ *(top)* and $P_1(T)$*(bottom)*.

2.2. Calibration using Willmore Impedance Bridge

The circuit shown in Fig.4 is selected for convenience in the experiment. Any voltage imbalance between *A* and *B*: V_{AB}, can not give any influence on the potential difference between *a* and *b*: V_{ab}, because the impedance bridge is balanced.

Suppose that the pendulum of the seismometer is locked, namely, there is not any secondary induced current in the circuit. The potential difference V_{ab} due to the electromotive force *E(t)* produces the current *i(t)* in the resistance *R*.

$$i = \frac{E(t)(1+R_0/R_s)}{R(1+R_0/R_s)+R_0} . \tag{7}$$

If the current in the coil resistance R_0 is *I*, that in the shunt resistance R_s must be *i-I* and these two resistors make the same potential difference $R_0I=R_s(i-I)$. Then,

$$I = \frac{i(t)}{1+R_0/R_s} = \frac{E(t)}{R(1+R_0/R_s)+R_0} . \tag{8}$$

This is the primary current input given to the seismometer by the function generator.

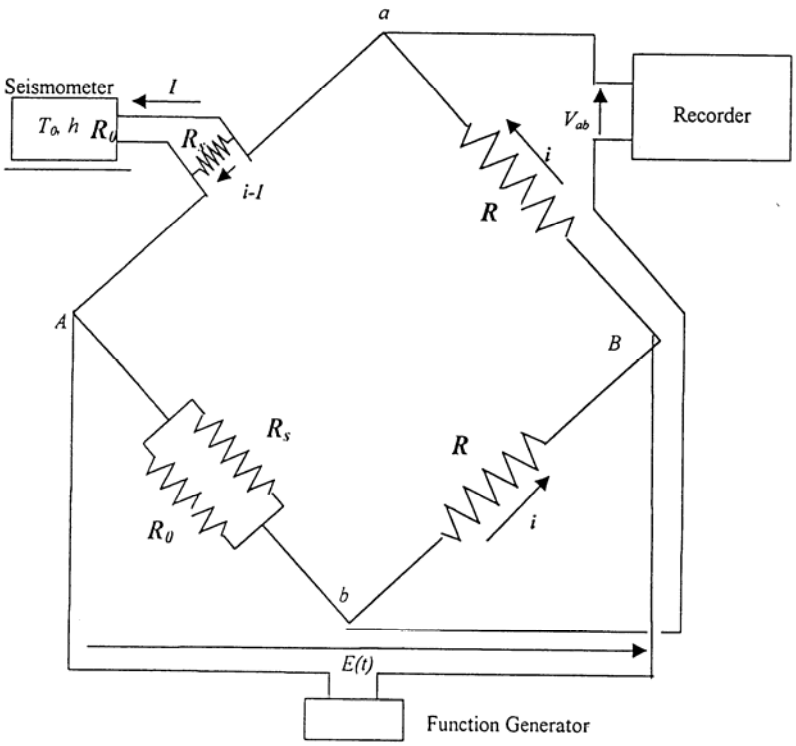


Fig.4 Circuit for Willmore impedance bridge (Redrawn after Aoki (1994)).

Note: (-) and (G) of the seismometer should be connected together with (-) of the recorder. (G) and (-) of the recorder should not be connected.

Release the pendulum. The current introduced to the seismometer's coil induces an electro-magnetic force that applies to the pendulum.

$$F_{em} = G \cdot I = \frac{GE(t)}{R(1 + R_0 / R_s) + R_0} \ , \tag{9}$$

where m is the weight of the pendulum mass for the linear motion type and the ratio of the inertial moment of the pendulum to the arm length of electromagnetic force for the rotation type. In the absence of input ground motion: $\ddot{y}(t) \equiv 0$.

This force must appear in the equation of motion as follows.

$$\ddot{x}(t) + 2\omega_0 h\dot{x}(t) + \omega^2{}_0 x(t) = \frac{-GE(t)}{m\{R(1 + R_0 / R_s) + R_0\}} - \ddot{y}(t) \ . \tag{10}$$

The velocity of mass motion $\dot{x}(t)$ produces the secondary electromotive force

$$E^s(t) = G \cdot \dot{x}(t) \ ,$$

and the secondary induced current $I^s(t)$ i.e., the output of the seismometer.

$$I^s(t) = \frac{E^s(t)}{R_0 + R'_s} = \frac{G}{R_0 + R'_s} \cdot \dot{x}(t) \ ,$$

where R'_s is total shunt resistance which consists of all resistors in the impedance bridge.

$$R'_s = \frac{1}{(1/R_s) + \left\{2R + (1/R_0 + 1/R_s)^{-1}\right\}^{-1}} = R_s \cdot \frac{(1 + R_0/R_s + R_0/2R)}{(1 + R_0/R_s)\cdot(1 + R_s/2R) + R_0/2R} \ .$$

R'_s tends to R_s at $R>>R_s>R_0$, and to $2R+R_0$ at infinite R_s.

The potential difference V_{aA}, produced by this current can be calculated as follows

$$\left\{2R + \left(R_0^{-1} + R_s^{-1}\right)^{-1}\right\} \cdot I_{ab} = R_s(I^s - I_{ab}) \ .$$

Then, $$I_{ab} = I^s \cdot \frac{R_s}{R_s + 2R + \left(R_0^{-1} + R_s^{-1}\right)^{-1}} \tag{11}$$

The potential difference V_{ab} is

$$V_{ab} = -2RI_{ab} = -2RI^s \cdot \frac{R_s}{R_s + 2R + \left(R_0^{-1} + R_s^{-1}\right)^{-1}} = -R''_s I^s \ ,$$

where

$$\mathrm{R_s}'' = 2\mathrm{R} \cdot \frac{\mathrm{R_s}}{\mathrm{R_s} + 2\mathrm{R} + \left(\mathrm{R_0}^{-1} + \mathrm{R_s}^{-1}\right)^{-1}} = \mathrm{R_s} \cdot \frac{1 + \mathrm{R_0}/\mathrm{R_s}}{(1 + \mathrm{R_0}/\mathrm{R_s}) + \left(\mathrm{R_s}^{-1} + 2\mathrm{R_0}\right)/2\mathrm{R}}$$
$$= 2\mathrm{R} \cdot \frac{1}{1 + 2\mathrm{R}/\mathrm{R_s} + \mathrm{R_0}/\left(\mathrm{R_0} + \mathrm{R_s}\right)} \ .$$

This also tends to R_s at $R>>R_s>R_0$, and to $2R$ at infinite R_s.

Then, $$V_{ab} = -\frac{GR''_s}{R_0 + R'_s} \cdot \dot{x}(t) \ . \tag{12}$$

2.2.1. Sinusoidal Current Input.

Assume the sinusoidal feature with angular frequency ω for all signals, i.e.

$x(t) = x_m exp\text{-}i\omega t),$

$y(t) = y_m\ exp\text{-}i\omega t),$

$E(t) = E_m\ exp(i\omega t),$

$V_{ab}(t) = (V_{ab})_m\ exp\text{-}i\omega t),$

The ratio of relative mass motion to the input voltage is derived in the following formula substituting them in Eq. (10).

$$\frac{x_m}{E_m} = \frac{-G}{m\{R(1+R_0/R_s)+R_0\}} \cdot \frac{1}{\{1-2ih(\omega_0/\omega)-(\omega_0/\omega)^2\}(-\omega^2)} . \qquad (13)$$

Eq.(12) can be changed in the following form, because the differentiation on time corresponds to the product with $-i\omega$ in this case.

$$\frac{(V_{ab})_m}{x_m} = \frac{-i\omega GR''_s}{R_0 + R'} .$$

Then, the ratio of the output voltage $(V_{ab})_m$ to the input voltage imbalance E_m is given as

$$\begin{aligned}\frac{(V_{ab})_m}{E_m} &= \frac{(V_{ab})_m}{x_m} \cdot \frac{x_m}{E_m} \\ &= \frac{-iR''_s}{R_0 + R'_s} \cdot \frac{G^2}{\omega m\{R(1+R_0/R_s)+R_0\}} \cdot \frac{1}{\{1-2ih(\omega_0/\omega)(\omega_0/\omega)^2\}} .\end{aligned} \qquad (14)$$

The absolute value is

$$\left|\frac{(V_{ab})_m}{E_m}\right| = \frac{R''_s}{R_0+R'_s} \cdot \frac{G^2}{\omega m\{R(1+R_0/R_s)+R_0\}} \cdot \frac{1}{\sqrt{\{1-(T/T_0)^2\}^2 + 4h^2(T/T_0)^2}} . \qquad (15.1)$$

The phase lag is

$$Arg\left(\frac{(V_{ab})_m}{E_m}\right) = -\frac{\pi}{2} - \tan^{-1}\left[-2h(T/T_0)/1-(T/T_0)^2\right] + 2N\pi . \qquad (15.2)$$

Define two functions $F_2(T)$ and $P_2{=}(T)$ as follows.

$$F_2(T) = \frac{\omega(R_0 + R'_s)\{R(1+R_0/R_s)+R_0\}}{R'_s} \cdot \left|\frac{(V_{ab})_m}{E_m}\right| = \frac{G^2}{m} \cdot \frac{1}{\sqrt{\{1-(T/T_0)^2\}^2 + 4h^2(T/T_0)^2}} . \qquad (16.1)$$

where $\omega=2\pi/T$. The high frequency asymptote of $F_2(T)$ gives the value of G^2/m.

$$\begin{aligned}P_2(T) &= Arg\left(\frac{(V_{ab})_m}{E_m}\right) + \frac{\pi}{2} \\ &= -\tan^{-1}\left[-2h(T/T_0)/\{1-(T/T_0)^2\}\right] + 2N\pi .\end{aligned} \qquad (16.2)$$

Note that $F_2(T)$ is almost same as $F_1(T)$ except the scaling factor, and $P_2(T)$ is the same function as $P_1(T)$. Then, the shape of $F_2(T)$ and $P_2(T)$ also can be seen in Fig.3.

2.2.2. Step Current Input.

Assume that the input voltage imbalance is a step function $E(t) = E_0 \cdot H(t)$, where $H(t)$ denotes the Heviside function. The time domain solution of the equation of motion

$$\ddot{x}(t) + 2\omega_0 h\dot{x}(t) + \omega_0{}^2 x(t) = \frac{-GE_0 H(t)}{m\{R(1+R_0/R_s)+R_0\}}\,, \tag{17}$$

is given by Kitsunezaki and Goto (1964) as

$$\dot{x}(t) = \begin{cases} -GE_0/m\{R(1+R_0/R_s)+R_0\}\cdot e^{-\omega_0 ht}/\omega_0\sqrt{1-h^2}\cdot\sin\left(\omega_0\sqrt{1-h^2}\cdot t\right), h<1.0, \\ -GE_0/m\{R(1+R_0/R_s)+R_0\}\cdot t\cdot e^{-\omega_0 t}\,, & h=1.0, \\ -GE_0/m\{R(1+R_0/R_s)+R_0\}\cdot\sinh\left(\omega_0\sqrt{h^2-1}\cdot t\right), & h>1.0. \end{cases} \tag{18}$$

Substitute it in Eq.(12) and the output voltage change is given as follows.

$$V_{ab}(t) = \begin{cases} \left(G^2E_0R''_s\right)/\left[m(R_0+R'_s)\{R(1+R_0/R_s)+R_0\}\right]\cdot e^{-\omega_0 ht}/\omega_0\sqrt{1-h^2}\cdot\sin\left(\omega_0\sqrt{1-h^2}\cdot t\right), h<1.0, \\ \left(G^2E_0R''_s\right)/\left[m(R_0+R'_s)\{R(1+R_0/R_s)+R_0\}\right]\cdot t\cdot e^{-\omega_0 t}\,, & h=1.0, \\ \left(G^2E_0R''_s\right)/\left[m(R_0+R'_s)\{R(1+R_0/R_s)+R_0\}\right]\cdot\sinh\left(\omega_0\sqrt{1-h^2}\cdot t\right), & h>1.0. \end{cases} \tag{19}$$

This shows the apparent period T_d, related to the natural one.

$$T_d = \begin{cases} T_0/\sqrt{1-h^2}\,, h<1.0, \\ T_0\,, & h=1.0, \\ T_0/\sqrt{h^2-1} & h>1.0. \end{cases}$$

The damping constant can be estimated by measuring the damping ratio, *i.e.*, the amplitude ratio of peaks and following troughs for h<1. When the damping constant h is close to 1.0, it is difficult to determine its value precisely by means of damping ratio. Kitsunezaki and Goto(1964) has given the relation of h with T_w, which is the time when record reduces its value to 0.2 times the maximum value.

Eq.(19) gives the amplitude of the first peak as follows,

$$(V_{ab})_{f.p.} = \frac{\left(G^2E_0R_s''\right)}{\left[m\{R(1+R_0/R_s)+R_0\}(R_0+R'_s)\right]\omega_0 P(h)}\,, \tag{20}$$

where

$$P(h) = \begin{cases} \exp\left\{h/\sqrt{1-h^2}\tan^{-1}\sqrt{1-h^2}/h\right\} h<1.0, \\ 2.718\,, & h=1.0, \\ \exp\left\{h/\sqrt{h^2-1}\tanh^{-1}\sqrt{h^2-1}/h\right\} h>1.0. \end{cases}$$

and the suffix $_{f.p.}$ denotes the amplitude of the first peak. The value of G^2/m is given by the absolute value of Eq.(20) in the following formula.

$$\frac{G^2}{m} = \frac{\left[\{R(1+R_0/R_s)+R_0\}(R_0+R'_s)\right]\omega_0 P(h)}{E_0R_s''}\cdot\left|(V_{ab})_{f.p.}\right|\,. \tag{21}$$

2.3. Calibration using Calibration Coil

If the seismometer has the second coil for calibration as show in Fig.5, the output current from the transducer coil can't be mixed with the input current given to the calibration coil. Then, any impedance bridge is not required. Electro motive force given to the second coil E_2 produces the input current I_2 in the second coil.

$$I_2 = E_2 / R_2,$$

where R_2 is the resistance of the calibration coil.

This current gives the electro-magnetic force

$$F_{em} = G_2 \cdot I_2, \tag{23}$$

where G_2 is the electro-dynamical constant of the calibration coil.

The equation of motion for the pendulum mass is

$$\ddot{x}(t) + 2\omega_0 h \dot{x}(t) + \omega_0{}^2 x(t) = \frac{-G_2 E_2(t)}{mR_2} \ . \tag{24}$$

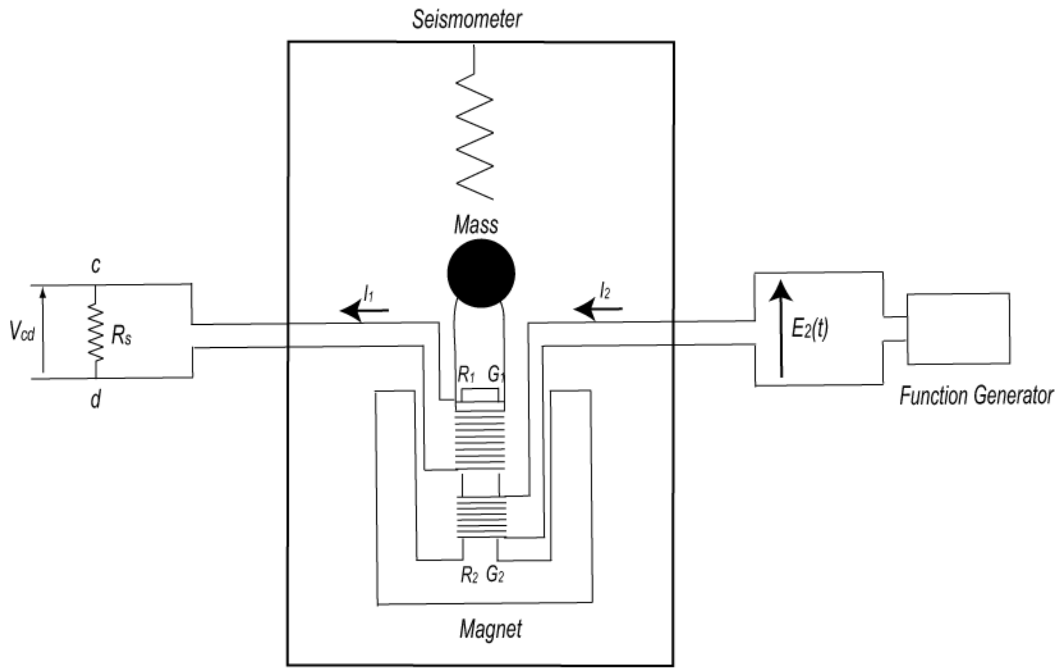


Fig.5 Circuit for the calibration coil.

2.3.1. Sinusoidal Current Input

Assume the sinusoidal feature with angular frequency ω for all signals. The equation of motion is solved in the form

$$\frac{x_m}{(E_2)_m} = \frac{G_2}{mR_2} \cdot \frac{1}{\{1 - 2ih(\omega_0/\omega) - (\omega_0/\omega)^2\}\omega^2} \ . \tag{25}$$

The ratio of output voltage to the mass displacement is given as

$$\frac{(V_{cd})_m}{x_m} = \frac{-i\omega G_1 R_s}{R_0 + R_s} \ .$$

Then,

$$\frac{(V_{cd})_m}{(E_2)_m} = \frac{(V_{cd})_m}{x_m} \cdot \frac{x_m}{(E_2)_m} = \frac{-i\omega G_1 G_2 R_s}{(R_0 + R_2) m R_2} \cdot \frac{1}{\{1 - 2ih(\omega_0/\omega)(\omega_0/\omega)^2\}\omega^2} \tag{26}$$

The absolute value is

$$\left|\frac{(V_{cd})_m}{(E_2)_m}\right| = \frac{R_s}{(R_0 + R_s)\omega R_2} \cdot \frac{G_1 G_2}{m} \cdot \frac{1}{\sqrt{\{1 - (T/T_0)^2\}^2 + 4h^2 (T/T_0)^2}} \ . \tag{27.1}$$

The phase lag is

$$Arg\left(\frac{(V_{cd})_m}{x_m}\right) = -\frac{\pi}{2} - \tan^{-1}\left[-2h(T/T_0)/1 - (T/T_0)^2\right] + 2N\pi \ . \tag{27.2}$$

Define two functions $F_3(T)$ and $P_3(T)$ as follows.

$$F_3(T) = \frac{(R_0 + R_s)\omega R_2}{R_s} \cdot \left|\frac{(V_{cd})_m}{(E_2)_m}\right| = \frac{G_1 G_2}{m} \cdot \frac{1}{\sqrt{\{1 - (T/T_0)^2\}^2 + 4h^2 (T/T_0)^2}} \ . \tag{28.1}$$

where $\omega = 2\pi/T$. The high frequency asymptote of $F_3(T)$ gives the value of G_1G_2/m.

$$P_3(T) = Arg\left(\frac{(V_{cd})_m}{x_m}\right) + \frac{\pi}{2} = -\tan^{-1}\left[-2h(T/T_0)/1 - (T/T_0)^2\right] + 2N\pi \ . \tag{28.2}$$

$P_3(T)$ is equal to $\pi/2$ at $T=T_0$

Note that $F_3(T)$ is almost same as $F_1(T)$ except of the value for high frequency asymptote, and $P_3(T)$ is the same function as $P_1(T)$. Then, the shape of $F_3(T)$ and $P_3(T)$ also can be seen in Fig.3.

Sensitivity of seismometer is defined as follows

$$G = lB \tag{29}$$

where l represents the length of coil wire within the magnetic field, B denotes the flux density of magnetic field. Then, the ratio of the electro-dynamical constants of two coils can be given by the ratio of their resistances.

$$G_1/G_2 = l_1B/l_2B = l_1/l_2 = R_0/R_2$$

If we have the value of m, we can estimate the value of G_1 and G_2 independently.

$$G_1 = \sqrt{(R_0/R_2) \cdot mF_3(0)} \ . \ G_2 = \sqrt{(R_2/R_0) \cdot mF_3(0)} \ . \tag{30}$$

2.3.2 Step Current Input

Assume that the input voltage imbalance is a step function $E_2(t)=E_0 \cdot H(t)$, where $H(t)$ denotes the Heviside function. The time domain solution of the equation of motion

$$\ddot{x}(t) + 2\omega_0 h\dot{x}(t) + \omega_0{}^2 x(t) = \frac{-G_2 E_0 H(t)}{mR_2} ,$$

is given as

$$\dot{x}(t) = \begin{cases} -G_2 E_0 / mR_2 \cdot e^{-\omega_0 ht} / \omega_0 \sqrt{1-h^2} \cdot \sin(\omega_0 \sqrt{1-h^2} \cdot t) \ , h<1.0, \\ -G_2 E_0 / mR_2 \cdot t \cdot e^{-\omega_0 t} \ , \qquad h=1.0, \\ -G_2 E_0 / mR_2 \cdot \sinh(\omega_0 \sqrt{h^2-1} \cdot t), \qquad h>1.0. \end{cases} \tag{31}$$

The natural period and the damping constant can be estimated in the way described above Consideration on the circuit which consists of the seismometer and the shunt resistance R_s gives

$$V_{cd}(t) = -\frac{G_1}{1+R_0/R_S} \cdot \dot{x}(t) , \tag{32}$$

Then,

$$V_{ab}(t) = \begin{cases} (G_1 G_2 E_0)/[mR_2(1+R_0/R_s)] \cdot e^{-\omega_0 ht} / \omega_0 \sqrt{1-h^2} \cdot \sin(\omega_0 \sqrt{1-h^2} \cdot t) \ , h<1.0, \\ (G_1 G_2 E_0)/[mR_2(1+R_0/R_s)] \cdot t \cdot e^{-\omega_0 t} \ , \qquad h=1.0, \\ (G_1 G_2 E_0)/[mR_2(1+R_0/R_s)] \cdot \sinh(\omega_0 \sqrt{h^2-1} \cdot t) \ , \qquad h>1.0. \end{cases}$$

The amplitude of the first peak is given

$$(V_{cd})_{f.p.} = \frac{G_1 G_2 E_0}{mR_2(1+R_0/R_s)\omega_0 P(h)} , \tag{33}$$

where the suffix *f.p.* denotes the amplitude of the first peak, $p(h)$is given by Eq.(20).

The value of G_1G_2/m is given in the following,

$$\frac{G_1 G_2}{m} = \frac{\left|(V_{cd})_{f.p.}\right| R_2(1+R_0/R_s)\omega_0 P(h)}{E_0} . \tag{34}$$

3. Examples of Application

3.1 Experiments on Shaking Table

The vertical component of the seismometer L-22D manufactured by Mark Product Ltd. is selected for the object of experiment. This is so old that its dynamic characteristics have been changed already. Moreover, the housing can be opened easily. These are enough for us to have doubt that the values of parameter in the specification sheet are unreliable.

Electro - Dynamic Test System in the Structural Testing Laboratory of the BRI consists of Function generator, Controller, Band Pass Filter, Shaker which gives the input motion to the tested seismometer, two Shaking tables, Vibrometer and Hydraulic unit. Shaker and Shaking table are located on the platform isolated from the rest of the building (Fig.7, Akashi, 1974). One of the shaking tables corresponds to vertical motion and another horizontal one. Each shaking table has a reference seismometer which is the differential transformer. The objective seismometer is fixed on the shaking table and its output is conserved on the recording paper simultaneously with the output of the reference seismometer.

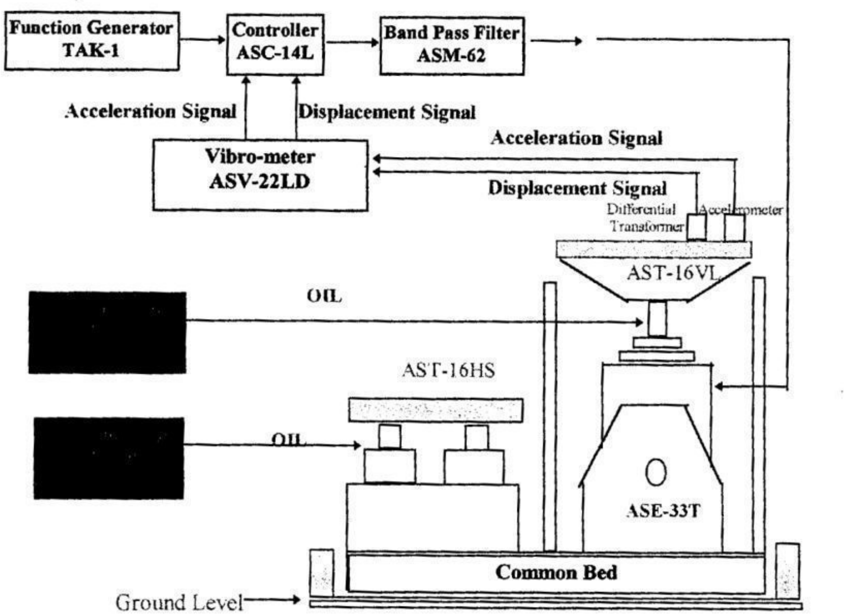


Fig.7 Electro-Dynamic Shaking Test System

At first, it is evaluated that the sensitivity of the reference seismometer is approximately 200 Volt per meter. The calibration process is the followings. Sinusoidal motion is provided to the shaking table by Controller and the amplitude of table's displacement relative to the flame is measured by one's eye with a scale. This amplitude is compared with the amplitude of the output voltage change of the reference seismometer. Thus, high accuracy can not be expected. At least an error of 5 % must be taken into account for the sensitivity of the reference seismometer.

Two sinusoidal curves are drawn on the recording paper which is the output voltage of the tested seismometer and that of the reference one. Besides, the timing signal is drawn by the inner clock of the recorder. Then, we can determine the period or the frequency of the signals, the amplitudes of two seismograms and the delay of the peak between two sinusoidal records (Fig.8).

The period T in second was calculated by the following.

$$T = D_s / D_c \quad , \qquad (42)$$

where $D_{s,}$ is the length of one cycle of the signal on the recording paper measured in meter, D_c the length measured in meter on the recording paper which corresponds to one second.

A similar procedure is applied to find the amplitude voltage values of the tested and the reference seismometers. The displacement amplitude $y_m(f)$ measured in meter of the shaking table is given by the following.

$$y_m(f) = \{A_{ref}(f) \cdot G_{rec}\}/G_{ref} \qquad (43)$$

where $A_{ref}(f)$ measured in meter is the amplitude of the output the reference seismometer on the recording paper, G_{rec}, the gain of the recorder measured Volt per meter, G_{ref} the sensitivity of the reference seismometer which is 200 Volt per meter in this case. The amplitude of the output voltage of the tested seismometer $e_m(f)$ measured in Volt e given as,

$$e_m(f)=A(f) \cdot G_{rec}{}^{v} , \qquad (44)$$

where $A(f)$ measured in meter is the amplitude of the output of the tested seismometer on the recording paper, the gain of the recorder measured in Volt per meter.

The function $P_1(f)$ measured in degree is given by the following.

$$P_1(f)=360-\{L(f)/D_s \cdot 360\}+90. \qquad (45)$$

where $L(f)$ is the delay measured in meter on the recording paper which is the length from the reference seismogram's peak to tested one's peak.

The measurements are conducted for five values of shunt resistance. Calculated $F_1(T)$ and $P_1(T)$ are plotted in Fig.9 in comparison with several theoretical curves. The value of G_1 (70 v/m/sec) is estimated from the high frequency asymptote of $F_1(T)$ and the natural period T_0(0.42 sec) from the intercept of $P_1(T)$ with 90 degree. Values of the damping constant are evaluated by comparison with theoretical curves. The correspondence of the damping constant to the shunt resistance is 0.38-infinite, 0.57-10 Kolm, 0.92-2 Kohm, 1.15-1 Kohm and 1.45- 200 Ohm.

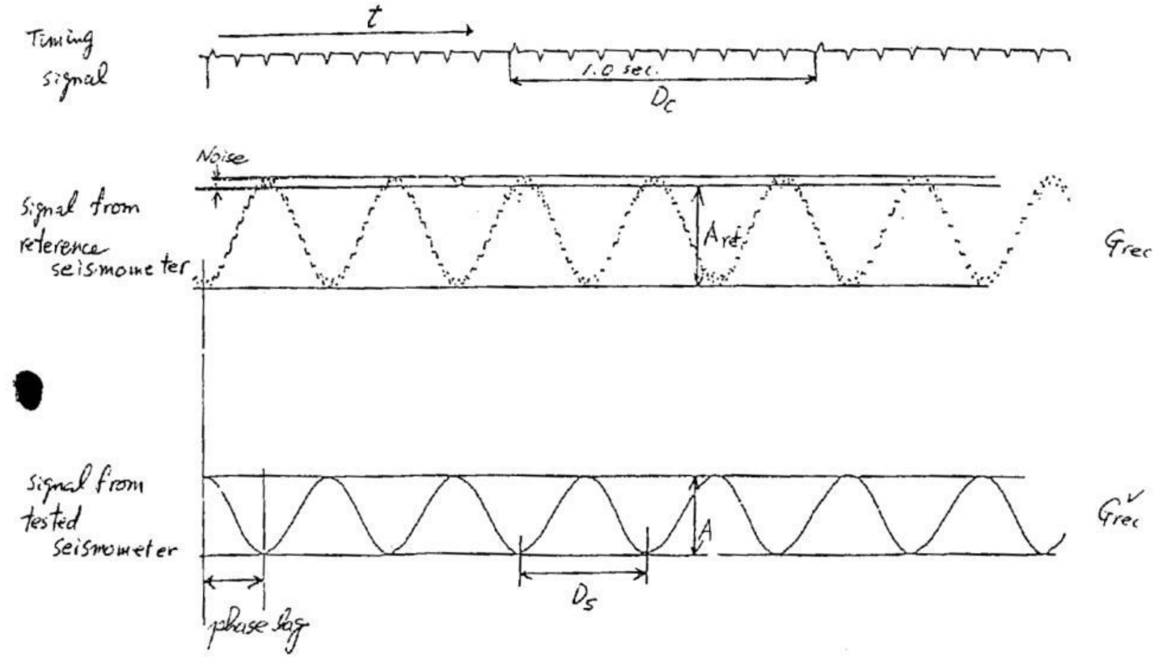


Fig.8 Measurement on recording paper for sinusoidal input

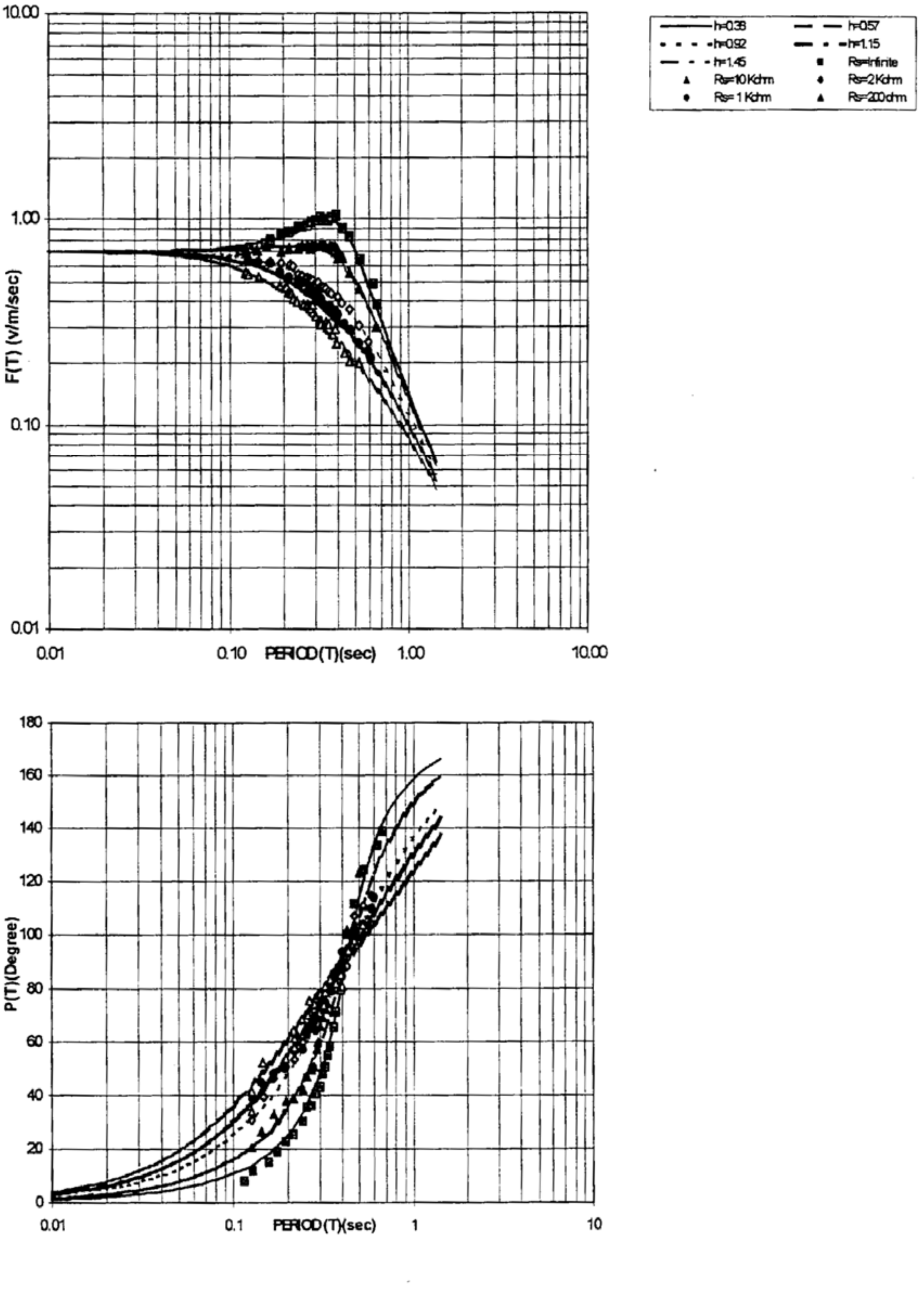


Fig.9 Calculated $F_1(T)$ and $P_1(T)$ plotted in comparison with several theoretical curves.

The damping constant h is given in the form

$$h = h_m + \frac{G^2}{2m\omega_0 (R_0 + R_s)} , \tag{46}$$

where m is the mass of the pendulum. Fig.10 shows the plot of h against $1/(R_0+R_s)$ for the data obtained above, where R_0 is equal to 2.25 Kohm for this experiment. The line in Fig.10 shows a good fitting to the plotted data, of which formula is

$$h = 0.38 + 2.4 \times 10^3 / (R_0 + R_s). \tag{47}$$

where two resistance are measured in ohm.

The slope of the line gives

$$G^2/m=2.4\times 10^3 \cdot 4\pi/T_0=7.18\times10^4(\mathrm{v}^2/\mathrm{Kg\ m}^2/\mathrm{sec}^2). \tag{48}$$

This and the value of G evaluated above give that of pendulum mass m

$$m= 0.068(\mathrm{kg}) =68(\mathrm{g})$$

The value of the shunt resistance which corresponds to h=0.64 can be evaluated by Fig.10. R_s=7.45 Kohm may give an appropriate value of the shunt resistance.

The estimated value of mass m=0.068 (Kg) slightly differs from that given in the specification sheet m=0.0728(Kg). In this experiment, the sensitivity of the reference seismometer is assumed to be 200 (v/m), and can have an error of, e.g., about 5%. If the sensitivity of the reference seismometer is assumed to be 193.6(v/m), the G obtained from the high frequency asymptote of F_1 *(f)* will be 72.3 (v/m/sec) and the mass m is 0.0728 (Kg) for the same measured data. Thus, this discrepancy of the value for m may be caused by the rough estimation of the sensitivity of the reference seismometer.

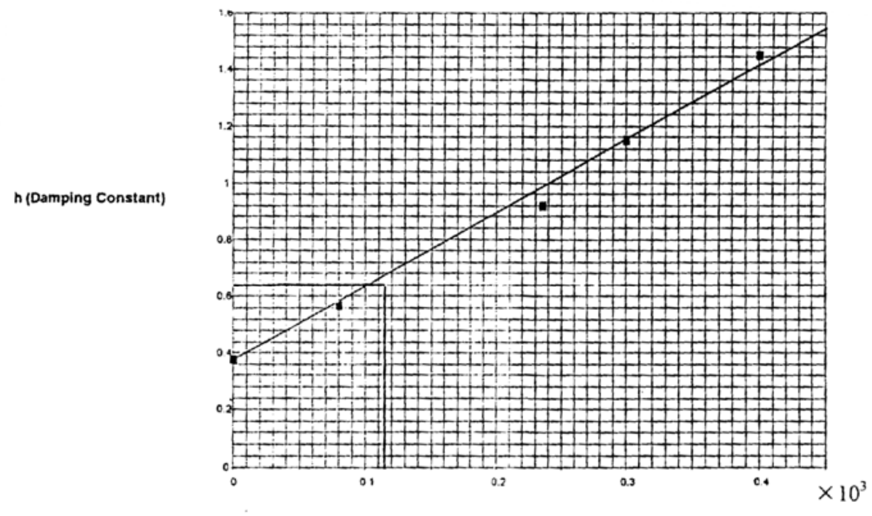


Fig.10 h against $1/(R_0+R_s)$ obtained in the calibration on Shaking Table

3.2. Experiment using Willmore Impedance Bridge

The same L-22D seismometer used above is the object of the experiment. Similar process of measurement on the recording paper as described above are conducted (See Fig.8) with impedance bridge (R=100 Kohm).

3.2.1. Sinusoidal Current Input

Then, $F_2(T)$ and $P_2(T)$ are obtained as shown in Fig.11. Fig.12 shows h against $1/(R_0+R_s)$for the data obtained above, where R_0 is equal to 2.25 Kohm for this experiment. The values of h and R_s are (0.38-infinitive, 0.57-10 Kohm, 0.71-5.1 Kohm, 0.92-2 Kohm, 1.15-1 Kohm, 1.45-200 Ohm). The line in Fig.12 is

$$h=0.38+2.45\times10^3/(R_0+R_s).$$

This is consistent for the values G^2/m =7.0 $\times10^4$ (v^2/ Kg m^2/sec^2)), T_0=0.42(sec) obtained from Fig.11.
The value of the shunt resistance which corresponds to h=0.64 can be evaluated by Fig.12 That is R_s=7.18 Kohm which is the appropriate value of the shunt resistance.

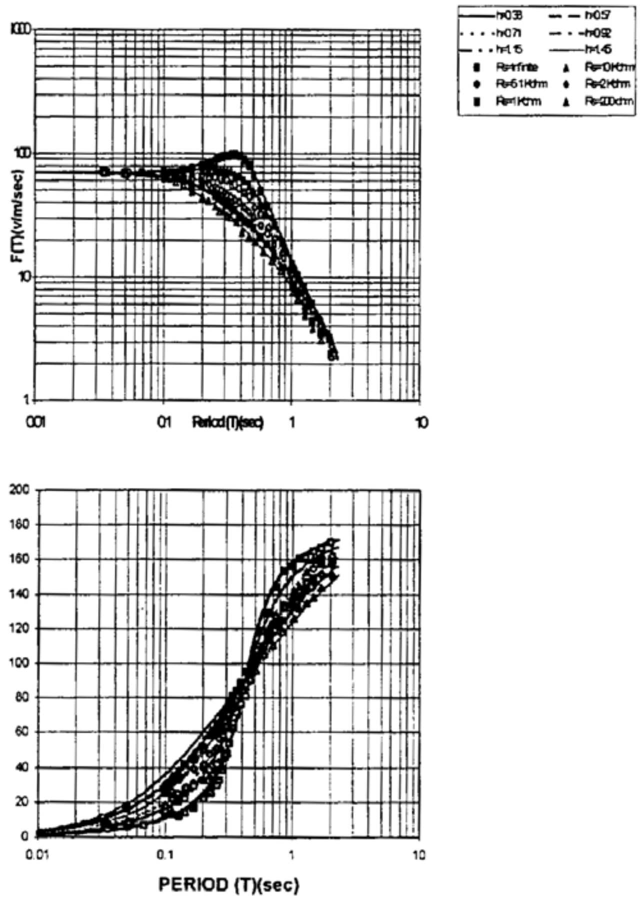


Fig.11 $F_2(T)$ and $P_2(T)$ in comparison with several theoretical curves.

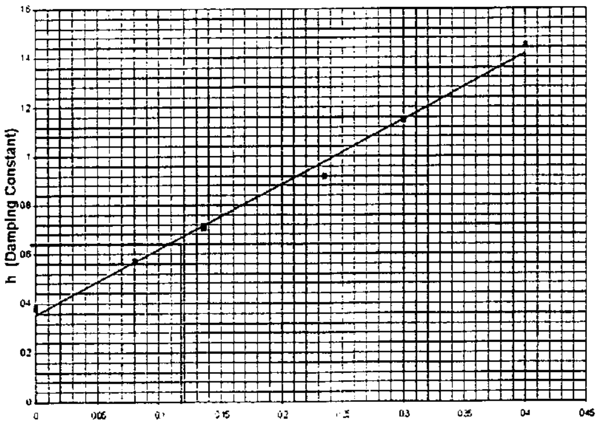


Fig. 12 h against $1/(R_0+R_s)$ for the result shown in Fig. 11.

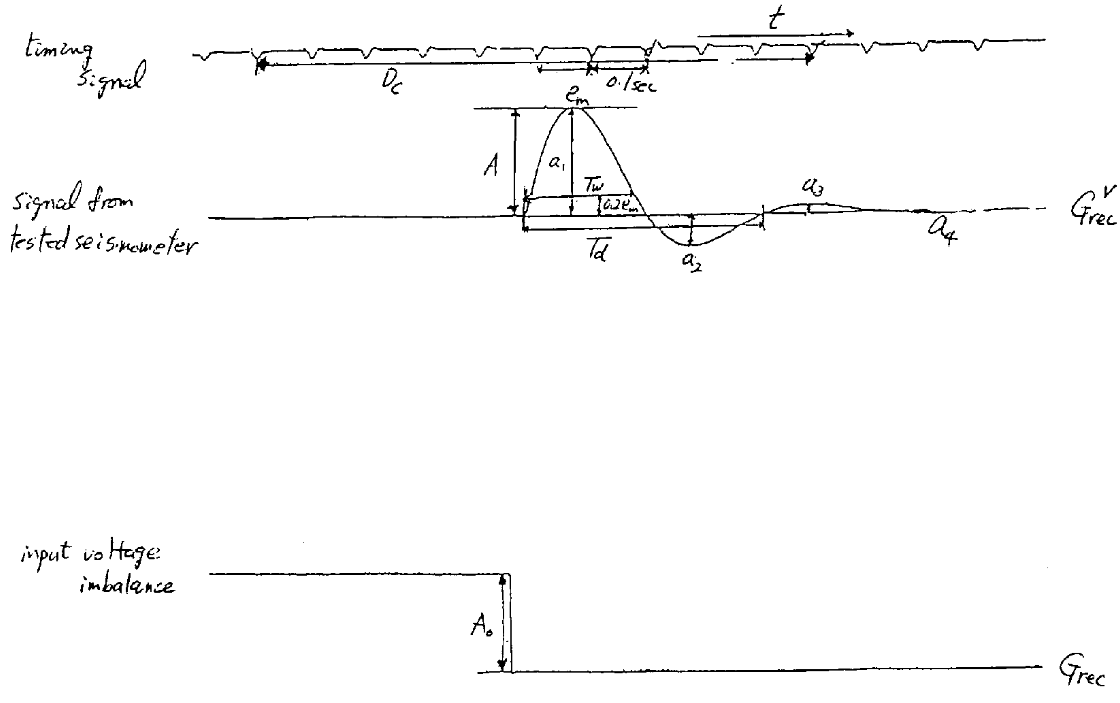


Fig.13 Measurement on recording paper for step current input

3.2.2. Step Current Input

Two wave trains are drawn on the recording paper which are V_{ab} and $V_{AB}=E_0$. Beside of them, the timing signal is drawn by the inner clock of the recorder.

The damping ratio v and damping factor h were calculated by the following formula.

$v=a_1/a_2=a_2/a_3=a_3/a_4$, $h=0.733\log_{10} v$

where a_1 is amplitude of the lust peak of the output voltage of the tested seismometer, a_2 is that of the second peak, etc.. For the overdamped case and the case h near to 1.0, the estimation used T_w is conducted. The time in that $E(t)$ takes the amplitude which is 0.2 times of the first peak T_w is also measured on the recording paper (Fig.13).

The apparent period T_d (sec) is calculated by the following formula with the measured values of D_s and D_c.

$T_d = D_s / D_c$,

where D_s is the length in meter of one cycle of the signal on the recording paper, D_c the length in meter on the recording paper which corresponds to one second. The natural period T_0 in second and the natural angular frequency ω_0 is calculated with the estimated values of h and T_d $(T_0 = \sqrt{1-h^2} \cdot T_d,\ \omega_0 = 2\pi / T_0)$.

The amplitude of the input voltage of the tested seismometer $E_0(f)$ is given ,

$$E_0=A_0 \cdot G_{rec}, \tag{49}$$

where A_0 is the amplitude of the step of input function measured in meter on the recording paper, G_{rec} the gain of the recorder in volt per meter. The amplitude of the output voltage of the tested seismometer $(V_{ab})_{f.p.}$ is given ,

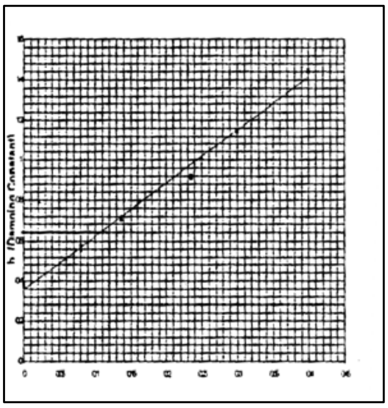
$$(V_{ab})_{f.p.} = A \cdot G_{rec}^{v} , \tag{50}$$

where A is the first peak's amplitude of the output of the tested seismometer on the recording paper, G^{v}_{rec} the gain of the recorder.

The obtained values of h and corresponding R_s are (0.38-infinite, 0.57-10 Kohm, 0.71-5.1 Kohm, 0.92-2 Kohm, 1.15 -1 Kohm, 1.45-200 Ohm). The calculated value of the natural period is $T_0=0.42$ second and the value of G^2/m is equal to $7.2\ X10^4$ (v^2/ Kg m^2/sec^2).

h against $1/(R_0+R_s)$ for the obtained data is shown in Fig.14, where R_0 is equal to 2.25 Kohm for this experiment. The line in Fig.14 is

$h=0.38+2.45 \times 10^3/(R_0+R_s)$.



This is consistent for the G^2/m obtained above from peak amplitude. The value of the shunt resistance which corresponds to h=0.64 can be evaluated by Fig.14. An appropriate value of the shunt resistance is given by R_s =7.18 Kohm.

The sensitivity calculated by $G^2/m=7.2X10^4$ (v^2/ Kg m^2/sec^2) and $m=0.0728$ (Kg) is $G=72.4$ (V/m/sec). This is acceptable compared with the value in the specification sheet ($G=75.7X10^4$).

3.3. Experiment using Calibration Coil

The vertical component of the seismometer L-4C-3D manufactured by Mark Product Ltd. is selected for the subject of practice. The weight of the pendulum mass is 0.96 (Kg) according to the specification sheet. The transducer coil resistance is 5.5 Kohm, where as the calibration coil resistance is 19 ohm.

3.3.1. Sinusoidal Current Input

Similar measurement on recording paper described in Fig.8 is conducted at various frequencies between 0.5 and 30.0 Hz with infinite value of shunt resistance. The results are plotted in Fig.15 with the theoretical curve of which parameters are G_1G_2/m=124.16 (v^2/ Kg m^2/sec^2), m=0.96 Kg., T_0=0.935 second and h=0.23.

These give

G_1= 185.7 (v/m/sec), G_2=0.642 (v/m/sec).

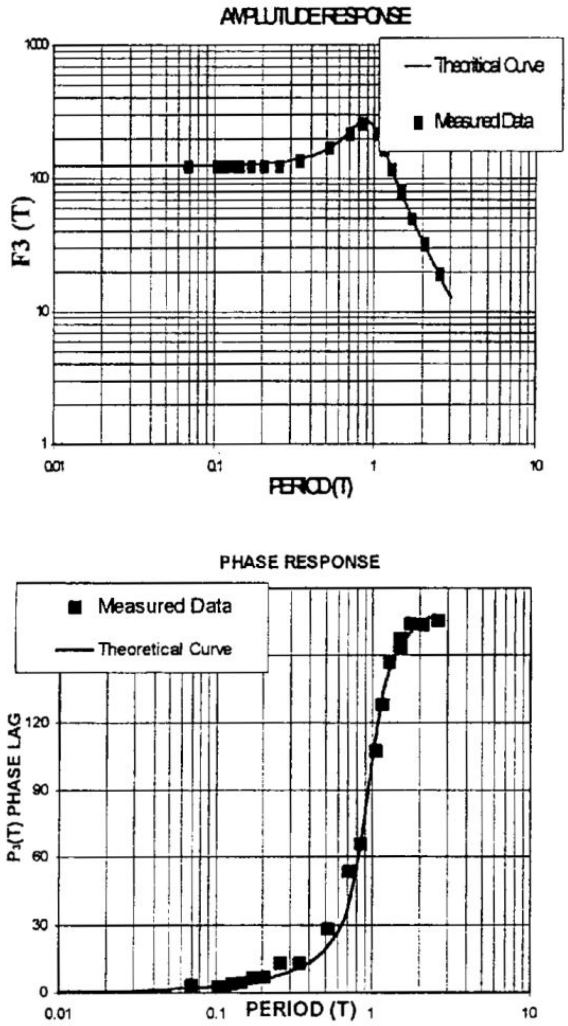


Fig.15 $F_3(T)$ and $P_3(T)$ in comparison with theoretical curve

3.3.2. Step Current Input

Similar measurement as described in Fig. 13 is conducted for nine values of shunt resistance. The obtained values of h and corresponding R_s are (0.23-infinitive, 0.29-100 Kohm, 0.37-51 Kohm, 0.52-20 Kohm 0.63-10 Kohm, 0.76-5.1 Kohm, 1.02-2 Kohm, 1.1-1 Kolm, 1.24- 510 Ohm). The natural period is $T_0=0.935$ second and the value of G_1G_2/m is equal to *124.14* (v^2/ Kg m^2/sec^2). h against $1/(R_0+R_s)$ for the data obtained above is plotted in Fig.16, where R_0 is equal to 5.5 Kohm for this experiment. The curve in Fig.16 is

$$h=0.23+5.95\ X10^3/\ (R_0+R_s)$$

This is consistent for the values $G_1G_2\ /m$ evaluated above. By using the value of mass in the specification sheet and those of coil resistances, the following two parameters are separated.

G_1= 185.74 (v/m/sec), G_2=0.642 (v/m/sec)

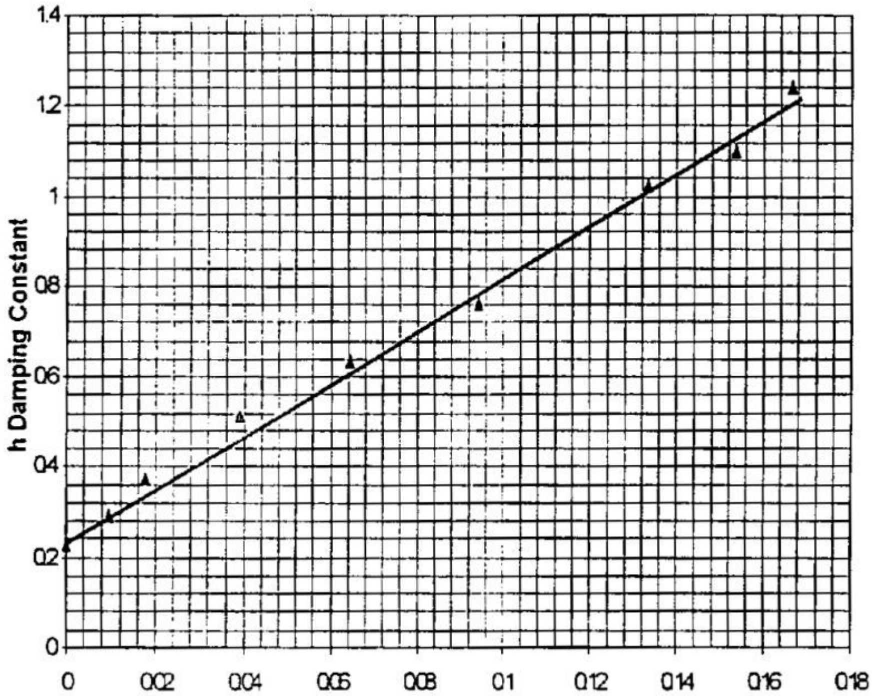


Fig16 h against $1/(R_0+R_s)$

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