Numerical simulation of tsunami and its application

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Goals

• To understand the basic tsunami physics, modeling and its limitations

• To simulate a tsunami by yourself and check the validation of its results through simulation exercises using a tsunami code *(Tohoku University’s Numerical Analysis Model for Investigation)*.
Contents

(1) Lecture of the tsunami modeling
   • Introduction of tsunami modeling
   • Differential equation of the long-wave model
   • Finite difference equation

(2) Practice of the tsunami simulation
   • How to use the tsunami-code
   • Practice exercises using the tsunami code
Introduction

Tsunami simulation is widely used for hazard assessments.

- Tsunami prediction
- Tsunami countermeasure
- Warning systems

2004 Indian Ocean tsunami
Modeling of the 2004 tsunami

Namkem, Thailand
Modeling of the 2004 tsunami

Namkem, Thailand

Tsunami current directions and velocity.

We can quantitatively estimate the impact of a tsunami flow from a numerical modeling.

In today's lecture, I am going to explain tsunami modeling from fundamental basis to its application.
Advanced development model

Landslide-induced tsunami
Example of landslide-induced tsunami

1792 Unzen erupted

Mayuyama was collapsed with the large earthquake and then, the landslide flowed into Ariake-sea accompanying a large tsunami

The landslide and tsunami killed 15,000 people in Shimabara and Higo
Outline of tsunami modeling

1. Derivation of governing equations

Differential equation
\[ \frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} = 0 \]
\[ \frac{\partial M}{\partial t} + gh \frac{\partial \eta}{\partial x} = 0 \]

2. Discretization of governing equations

Finite difference method
\[ \frac{\eta^{k+1} - \eta^k}{\Delta t} + \frac{M_{i+1/2} - M_{i-1/2}}{\Delta x} = 0 \]
\[ \frac{M^{k+1/2} - M^{k-1/2}}{\Delta t} + gh_i \frac{\eta_{i+1} - \eta_i}{\Delta x} = 0 \]

The differential equations could not be computed directly.

3. Programming using fortran
Outline of tsunami modeling

1. Derivation of governing equations

Differential equation
\[
\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} = 0
\]
\[
\frac{\partial M}{\partial t} + gh \frac{\partial \eta}{\partial x} = 0
\]

2. Discretization of governing equations

Finite difference method
\[
\eta^{k+1} - \eta^k \frac{\Delta t}{\Delta t} + \frac{M_{i+1/2} - M_{i-1/2}}{\Delta x} = 0
\]
\[
\frac{M^{k+1/2} - M^{k-1/2}}{\Delta t} + gh_i \frac{\eta_{i+1} - \eta_i}{\Delta x} = 0
\]

The differential equations could not be computed directly.

3. Programming using fortran
1. Differential equation (Long-wave approximation)

- Long-wave approximation
  1. Long wave (wave height $\ll$ wave length)
  2. Vertical acceleration of water particle $\approx 0$
  3. The water pressure is gravitational pressure
1. Differential equation (Long-wave approximation)

- **Long-wave approximation**
  1. Long wave (wave height << wave length)
  2. Vertical acceleration of water particle $\approx 0$
  3. The water pressure is gravitational pressure

![Diagram of wave particle movement]

- (Movement of water particle)

  - Long wave
    - Vertical velocity = max
    - Slow change
  - Short wave
    - Vertical velocity = 0

- Several meters

- Several kilometers

- Long-wave
  - long and thin ellipsoid

- Short wave
1. Differential equation (Long-wave approximation)

- **Long-wave approximation**
  1. Long wave (wave height \(<<\) wave length)
  2. Vertical acceleration of water particle \(\approx 0\)
  3. The water pressure is gravitational pressure

\(<\text{Movement of water particle}\rangle

\(<\text{Several kilometers}\rangle

\(<\text{Several meters}\rangle

\(<\text{Long-wave}\rangle

\(<\text{Short wave}\rangle

\(<\text{long and thin ellipsoid}\rangle
1. Differential equation (Long-wave approximation)

Movement of water particle

Horizontal velocity is vertically constant

Vertical distribution of wave pressure
1. Differential equation (Long-wave approximation)

- Long-wave approximation

1. Long wave (wave height $<<$ wave length)
2. Vertical acceleration of water particle $\approx 0$
3. The water pressure is gravitational pressure
Governing equations
(Differential equations)
1. Differential equation (Governing equations)

- The continuity equation of incompressible fluid

**Law of conservation of mass**

\[
\rho \left\{ (u \Delta x \Delta t) + (w \Delta z \Delta t) \right\} = \rho \left\{ (u + \frac{\partial u}{\partial x}) \Delta x \Delta t + (w + \frac{\partial w}{\partial z}) \Delta z \Delta t \right\}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0
\]
1. Differential equation (Governing equations)

- The momentum equations of incompressible fluid

**Momentum condition:** Navier-stokes equation

**Horizontal direction**

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial z^2} \right)
\]

- Horizontal acceleration of water particle
- Pressure term
- Viscosity term

**Vertical direction**

\[
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} = g + \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left( \frac{\partial^2 w}{\partial z^2} \right)
\]

- Vertical acceleration of water particle
- Pressure term
- Viscosity term

**Newton's second law**

\[ F = ma \]

- \( u \): Horizontal velocity
- \( w \): Vertical velocity
- \( p \): Water pressure
- \( \nu \): Viscosity coefficient
- \( g \): Gravity acceleration
1. Differential equation (Governing equations)

**Governing equation**

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} &= 0 \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} &= F_x - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial z^2} \right) \\
&= F_z - \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left( \frac{\partial^2 w}{\partial z^2} \right)
\end{align*}
\]

These are fundamental equations for not only tsunami but also all incompressible fluid.

Direct numerical simulation
(Vof, smac, etc.)

- Computationally expensive
- Long-wave approximation

(Movement of water particle)
The long-wave equation
Flow for long-wave model

Actually long-wave model is derived from solving boundary value problems.

**Governing equations**

- Long-wave approximation (small vertical velocity)

And then, I integrate governing equations in the vertical direction to remove vertical component from governing equation.

**Integration of governing equations in the vertical direction**

**Boundary conditions**

1. Surface boundary condition
2. Bottom boundary condition

- Continuity equation
- Navier-Stokes equation

Long-wave

Horizontal velocity
1. Differential equation

(① Boundary conditions)

- Surface boundary ($Z=\eta$)
  
  a) Dynamic boundary condition

  $$P_{surface} = 0$$

  b) Kinematic boundary condition

  $$\left( \frac{\partial \eta}{\partial t} + u_{surface} \frac{\partial \eta}{\partial x} \right) \times \Delta t = w_{surface} \times \Delta t$$

  - $\eta$: surface elevation (m)
  - $u$: horizontal velocity (m/s)
  - $w$: vertical velocity (m/s)

  \[ \text{Rise velocity of water surface} \]
  \[ \text{Vertical velocity of water particle at surface} \]
1. Differential equation (① Boundary conditions)

- Bottom boundary (Z= -h)

  c) Kinematic boundary condition

\[
\tan \theta = \frac{w_b}{u_b} = -\frac{\partial h}{\partial x}
\]

\{ 
\begin{align*}
  h &: \text{water depth (m)} \\
  u &: \text{Horizontal velocity (m/s)} \\
  w &: \text{vertical velocity (m/s)}
\end{align*}
\]

The ratio between vertical velocity and horizontal velocity at bottom is equal to the gradient of bottom.

The bottom of water must flow along bottom bed.
Flow of long-wave approximation

Governing equations

Boundary equations

Long-wave approximation
(small vertical velocity)

Integration of governing equations in the vertical direction

I integrate governing equations in the vertical direction to remove vertical component from governing equation.
1. Differential equation

(2) Integration of continuity equation

**Continuity equation**

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0
\]

Integration in vertical direction

\[
\int_{-h}^{h} \frac{\partial u}{\partial x} \, dz + \int_{-h}^{h} \frac{\partial w}{\partial z} \, dz = 0
\]
1. Differential equation
(② Integration of continuity equation)

Continuity equation

\[ \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \]

Integration in vertical direction

\[ \int_{-h}^{\eta} \frac{\partial u}{\partial x} \, dz + \int_{-h}^{\eta} \frac{\partial w}{\partial z} \, dz = \int_{-h}^{\eta} \frac{\partial u}{\partial x} \, dz + [w]_{-h}^{\eta} = \]
1. Differential equation

(2) Integration of continuity equation

**Continuity equation**

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0
\]

**Integration in vertical direction**

\[
\int_{-h}^{\eta} \frac{\partial u}{\partial x} \, dz + \int_{-h}^{\eta} \frac{\partial w}{\partial z} \, dz = \int_{-h}^{\eta} \frac{\partial u}{\partial x} \, dz + \left[ w \right]_{-h}^{\eta} = \int_{-h}^{\eta} \frac{\partial u}{\partial x} \, dz + w(x, \eta, t) - w(x, -h, t)
\]

Vertical velocity at surface boundary

Vertical velocity at bottom boundary

**still-water level** = \(0\)

**Continuity equation**

Navie-stokes equation

\[\tan \theta = \frac{\partial h}{\partial x}\]
1. Differential equation

(② Integration of continuity equation)

**Continuity equation**

\[ \int_{\text{Surface}} \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \, dz = \int_{-h}^{n} \frac{\partial u}{\partial x} \, dz + w(x, \eta, t) - w(x, -h, t) \]

\[ = \int_{-h}^{n} \frac{\partial u}{\partial x} \, dz + \left( \frac{\partial \eta_s}{\partial t} + u_s \frac{\partial \eta_s}{\partial x} \right) - \left( -u_b \frac{\partial h}{\partial x} \right) \]

\[ \int_{-h}^{n} \frac{\partial u}{\partial x} \, dz = \frac{\partial}{\partial x} \int_{-h}^{n} u \, dz - u_s \frac{\partial \eta}{\partial t} - u_b \frac{\partial h}{\partial t} \]

**Boundary conditions**

\[ w_s = \left( \frac{\partial \eta_s}{\partial t} + u_s \frac{\partial \eta_s}{\partial x} \right) \]
\[ w_b = -\frac{\partial h}{\partial x} \]

**Leibniz integral rule**

\[ \frac{\partial}{\partial x} \int_{a(x)}^{b(x)} Q(x, y) \, dz = \]

\[ \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} Q(x, y) \, dz + Q(x, \beta(x)) \frac{\partial \beta(x)}{\partial x} - Q(x, \alpha(x)) \frac{\partial \alpha(x)}{\partial t} \]
1. Differential equation
(\( \textcircled{2} \) Integration of continuity equation)

From long-wave approximation, horizontal velocity is vertically constant.

\[
\frac{\partial}{\partial t} \int_{-h}^{\eta} udz = \frac{\partial}{\partial t} (uD)
\]

\( D \): Depth from bottom to surface (\( = \eta + h \) )

\( M \): Flux in the x-direction

**Continuity equation in long-wave model**

\[
\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} = 0
\]

\( \eta \): Surface elevation

\( M \): Flux in the x-direction
1. Differential equation
(2 Integration of momentum equation)

Momentum equation in vertical direction

\[ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} = g - \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left( \frac{\partial^2 w}{\partial z^2} \right) \]

In the long wave condition, vertical acceleration of water particle is expected to be small.

\[ 0 = g - \frac{1}{\rho} \frac{\partial p}{\partial z} \]
1. Differential equation

(2) Integration of momentum equation

**Indefinite Integration in vertical direction**

\[ 0 = g - \frac{1}{\rho} \frac{\partial p}{\partial z} \quad \rightarrow \quad 0 = \int \left( g - \frac{1}{\rho} \frac{\partial p}{\partial z} \right) \, dz \quad \rightarrow \quad 0 = \rho \int g \, dz - \int \frac{\partial p}{\partial z} \, dz \]

\[ 0 = \rho g z - (p(x, z) + c) \quad (C \text{ is a constant of integration}) \]

From kinematic boundary condition \( p_{surface} = 0 \)

\[ p(x, \eta) = \rho g \eta - c \quad \rightarrow \quad c = \rho g \eta \]

\[ p(x, z) = \rho g (\eta - z) \]

Gravitational pressure
1. Differential equation

(2) Integration of momentum equation

**Momentum equation in horizontal direction**

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial z^2} \right) \\
\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial wu}{\partial z} &= -g \frac{\partial \eta}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial z^2} \right)
\end{align*}
\]

\[p = \rho g (\eta - z)\]

**Integration in vertical direction**

\[
\int_{\text{bottom}}^{\text{surface}} \left( \frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial wu}{\partial z} \right) dz = \int_{\text{bottom}}^{\text{surface}} \left( -g \frac{\partial \eta}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial z^2} \right) \right) dz
\]
1. Differential equation

(2) Integration of momentum equation

**Integration in vertical direction**

*(Left side terms)*

\[
\begin{align*}
\int_{\text{bottom}}^{\text{surface}} \left( \frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial w u}{\partial z} \right) dz &= \frac{\partial}{\partial x} \int_{-h}^{\eta} u dz - u_{\text{surface}} \frac{\partial \eta}{\partial t} - u_{\text{bottom}} \frac{\partial h}{\partial t} \\
\int_{\text{bottom}}^{\text{surface}} \frac{\partial u^2}{\partial x} dz &= \frac{\partial}{\partial x} \int_{-h}^{\eta} u^2 dz - u_{\text{surface}}^2 \frac{\partial \eta}{\partial x} - u_{\text{bottom}}^2 \frac{\partial h}{\partial x} \\
\int_{\text{bottom}}^{\text{surface}} \frac{\partial w u}{\partial z} dz &= [u w]_{\text{bottom}} \\
&= u_{\text{surface}} \left( \frac{\partial \eta}{\partial t} + u_{\text{surface}} \frac{\partial \eta}{\partial x} \right) + u_{\text{bottom}}^2 \frac{\partial h}{\partial x}
\end{align*}
\]

**Leibniz integral rule**

\[
\begin{align*}
\frac{\partial}{\partial x} \int_{\alpha(x)}^{{\beta(x)}} Q(x, y) dy &= \int_{\alpha(x)}^{{\beta(x)}} \frac{\partial Q(x, y)}{\partial x} dy + \int_{\alpha(x)}^{{\beta(x)}} Q(x, \beta(x)) \frac{\partial \beta(x)}{\partial x} - Q(x, \alpha(x)) \frac{\partial \alpha(x)}{\partial x}
\end{align*}
\]
1. Differential equation

\( \int \frac{\partial}{\partial t} \int_{-h}^{\eta} u \, dz = \frac{\partial}{\partial t} (uD) \)

\( \int \frac{\partial}{\partial t} \int_{-h}^{\eta} u^2 \, dz = \frac{\partial}{\partial x} (u^2 D) \)

\( D \) : Depth from bottom to surface ( \( = \eta + h \) )

\[ \int_{\text{bottom}}^{\text{surface}} \frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial wu}{\partial z} \, dz = \frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{D} \right) \]
1. Differential equation

(2) Integration of momentum equation

Integration in vertical direction

\[
\int_{\text{surface}} \frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial wu}{\partial z} \, dz = \int_{\text{bottom}} - g \frac{\partial \eta}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial z^2} \right) \, dz
\]

\[
= \frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{D} \right)
\]
1. Differential equation

(2) Integration of momentum equation

Integration in vertical direction

(Right side terms)

\[
\int_{\text{surface}} \left( -g \frac{\partial \eta}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial z^2} \right) \right) dz \]

\[
\int_{\text{bottom}} \left( -g \frac{\partial \eta}{\partial x} \right) dz = -gD \frac{\partial \eta}{\partial x} 
\]

\[
\int_{\text{surface}} \nu \frac{\partial^2 u}{\partial z^2} dz = \left[ \nu \frac{\partial u}{\partial z} \right]_{\text{surface}} - \left[ \nu \frac{\partial u}{\partial z} \right]_{\text{bottom}} 
\]

\[
= \nu \frac{\partial u_{\text{surface}}}{\partial z} - \nu \frac{\partial u_{\text{bottom}}}{\partial z} = \frac{1}{\rho} \left( \tau_{\text{surface}} - \tau_{\text{bottom}} \right) 
\]

Horizontal velocity is vertically constant

Long-wave

Horizontal velocity

Manning formula

\[
\tau_{\text{bottom}} = \frac{\rho g n^2}{D^{7/3}} M |M| 
\]
1. Differential equation

(2) Integration of momentum equation

**Left side terms**

\[
\int_{\text{bottom}}^{\text{surface}} \frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial w u}{\partial z} \, dz = \frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{D} \right)
\]

**Right side terms**

\[
\int_{\text{bottom}}^{\text{surface}} -g \frac{\partial \eta}{\partial x} \, dz = -gD \frac{\partial \eta}{\partial x}
\]

\[
\int_{\text{bottom}}^{\text{surface}} \nu \left( \frac{\partial^2 u}{\partial z^2} \right) \, dz = -\frac{gn^2}{D^{7/3}} M |M|
\]

**Momentum equation in non-linear long-wave model**

\[
\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{D} \right) + gD \frac{\partial \eta}{\partial x} + \frac{gn^2}{D^{7/3}} M |M| = 0
\]

When I summarize all terms, the momentum equation in non-linear long-wave model is derived.
1. Differential equation
(Nonlinear long-wave model)

**Continuity equation**

\[
\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} = 0
\]

**Momentum equations (Nonlinear long-wave model)**

\[
\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{D} \right) + gD \frac{\partial \eta}{\partial x} + \frac{gn^2}{D^{7/3}} M |M| = 0
\]

- **Advection term (Nonlinear term)**
- **Friction term**
- **Gravity term**

**Linear long-wave model**

\[
\frac{\partial M}{\partial t} + gh \frac{\partial \eta}{\partial x} = 0
\]
1. Differential equation (Nonlinear long-wave model)

**Ratio of each terms to local change term**

- **Gravity term**
- **Friction term**
- **Advection term**

More than 50 m depth

Imamura et al., (1986)
The limitation of long-wave model

Long wave approximation
1. Long wave (wave height $\ll$ wave length)
2. Vertical acceleration of water particle $\equiv 0$
   (Horizontal velocity is vertically constant)
3. The water pressure is gravitational pressure

Long-wave model cannot represent phenomena in which the vertical acceleration is needed such as wave breaking, water splash etc.
Finite difference method
Outline of tsunami modeling

1. Derivation of governing equations

Differential equation
\[
\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} = 0
\]
\[
\frac{\partial M}{\partial t} + gh \frac{\partial \eta}{\partial x} = 0
\]

2. Discretization of governing equations

Finite difference method
\[
\frac{\eta^{k+1} - \eta^k}{\Delta t} + \frac{M_{i+1/2} - M_{i-1/2}}{\Delta x} = 0
\]
\[
\frac{M^{k+1/2} - M^{k-1/2}}{\Delta t} + gh_i \frac{\eta_{i+1} - \eta_i}{\Delta x} = 0
\]
The differential equations could not be computed directly.

3. Programming using fortran

discretized value

\[\Delta x\]
Basic concept

1. Differential equation

The differential of a function means tangential line.

\[
\frac{\partial f(x)}{\partial x} \approx \frac{f(x + \Delta x) - f(x)}{\Delta x}
\]

2. Finite difference equation

The differential is approximated by the tangential line from discretized values.
The finite difference equation

**Forward difference**

\[
\frac{\partial f(x)}{\partial x} \approx \frac{f_{i+1} - f_i}{\Delta x}
\]

Low accuracy, stable (M>0)

**Backward difference**

\[
\frac{\partial f(x)}{\partial x} \approx \frac{f_i - f_{i-1}}{\Delta x}
\]

Low accuracy, stable (M<0)

**Centered difference**

\[
\frac{\partial f(x)}{\partial x} \approx \frac{f_{i+1/2} - f_{i-1/2}}{\Delta x}
\]

High accuracy, including unstable error
Practice (1)

Discretize the following equation using (1) backward and (2) centered difference for spatial grid and backward difference for time grid.

\[ \frac{\partial u}{\partial t} = c \frac{\partial u}{\partial x} \]

- \( t \): time
- \( x \): distance
- \( C \): wave speed
Practice (1)

Discretize the following equation using backward and centered difference for spatial grid and backward difference for time grid.

\[
\frac{\partial u}{\partial t} = c \frac{\partial u}{\partial x}
\]

- **t**: time
- **x**: distance
- **C**: wave speed

**Backward difference**

\[
\frac{u_i^k - u_i^{k-1}}{\Delta t} = c \frac{u_i^k - u_{i-1}^k}{\Delta x}
\]

**Centered difference**

\[
\frac{u_i^k - u_i^{k-1}}{\Delta t} = c \frac{u_{i+1}^k - u_{i-1}^k}{\Delta x}
\]
Practice (2)

Simulate the equation using Excel in the following conditions

① $dx=0.2 \text{ (m)}$, $dt=0.2 \text{ (sec)}$, $c=1.0 \text{ (m/s)}$

② $\text{Area} = 0 \sim 10 \text{ (m)}$, $\text{Total time} = 5 \text{ (sec)}$

③ Boundary($\text{sin wave with 1 m amplitude and 1.0 sec period}$) => $\sin(2 \times 3.14/1.0)$
Practice (2)

(2) Simulate the equation using Excel using the following conditions

① \( dx=0.2 \text{ (m)} \quad dt=0.2 \text{ (sec)} \quad c=1.0 \text{ (m/s)} \)

② Area = 0\(\sim\) 10 (m), Total time = 5 (sec)

③ Boundary(sin wave with 1 m amplitude and 1.0 sec period) \(\Rightarrow\) \(\sin(2\times3.14/1.0)\)

Step 1 make the following table

<table>
<thead>
<tr>
<th>Distance</th>
<th>time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0.2</td>
<td>0.4</td>
<td>...</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(2) Simulate the equation using Excel using the following conditions

① $dx=0.2$ (m) $dt=0.2$ (sec) $c=1.0$ (m/s)
② Area = 0~10 (m), Total time = 5 (sec)
③ Boundary(sin wave with 1 m amplitude and 1.0 sec period) $\Rightarrow \sin(2 \times 3.14/1.0)$

Step 2 Input the initial conditions at time 0

<table>
<thead>
<tr>
<th>Distance</th>
<th>time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>...</td>
<td>5</td>
</tr>
</tbody>
</table>
Practice (2)

(2) Simulate the equation using Excel using the following conditions

① $dx=0.2 \text{ (m)}$  $dt=0.2 \text{ (sec)}$  $c=1.0 \text{ (m/s)}$

② Area = $0 \sim 10 \text{ (m)}$, Total time = 5 (sec)

③ Boundary(sin wave with 1 m amplitude and 1.0 sec period) => $\sin(2\times\frac{3.14}{1.0})$

Step 3 Input the boundary condition

<table>
<thead>
<tr>
<th>Distance</th>
<th>time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>0</td>
<td>0.2</td>
<td>0.4</td>
<td>...</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(2) Simulate the equation using Excel using the following conditions

1. \( dx = 0.2 \text{ (m)} \quad dt = 0.2 \text{ (sec)} \quad c = 1.0 \text{ (m/s)} \)

2. Area = 0～10 (m), Total time = 5 (sec)

3. Boundary\((\text{sin wave with 1 m amplitude and 1.0 sec period}) \Rightarrow \text{sin}(2\times3.14/1.0)\)

Step 4 Input the difference equation

<table>
<thead>
<tr>
<th>Distance</th>
<th>time</th>
<th>Distance</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>time</th>
<th>Distance</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.2</td>
<td>0</td>
<td>0.2</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ u_{11} = u_{01} - c \frac{dt}{dx} (u_{01} - u_{00}) \]
## Practice (2)

**(2) Simulate the equation using Excel using the following conditions**

1. \( dx=0.2 \) (m) \( dt=0.2 \) (sec) \( c=1.0 \) (m/s)
2. Area = 0~10 (m), Total time = 5 (sec)
3. Boundary(sin wave with 1 m amplitude and 1.0 sec period) \( \Rightarrow \) \( \sin(2*3.14/1.0) \)

### Step 4 Input the difference equation

<table>
<thead>
<tr>
<th>Distance</th>
<th>time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0.2</td>
<td>0.4</td>
<td>... 5</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>=( \sin(2*3.14/1.0 \times \text{time1}) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>0</td>
<td>( u11=\text{u01}-c \times \frac{dt}{dx} \times (\text{u01}-\text{u00}) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Staggered grid

(Special grid)

\[ M_{i-1/2} \rightarrow \eta_i \rightarrow M_{i+1/2} \]

\[ \eta_{i-1/2} \quad \text{i-1/2} \quad \eta_{i+1/2} \quad \text{i+1/2} \]

(Time grid)

\[ \eta^k \quad \text{K} \quad \eta^{k+1} \quad \text{K+1} \]

\[ M^k \quad \text{K} \quad M^{k+1/2} \quad \text{K+1/2} \]

Water elevation is defined at center of grid

Water flux is defined at edge of grid

there is dislocation between \( \eta \) and \( M \)
The finite difference equation (Linear term)

**Continuity equation**

\[
\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} = 0 \quad (1)
\]

\[
\frac{\eta^{k+1} - \eta^k}{\Delta t} + \frac{M_{i+1/2} - M_{i-1/2}}{\Delta x} = 0
\]

**Momentum equation**

\[
\frac{\partial M}{\partial t} + gh \frac{\partial \eta}{\partial x} = 0 \quad (2)
\]

\[
\frac{M^{k+1/2} - M^{k-1/2}}{\Delta t} + gD_{i+1/2} \frac{\eta_{i+1} - \eta_i}{\Delta x} = 0
\]

Special grid for (1), (2)

Center of spatial grid for (1), (2)

Time grid

Center of time grid for (1), (2)
The finite difference equation

**Continuity equation**

\[
\eta^{k+1} - \eta^k + \frac{M_{i+1/2} - M_{i-1/2}}{\Delta x} = 0
\]

\[
\eta^{k+1} = \eta^k - \frac{\Delta t}{\Delta x} \left( M_{i+1/2} - M_{i-1/2} \right)
\]

**Momentum equation**

\[
\frac{M^{k+1/2} - M^{k-1/2}}{\Delta t} + gD_{i+1/2} \frac{\eta_{i+1} - \eta_i}{\Delta x} = 0
\]

\[
M^{k+1/2} = M^{k-1/2} - gD_{i+1/2} \frac{\Delta t}{\Delta x} (\eta_{i+1} - \eta_i)
\]
The finite difference equation

**Continuity equation**

\[ \eta^{k+1} = \eta^k - \frac{\Delta t}{\Delta x} (M_{i+1/2} - M_{i-1/2}) \]

**Momentum equation**

\[ M^{k+1/2} = M^{k-1/2} - gh_{i+1/2} \frac{\Delta t}{\Delta x} (\eta_{i+1} - \eta_i) \]
Practice (2)

- Write the leap-frog method on the flowing grid
Practice (2)

- Write the leap-frog method on the flowing grid
Practice (2)

- Write the leap-frog method on the flowing grid

\[ \eta^{k+1} = \eta^k - \Delta t \left( \frac{M_{i+1/2} - M_{i-1/2}}{\Delta x} \right) \]

Initial conditions
Practice (2)

- Write the leap-frog method on the flowing grid

\[
\begin{align*}
M^{k+1/2} &= M^{k-1/2} - gD_{i+1/2} \frac{\Delta t}{\Delta x} (\eta_{i+1} - \eta_i) \\
\eta^{k+1} &= \eta^k - \frac{\Delta t}{\Delta x} (M_{i+1/2} - M_{i-1/2})
\end{align*}
\]

Moment equation
Continuity equation

Initial conditions
Practice (2)

- Write the leap-frog method on the flowing grid

\[
M^{k+1/2} = M^{k-1/2} - gD_{i+1/2} \frac{\Delta t}{\Delta x} (\eta_{i+1} - \eta_i)
\]

Moment equation

\[
\eta^{k+1} = \eta^k - \frac{\Delta t}{\Delta x} (M_{i+1/2} - M_{i-1/2})
\]

Continuity equation

Initial conditions
Practice (2)

- Write the leap-frog method on the flowing grid

Moment equation
\[ M_{k+1/2} = M_{k-1/2} - gD_{i+1/2} \frac{\Delta t}{\Delta x} (\eta_{i+1} - \eta_i) \]

Continuity equation
\[ \eta_{k+1} = \eta_k - \frac{\Delta t}{\Delta x} (M_{i+1/2} - M_{i-1/2}) \]

Initial conditions
Practice (2)

- Write the leap-frog method on the flowing grid

\[ M^{k+1/2} = M^{k-1/2} + D_{i+1/2} \frac{\Delta t}{\Delta x} (\eta_{i+1} - \eta_i) \]

\[ \eta^{k+1} = \eta^k - \frac{\Delta t}{\Delta x} (M_{i+1/2} - M_{i-1/2}) \]

**Initial conditions**

**Moment equation**

**Continuity equation**
Practice (2)

• Write the leap-frog method on the flowing grid

\[ M^{k+1/2} = M^{k-1/2} - gD_{i+1/2} \frac{\Delta t}{\Delta x} (\eta_{i+1} - \eta_i) \]

\[ \eta^{k+1} = \eta^k - \frac{\Delta t}{\Delta x} (M_{i+1/2} - M_{i-1/2}) \]

Boundary conditions

Moment equation

Continuity equation

Initial conditions
Practice (2)

- Write the leap-frog method on the flowing grid

Boundary conditions

Moment equation

\[ M_{k+1}^{1/2} = M_{k-1/2}^{1/2} - gD_{i+1/2} \frac{\Delta t}{\Delta x} (\eta_{i+1} - \eta_i) \]

Continuity equation

\[ \eta_{k+1} = \eta_{k} - \frac{\Delta t}{\Delta x} (M_{i+1/2} - M_{i-1/2}) \]

Initial conditions
Practice (2)

- Write the leap-frog method on the flowing grid

\[
\begin{align*}
M_{i+1/2}^{k+1/2} &= M_{i-1/2}^{k-1/2} - gD_{i+1/2} \frac{\Delta t}{\Delta x} (\eta_{i+1} - \eta_i) \\
\eta_{i+1} &= \eta_i - \Delta t \left( M_{i+1/2}^{k+1} - M_{i-1/2}^{k-1} \right)
\end{align*}
\]
Practice (2)

• Write the leap-frog method on the flowing grid

![Spatial grid](i \rightarrow j)

![Time grid](1 \rightarrow 2 \rightarrow 3 \rightarrow 4)

![Water elevation](Δη = Δη + Δt \cdot \text{Flux})

![Boundary conditions](\text{Moment equation})

\[ M^{k+1/2} = M^{k-1/2} - gD_{i+1/2} \frac{Δt}{Δx} (\eta_{i+1} - \eta_i) \]

![Continuity equation](\text{Initial conditions})

\[ \eta^{k+1} = \eta^k - \frac{Δt}{Δx} (M_{i+1/2} - M_{i-1/2}) \]
The finite difference equation
(Nonlinear term)

Nonlinear terms cause unstable effect in some cases. Thus, more stable scheme is used for nonlinear terms.

**Upwind difference scheme**

\[
\frac{\partial}{\partial x} \left( \frac{M^2}{D} \right) 
\]

**Backward difference** (M > 0)

\[
\frac{\partial}{\partial x} \left( \frac{M^2}{D} \right) \approx \frac{1}{\Delta x} \left( \frac{M_{i+1/2}^2}{D_{i+1/2}} - \frac{M_{i-1/2}^2}{D_{i-1/2}} \right)
\]

**Forward difference** (M < 0)

\[
\frac{\partial}{\partial x} \left( \frac{M^2}{D} \right) \approx \frac{1}{\Delta x} \left( \frac{M_{i+3/2}^2}{D_{i+3/2}} - \frac{M_{i+1/2}^2}{D_{i+1/2}} \right)
\]

**Special grid**

- **M_{i-1/2}**, **M_{i+1/2}**, **M_{i+3/2}**

  - **i-1/2**, **η_i**, **M_{i-1/2}**
  - **i+1/2**, **η_{i+1}**, **M_{i+1/2}**
  - **i+3/2**, **M_{i+3/2}**

- **M_{i-1/2}**, **M_{i+1/2}**, **M_{i+3/2}**

  - **i-1/2**, **M_{i-1/2}**
  - **i+1/2**, **η_i**, **M_{i+1/2}**
  - **i+3/2**, **η_{i+1}**, **M_{i+3/2}**
The finite difference equation of tsunami model

**Continuity equation**

\[
\eta^{k+1} = \eta^k - \frac{\Delta t}{\Delta x} (M_{i+1/2} - M_{i-1/2})
\]

**Momentum equation**

\[
M_i^{k+1/2} = M_i^{k-1/2} - gD_i \frac{\eta_{i+1} - \eta_i}{\Delta x} \Delta t - \left( \lambda_1 \frac{M_{i+3/2}^2}{D_{i+3/2}} + \lambda_2 \frac{M_{i+1/2}^2}{D_{i+1/2}} + \lambda_3 \frac{M_{i-1/2}^2}{D_{i-1/2}} \right)
\]

\[
\begin{cases}
M_{i+1/2} \geq 0 & \lambda_1 = 0, \quad \lambda_2 = 1, \quad \lambda_3 = -1 \\
M_{i+1/2} < 0 & \lambda_1 = 1, \quad \lambda_2 = -1, \quad \lambda_3 = 0
\end{cases}
\]
Inundation model
Inundation model

Momentum equation

\[ M^{k+1/2} = M^{k-1/2} - gD_{i+1/2} \frac{\eta_{i+1} - \eta_i}{\Delta x} \Delta t - \left( \lambda_1 \frac{M_{i+3/2}^2}{D_{i+3/2}} + \lambda_2 \frac{M_{i+1/2}^2}{D_{i+1/2}} + \lambda_3 \frac{M_{i-1/2}^2}{D_{i-1/2}} \right) \]

D (total depth) in momentum equation is defined at the edge of grid,
But estimated D(total depth) from continuity equation is center of grid

IF \( Di = 0 \) (dry condition)

\[ \begin{align*}
\text{Di-1/2} &= 0.5 \times (Di + Di-1) \\
\text{Di+1/2} &= 0.5 \times (Di+1 + Di)
\end{align*} \]

⇒ Inundation model
Inundation model

- There are 6 types of inundation patterns in the difference scheme.

Pattern (1)

Pattern (2)

Pattern (3)

Pattern (4)

Pattern (5)

Pattern (6)
**Inundation model**

### Algorism of inundation model

1. **Branch 1**
   - Is there water in left side?
     - No
     - **Branch 4**
       - Is there water in right side?
         - No
         - **Pattern 6**
           - No water! Water flux $M = 0$
         - Yes
         - **Pattern 5**
           - Water can not move! Water flux $M = 0$
       - Yes
       - **Pattern 4**
         - Depth at $i$
           - $D_i = Z(i+1) + H(i)$
         - Water can not move! Water flux $M = 0$
   - Yes
   - **Pattern 2**
     - Depth at $i$
       - $D_i = Z(i) + H(i+1)$
     - Water flux $M = 0$
2. **Branch 2**
   - Is there water in right side?
     - No
     - **Branch 3**
       - Is water level at LEFT side higher than the ground level at RIGHT side?
         - No
         - **Pattern 1**
           - Depth at $i$
             - $D_i = (DZ(i) + DZ(i+1))/2$
         - Yes
         - **Pattern 3**
           - Water can not move! Water flux $M = 0$
       - Yes
       - **Pattern 4**
         - Depth at $i$
           - $D_i = Z(i) + H(i+1)$
         - Water can not move! Water flux $M = 0$
   - Yes
   - **Pattern 5**
     - Water can not move! Water flux $M = 0$
Nesting model
It is better to use detailed grid mesh. But it takes time to compute the detailed grid mesh. So we use changing grid systems.
Nesting grid system

Small region receives interpolated water flux M,N from large region.

Large region receives averaged water level η from small region.
Nesting grid system

(1) Large region to Small region

$$m_1 = \frac{2 \times M_i + M_{i+1}}{3}$$
$$m_2 = \frac{M_i + 2 \times M_{i+1}}{3}$$

Algorism

- Continuity equation of small region
- Insert the averaged $\eta$ from small to Large region
- Momentum equation of Large region
- Insert the interpolated $M$ and $N$ from Large to small region

(2) Large region to Small region

$$\eta_i = \frac{\eta_1 + \eta_2 + \eta_3 + \cdots + \eta_n}{n}$$
Exercise
Tsunami simulation

Original Fortran code:
TUNAMI (Tohoku University’s Numerical Analysis Model for Investigation)

Governing equations:
Linear / Nonlinear long-wave / Dispersive wave model

Coordinate systems:
Spherical (lat, long) / Cartesian (meter) coordinate system

Required data:
Bathymetry data (Digital elevation model)
$\rightarrow$ GEBCO
Numerical condition

Temporal grid size: CFL condition

Velocity of numerical information\[ \sqrt{2} \frac{\Delta x}{\Delta t} \]

Velocity of tsunami wave\[ \sqrt{gh} \]

\[ \sqrt{2} \frac{\Delta x}{\Delta t} > \sqrt{gh} \rightarrow \Delta t < \Delta x \sqrt{\frac{2}{gh}} \]

In nonlinear case, temporal grid size should be much smaller than CFL-condition
How to estimate required spatial grid size

**Numerical domain**

$\lambda < \frac{\lambda}{a}$  \hspace{1cm} (a $\approx$ 20)

\[ \lambda = \sqrt{gh} \times T \]

$\Delta x$ is grid size, $h$ is water depth, $T$ is wave period

Here, we estimate wave period $T$ from the condition at tsunami source (fault)

$T = \frac{\lambda_0}{\sqrt{gh_0}}$

\[ \lambda_0 = 2 \times \cos \delta \times W \]

$W \times \cos \delta$

$\lambda_0$ is wave length at tsunami source, $h_0$ is water depth at tsunami source, $\delta$ is dip of fault, $W$ is fault width

$\Delta x_{\text{max}} = \sqrt{gh_{\text{min}}} \times T / a$

One-twentieth of wave length
**Numerical condition**

### Numerical domain

\[ h_{\text{min}} = \frac{(a\Delta x)^2 h_0}{\lambda_0^2} \]

- **Step 1**: Set \( \Delta x \) in region 1
- **Step 2**: Calculate \( h_{\text{min}} \) in region 1
- **Step 3**: Determine area of region 2

**Example** (\( W=50 \text{ km}, \delta=10\text{deg}, h_0=5 \text{ km} \))

<table>
<thead>
<tr>
<th>Region</th>
<th>( \Delta x )</th>
<th>( h_{\text{min}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1850</td>
<td>2823</td>
</tr>
<tr>
<td>2</td>
<td>617</td>
<td>314</td>
</tr>
<tr>
<td>3</td>
<td>206</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>69</td>
<td>4</td>
</tr>
</tbody>
</table>

Nonlinear region (< \( h=50 \text{m} \))

It is better to extend more wide area
Tsunami source model

Manshinha & Smylie (1974)
Exercise
Software installation

1. GMT (The general mapping tools, http://gmt.soest.hawaii.edu/)
   Editor tool for Bathymetry data (GMT_basic_install.exe)

2. Ghostscript and GSView
   Viewer for postscript format (gs863w32full-gpl, gsv49w32.exe)

3. Gnuplot 4.6
Bathymetry data

• GEBCO (General Bathymetric Chart of the Oceans)
  Web page: http://www.gebco.net/
  Download: https://www.bodc.ac.uk/data/online_delivery/gebco/

• Grid size: 1 minute (2003 released and 2008 updated)
  30 second (2009 released)

• Format: NetCDF (Binary data)

• Software: Grid display software (provided by GEBCO)
  GMT (http://gmt.soest.hawaii.edu/)
Exercise 1 (GMT)

Clip the bathymetry data around Indian Ocean

Requirement: Make two different grid size data => 4 min

Data size should be Less than 400 × 400 Grid

2004 Indian Ocean tsunami

(C) Google earth
Example (Indonesia)

4 min (75/100/-10/20)

4 min (your target area)

Chile
Myanmar
Papua New Guinea
Philippines
Etc..
TUNAMI MODEL

Example 1 (.. ¥TUNAMI MODEL¥example1(GAUSS))

(1) Binary data: Tunami-32bit.exe
(2) Cntrl folder: cntrl.fil, domains.cnt, faults.int, options.opt
(3) Input folder: bathymetry data (ASCII data)
(4) Output folder:
Flow

1. Put the bathymetry data into “input folder”

2. Set numerical condition in the following files:
   - Change cntrl.fil ・・・ to set file name blow
   - Change ***.cnt ・・・ to set numerical condition
   - Change ***.int ・・・ to set tsunami source
   - Change ***.opt ・・・ to do simple case test etc..

3. Click TUNAMI.exe
Exercise 2

Gaussian distribution

Initial elevation: Gaussian distribution
Water Depth: constant (=5000m)
Governing equation: Linear long-wave model
\( X,y\)-grid\((300,300),dx=3000.0,dt=5.0 \)

\[ Z = \frac{1}{2\pi} e^{-\frac{(x^2+y^2)}{2}} \]
Exercise 3

Sine wave propagation in a channel

Initial elevation: Sine wave

Water Depth: Channel.dat

Governing equation:

Nonlinear long-wave model

dx=10.0, dt=0.2, x-size=11, y-size=3000

A=5.0, Period=600.0, Duration=300.0
Exercise 4

Tsunami propagation in real bathymetry

Tsunami source: 2004 Indian Ocean tsunami

Maximum tsunami height

Fault Parameter of 2004 event

<table>
<thead>
<tr>
<th>No</th>
<th>Slip</th>
<th>Length</th>
<th>Width</th>
<th>Depth</th>
<th>Strike</th>
<th>Dip</th>
<th>Slip angle</th>
<th>Lat</th>
<th>Long</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>400</td>
<td>150</td>
<td>10</td>
<td>358</td>
<td>15</td>
<td>90</td>
<td>92.5</td>
<td>6.5</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>500</td>
<td>150</td>
<td>10</td>
<td>329</td>
<td>15</td>
<td>90</td>
<td>94.8</td>
<td>2.5</td>
</tr>
</tbody>
</table>
Exercise 4.1

Set tide-gauge data and make tide records

Tsunami source: 2004 Indian Ocean tsunami

Tide-record at SIBOLGA (98.76667 1.75) in 2004 Indian Ocean tsunami
Exercise 5

Nesting in spherical coordinate system

Tsunami source: 2004 Indian Ocean tsunami
Clip bathymetry

4 min (75/100/-10/20)

2 min

$h_{\text{min}} \approx 400 \text{ m}$
Numerical condition

**Numerical domain**

\[ h_{\text{min}} = \frac{(a\Delta x)^2 h_0}{\lambda_0^2} \]

Step 1: Set \( \Delta x \) in region 1
Step 2: Calculate \( h_{\text{min}} \) in region 1
Step 3: Determine area of region 2

**Example** (\( W=50 \text{ km}, \delta=10\text{deg}, h_0=5 \text{ km} \))

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<td>69</td>
<td>4</td>
</tr>
</tbody>
</table>

Nonlinear region (< \( h=50\text{m} \))
It is better to extend more wide area
Degree to meter

Simple estimation

\[ \Delta x = \Delta s \times R \times \cos \left( \text{latitude} \times \frac{\pi}{180} \right) \]

【\( \Delta x \) in meter】

<table>
<thead>
<tr>
<th>Grid size/ Latitude(°)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 min</td>
<td>1854</td>
<td>1847</td>
<td>1826</td>
<td>1743</td>
</tr>
<tr>
<td>2 min</td>
<td>3709</td>
<td>3695</td>
<td>3652</td>
<td>3485</td>
</tr>
<tr>
<td>3 min</td>
<td>5563</td>
<td>5542</td>
<td>5479</td>
<td>5228</td>
</tr>
<tr>
<td>4 min</td>
<td>7417</td>
<td>7389</td>
<td>7305</td>
<td>6971</td>
</tr>
</tbody>
</table>

【Water depth: \( h_{\text{min}} \)】

Indian Ocean tsunami (\( W=150\text{km}, \text{dip}=15\text{deg}, h_0=1000\text{ m} \) )

<table>
<thead>
<tr>
<th>Grid size/ Latitude(°)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>20</th>
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<tbody>
<tr>
<td>1 min</td>
<td>25</td>
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<td>22</td>
</tr>
<tr>
<td>2 min</td>
<td>98</td>
<td>98</td>
<td>95</td>
<td>87</td>
</tr>
<tr>
<td>3 min</td>
<td>221</td>
<td>219</td>
<td>214</td>
<td>195</td>
</tr>
<tr>
<td>4 min</td>
<td>393</td>
<td>390</td>
<td>381</td>
<td>347</td>
</tr>
</tbody>
</table>
Exercise 6

Nesting in spherical coordinate system for your target area

Step 1 Clip your target area
(4min and 2 min and 1 min)

Step 2 Set control files

Step3 Make animation and figures
(Water propagation and Maximum elevation)