2. Refraction/Wide-Angle Reflection Seismology

- 2.1 The Slope-Intercept Time Method
- 2.2 Asymptotic Ray Theory
  - Elastodynamic equation
  - High-frequency solutions
  - Basic equations of the ray method
    - Eikonal equation
    - Transport equation

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2. Refraction/Wide-Angle Reflection Seismology

- 2.3 Ray Tracing
  - Fermat's principle
  - Euler's equations
  - Examples
- 2.4 Travel Time Inversion
  - Time-term method
  - Travel time inversion

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2. Refraction/Wide-Angle Reflection Seismology

- 2.5 Reflectivity method
  - Displacement potentials
  - Displacement-strss vector and potential vector
  - Boundary conditions and layer matrix
  - Integral representation

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2.1 The Slope-Intercept Time Method

- Horizontally two-layered case (e.g. Palmer, 1986)
- Dipping layer case (e.g. Palmer, 1986)
- Horizontally multi-layered case (Kennet, 1976)

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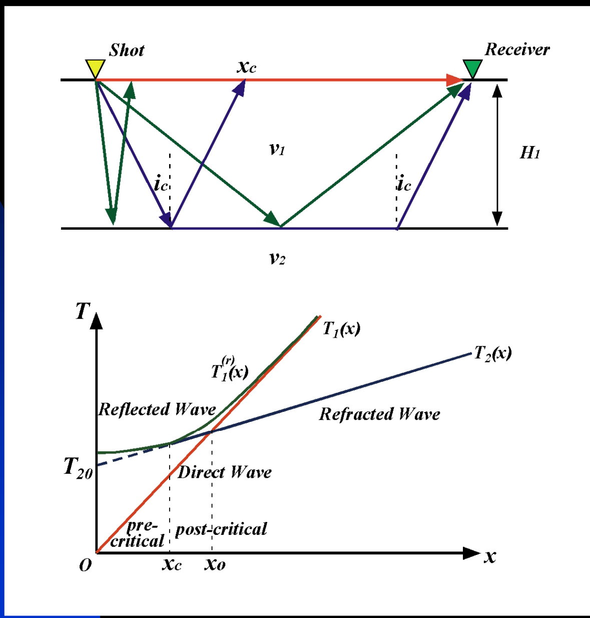

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Direct wave : $T_1(x) = x / v_1$.

Refracted wave : $T_2(x) = x / v_2 + T_{20}$,

where $T_{20} = 2H_1(1 - v_1^2 / v_2^2)^{1/2} / v_1$ (intercept time).

Reflected wave : $T_1^{(r)}(x) = 2(H_1^2 + x^2 / 4)^{1/2} / v_1$.

Critical angle : $i_c = \sin^{-1}(v_1 / v_2)$.

Critical distance : $x_c = 2H_1 \tan i_c = 2H_1 / (v_2^2 / v_1^2 - 1)^{-1/2}$.

Crossover distance : $x_0 = 2H_1 (v_2 + v_1)^{1/2} / (v_2 - v_1)^{1/2}$.

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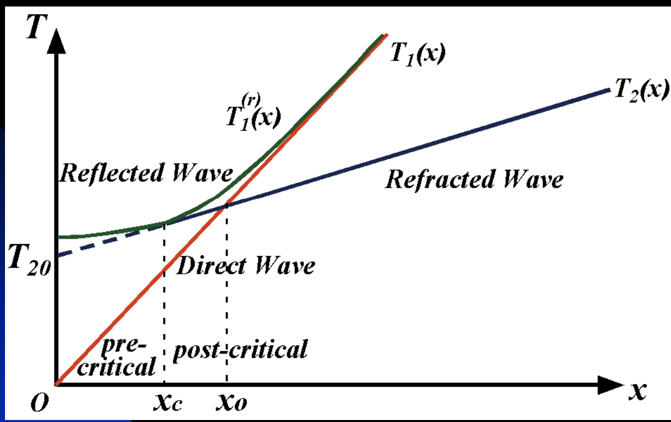
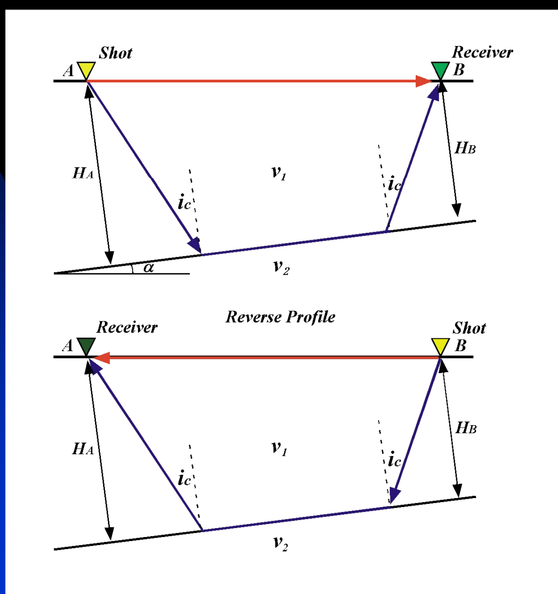
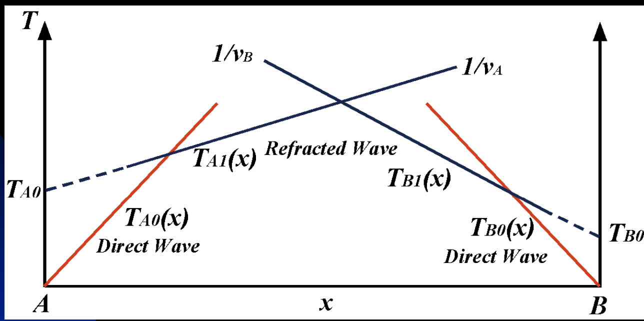


(1) Slope of direct wave : $1/v_1$ ---> v_1.

(2) Slope of refracted wave : $1/v_2$ ---> v_2.

(3) Intercept time : H_1 using v_1 and v_2.





$T_{A0}(x) = x / v_1,$

$T_{A1}(x) = x / v_A + T_{A0},$

where $v_A = v_1 / \sin(i_c - \alpha)$ and $T_{A0} = 2H_A(1 - v_1^2 / v_2^2)^{1/2} / v_1.$

$T_{B0}(x) = x / v_1,$

$T_{B1}(x) = x / v_B + T_{B0},$

where $v_B = v_1 / \sin(i_c + \alpha)$ and $T_{B0} = 2H_B(1 - v_1^2 / v_2^2)^{1/2} / v_1.$

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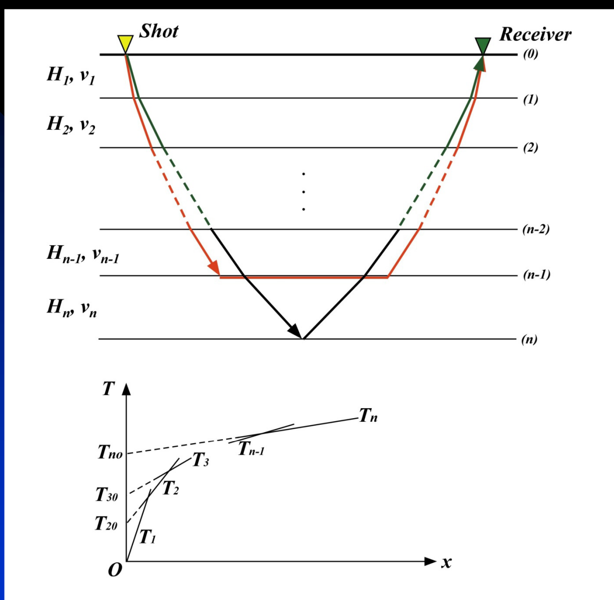

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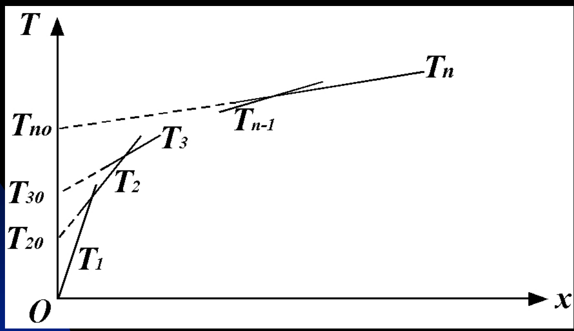

Travel time of the n-th segment : $T_n(x) = x / v_n + T_{n0,}$

where $T_{n0} = \sum_{j=1}^{n-1} 2H_j (1 - v_j^2 / v_n^2)^{1/2} / v_j$.

(1) Slope : $1/v_n$ --> v_n.

(2) Intercept time : T_{n0} --> H_{n-1}

with aids of $v_j\,(j = 1,..,n-1)$ and $H_j\,(j = 1,..,n-2)$.

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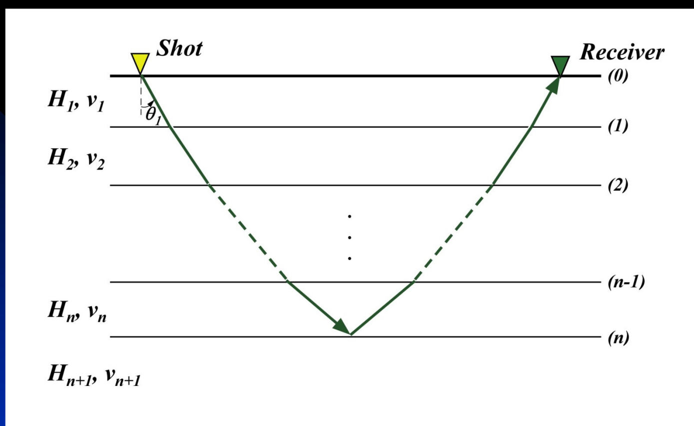

$$T_n^{(r)}(p) = 2\sum_{j=1}^{n} H_j / v_j (1 - p^2 v_j^2)^{1/2},\ x(p) = 2\sum_{j=1}^{n} p v_j H_j / (1 - p^2 v_j^2)^{1/2},$$

where $p = \sin(\theta_1) / v_1$.

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Exact

$$(T_1^{(r)}(x))^2 = (2H_1/v_1)^2 + x^2/v_1^2 \quad \text{for } n = 1.$$

Approximation

$$(T_n^{(r)}(x))^2 = (T_{n0}^{(r)})^2 + x^2/\bar{V}_n^2,$$

where $T_{n0}^{(r)} = 2\sum_{j=1}^{n} H_j/v_j = 2\sum_{j=1}^{n} \Delta t_j$,

and $\bar{V}_n^2 = 2\sum_{j=1}^{n} v_j H_j / \sum_{j=1}^{n} H_j/v_j = 2\sum_{j=1}^{n} v_j^2 \Delta t_j / \sum_{j=1}^{n} \Delta t_j,$

with $\Delta t_j = H_j/v_j$.

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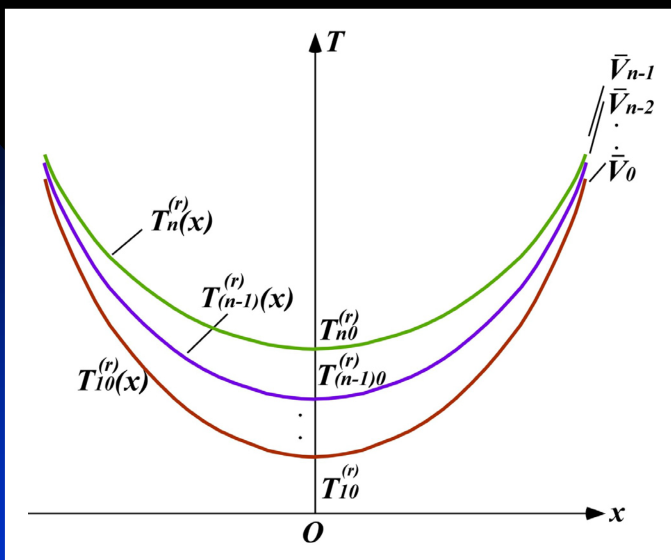

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$$T_{n0}^{(r)}\overline{V}_n^2 - T_{(n-1)0}^{(r)}\overline{V}_{n-1}^2 = 2v_n^2\Delta t_n = v_n^2(T_{n0}^{(r)} - T_{(n-1)0}^{(r)}),$$

$$T_{n0}^{(r)}\overline{V}_n^2 - T_{(n-1)0}^{(r)}\overline{V}_{n-1}^2 = 2H_n v_n$$

$$\hat{V}_n^2 = (T_{n0}^{(r)}\overline{V}_n^2 - T_{(n-1)0}^{(r)}\overline{V}_{n-1}^2)/(T_{n0}^{(r)} - T_{(n-1)0}^{(r)}) \to v_n^2,$$

$$\hat{H}_n = (T_{n0}^{(r)}\overline{V}_n^2 - T_{(n-1)0}^{(r)}\overline{V}_{n-1}^2)/2\hat{V}_n \to H_n$$

or

$$\hat{H}_n = (T_{n0}^{(r)} - T_{(n-1)0}^{(r)})\hat{V}_{n-1}/2 \to H_n.$$

$\overline{V}_n$: RMS velocity, $\hat{V}_n$: Interval velocity.

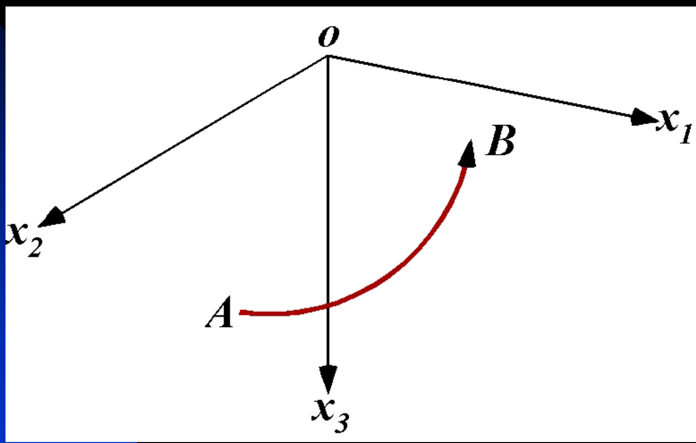
Kennet, 1976

2.2 Asymptotic Ray Theory

- Elastodynamic equation
- High-frequency solutions
- Basic equations of the ray method
  - Eikonal equation
  - Transport equation

(Cerveny and Ravindra, 1971; Cerveny, Molotkov, & Psencik, 1977, Cerveny, 1985)

Geometry and coordinate system



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Elastodynamic Equation

$$\sigma_{ij,j} = \rho u_{i,tt},$$

$$\text{with} \quad \sigma_{ij} = \lambda \delta_{ij} u_{k,k} + \mu(u_{i,j} + u_{j,i}).$$

High-Frequency Solutions

$$u_i = \exp[-i\omega(t-\tau)] \sum_{n=0}^{\infty} U_i^{(n)} (-i\omega)^{-n}.$$

$$\rightarrow \; u_i = U_i \exp[-i\omega(t-\tau)].$$

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$$(i\omega)^2 N_i(\vec{U}) + (i\omega) M_i(\vec{U}) \doteq 0,$$

where

$$N_i(\vec{U}) = U_j[(\lambda+\mu)\tau_{,i}\tau_{,j} + \mu\delta_{ij}\tau_{,k}\tau_{,k} - \rho\delta_{ij}],$$

$$M_i(\vec{U}) = (\lambda+\mu)(\tau_{,i}U_{j,j} + \tau_{,j}U_{j,i} + U_j\tau_{,ij}) + \mu(2\tau_{,j}U_{i,j} + U_i\tau_{,jj})$$
$$+\lambda_{,i}\tau_{,j}U_j + \mu_{,j}(\tau_{,j}U_i + \tau_{,i}U_j).$$

$$N_i(\vec{U}) = 0,$$
$$M_i(\vec{U}) = 0.$$

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Eikonal Equation

$$N_i(\vec{U}) = 0.$$

$$\rightarrow\ U_j(\Gamma_{ij} - \delta_{ij}) = 0.$$

where $\Gamma_{ij} = \dfrac{\lambda+\mu}{\rho}\tau_{,i}\tau_{,j} + \dfrac{\mu}{\rho}\delta_{ij}\tau_{,k}\tau_{,k}.$

$$\det(\Gamma_{ij} - \delta_{ij}) = 0.$$

$$\rightarrow\ \frac{\mu}{\rho}\tau_{,k}\tau_{,k} = V_S^2\tau_{,k}\tau_{,k} = 1,\quad \frac{\lambda+2\mu}{\rho}\tau_{,k}\tau_{,k} = V_P^2\tau_{,k}\tau_{,k} = 1.$$

Eikonal Equation

$$\tau_{,k}\tau_{,k} = 1/V_S^2,\quad \tau_{,k}\tau_{,k} = 1/V_P^2.$$

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Transport Equation

$$M_i(\vec{U}) = 0.$$

$$\rightarrow \vec{U} = U_1^{(q)}\mathbf{e}_1 + U_2^{(q)}\mathbf{e}_2 + U_3^{(q)}\mathbf{t}.$$

$$\vec{U} = U_3^{(q)}\mathbf{t} \qquad \text{for P waves,}$$

$$\vec{U} = U_1^{(q)}\mathbf{e}_1 + U_2^{(q)}\mathbf{e}_2 \qquad \text{for S waves.}$$

$$M_i(\vec{U})\bullet\mathbf{t} = 0 \qquad \text{for P waves,}$$

$$M_i(\vec{U})\bullet\mathbf{e}_1 = M_i(\vec{U})\bullet\mathbf{e}_2 = 0 \qquad \text{for S waves.}$$

$$\rightarrow \frac{dU_i^{(q)}}{ds} + \frac{1}{2}U_i^{(q)}\{v\nabla^2\tau + \frac{d}{ds}\ln(\rho v^2)\} = 0.$$

$$\rightarrow \frac{dU_i^{(q)}}{ds} + \frac{1}{2}U_i^{(q)}\left(\frac{v}{J}\frac{d}{ds}\left(\frac{J}{v}\right) + \frac{d}{ds}\ln(\rho v^2)\right) = 0,$$

$$\rightarrow \frac{dU_i^{(q)}}{ds} + \frac{1}{2}U_i^{(q)}\frac{d}{ds}\ln(J\rho v) = 0,$$

$$\rightarrow \frac{d}{ds}\left(\sqrt{J\rho v}U_i^{(q)}\right) = 0.$$

$$U_i^{(q)}(s) = \frac{\Psi_1}{[v(s)\rho(s)J(s)]^{1/2}} \cdot$$ **Geometrical spreading**

Ray Tube

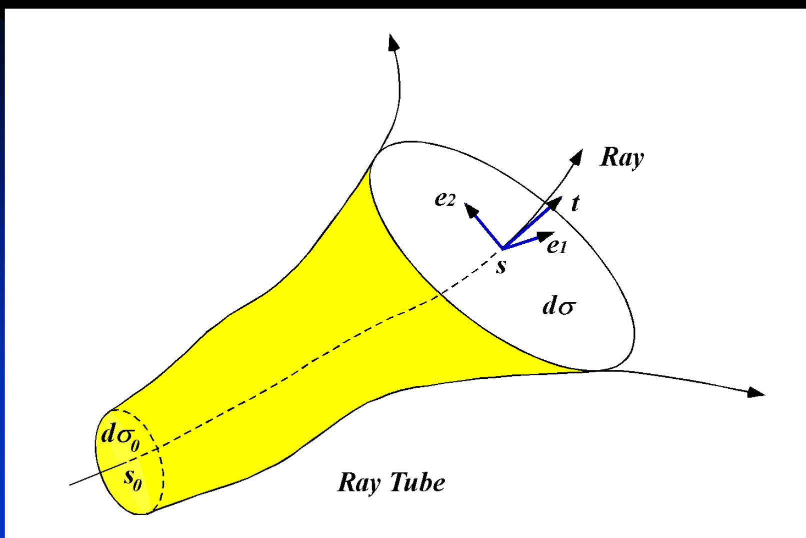
$$J = \begin{vmatrix} \partial x_1/\partial \gamma_1 & \partial x_2/\partial \gamma_1 & \partial x_3/\partial \gamma_1 \\ \partial x_1/\partial \gamma_2 & \partial x_2/\partial \gamma_2 & \partial x_3/\partial \gamma_2 \\ \partial x_1/\partial s & \partial x_2/\partial s & \partial x_3/\partial s \end{vmatrix}.$$

$$d\vec{\sigma}_w = \left(\frac{\partial \vec{x}}{\partial \gamma_1} \times \frac{\partial \vec{x}}{\partial \gamma_2} \right) d\gamma_1 d\gamma_2,$$

$$d\sigma = d\vec{\sigma}_w \bullet \mathbf{t} = J d\gamma_1 d\gamma_2.$$

Cerveny(1985)

Geometrical spreading

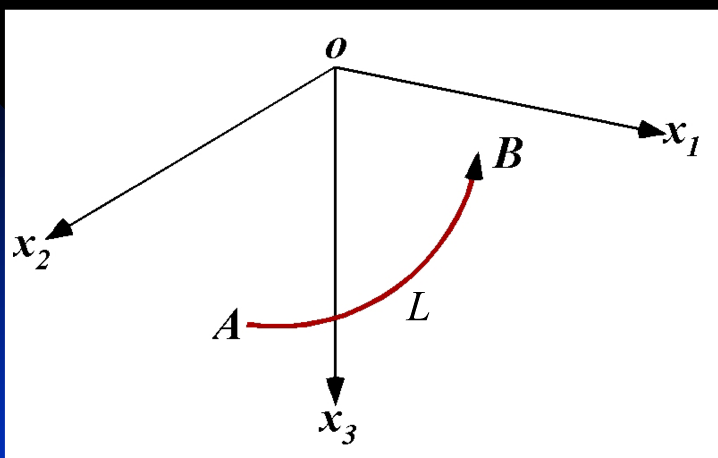


2.3 Ray Tracing

- Fermat’s principle
- Euler’s equations

  (e.g. Courant & Hilbert, 1966; Julian and Gubbins,1977)

- Examples (Iwasaki, 1988)



travel time : $T = \int_L ds / v.$

Eikonal equation : $\tau_{,k}\tau_{,k} = 1/v^2$.

Fermat's principle : $T = \int_L ds/v \rightarrow$ stationary.

$\rightarrow$ Euler's equations.

$$\frac{d}{ds}\left[\frac{1}{v}\left(\frac{d\mathbf{r}}{ds}\right)\right] = \nabla\left(\frac{1}{v}\right),$$

or

$$\frac{d}{dt}\left[\frac{1}{v^2}\left(\frac{d\mathbf{r}}{dt}\right)\right] = v\nabla\left(\frac{1}{v}\right) = -\frac{1}{v}\nabla v,$$

where $ds = vdt$.

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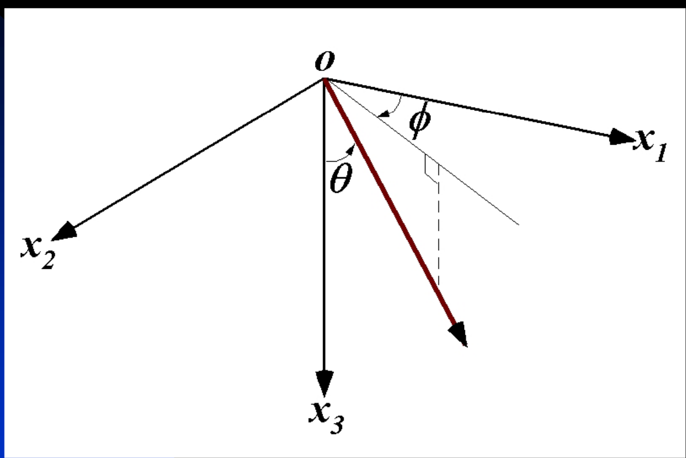

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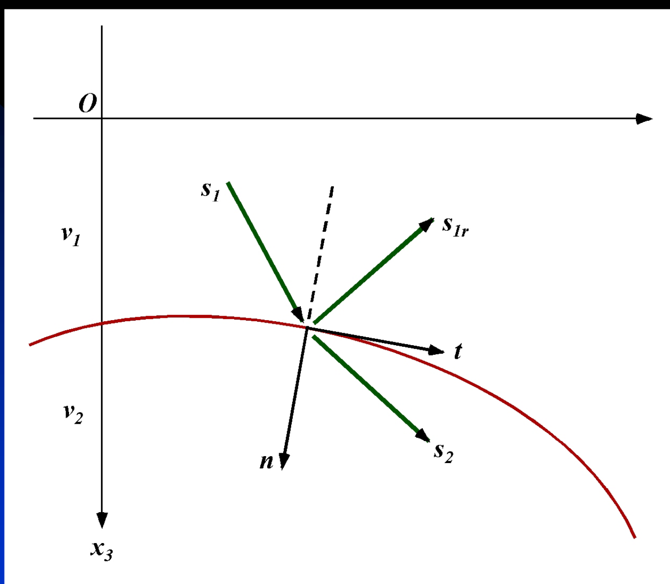
$$\frac{d\mathbf{r}}{dt} \equiv \mathbf{r}' = \begin{bmatrix} v\cos\phi\sin\theta \\ v\sin\phi\sin\theta \\ v\cos\theta \end{bmatrix},$$

and

$$\phi' = (v_{,1}\sin\phi - v_{,2}\cos\phi)/\sin\theta.$$

$$\theta' = -(v_{,1}\cos\phi + v_{,2}\sin\phi)\cos\theta + v_{,3}\sin\theta.$$



$$\mathbf{s} \equiv \frac{\mathbf{r}'}{v^2} \text{ (slowness vector).}$$

$$s^{(t)} = \mathbf{s}\cdot\mathbf{t} \text{ (tangential), } s^{(n)} = \mathbf{s}\cdot\mathbf{n} \text{ (normal).}$$

$$\text{Snell's law : } \mathbf{s}_1\cdot\mathbf{t} = \mathbf{s}_2\cdot\mathbf{t} = \mathbf{s}_{1r}\cdot\mathbf{t}.$$

$$\rightarrow \quad s_2^{(t)} = s_1^{(t)},$$

$$s_2^{(n)} = \left[1/v_2^2 - (s_2^{(t)})^2\right]^{1/2}\left[s_1^{(n)}/\left|s_1^{(n)}\right|\right] \qquad \text{(refraction)}$$

and

$$s_{1r}^{(t)} = s_1^{(t)},\ s_{1r}^{(n)} = -s_1^{(n)} \qquad \text{(reflection).}$$

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