Global Seismological Observation Course

Instrumentation & Observation (1)

Seismometer

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Contents

I_{JMA} and M_{JMA}

+JMA Magnitude

Magnitude: a conventional scale of earthquakes that describe how big the event itself.
There are many definitions for magnitude scale, e. g., M_L, M_s, M_b, and M_{JMA}

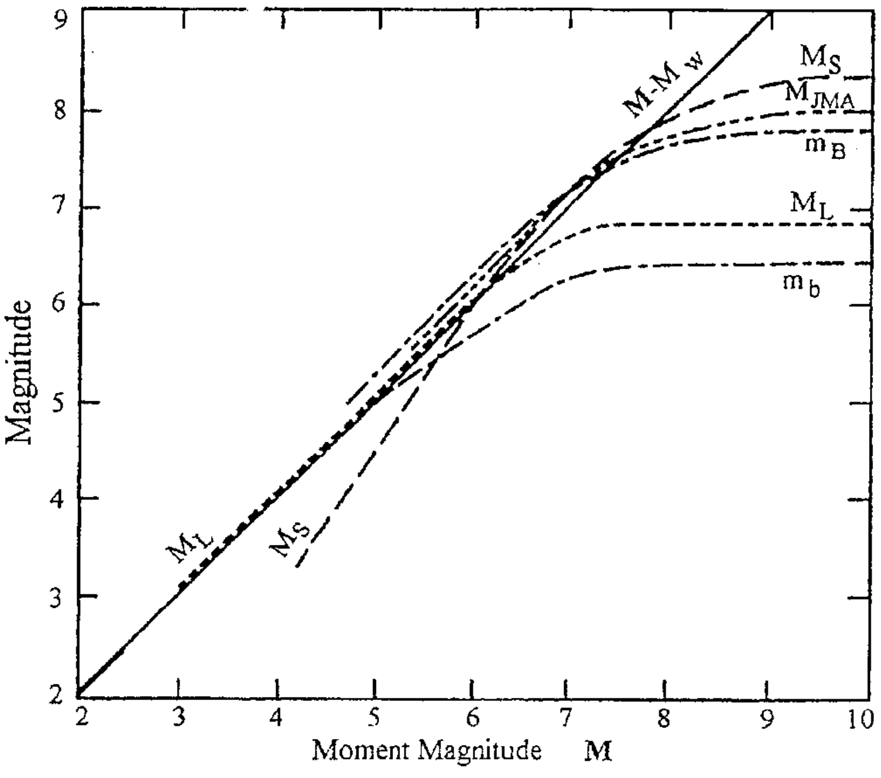


Figure 14 Magnitudes from different scales plotted against moment magnitude M. (Modified from Heaton et al., 1986)

History of Intensity Scale

Rossi-Forel Scale:

1883. Rossi & Forel (10 scale)

Modified Melcali scale:

????. Mercalli (10 scale)

1902. Modified by Cancani (12 scale)

1931. Modified by Wood & Neuman

1956. Modified by Richter

MSK scale:

1964. Medvedev, Sponheuer & Karnik

European Macroseismic Scale

1998 EMS98 by CONSEIL DEL'EUROPE

http://www.gfz-potsdam.de/pb5/pb53/projekt/ems/

JMA scale:

1884. Sekiya (4 scale)

1898. JMA (7 scale)

1936. Modified.

1949. Modified based on Fukui Eq. (8 scale)

1996. Modified for automated determination

Intensity Scale

Input or Output ?

Input
Ground Motion

⇒

System
Human Being
Building & Houses

⇒

Output
Feelings
Damages

JMA Intensity Scale is defined as Input, however, designed to keep high correlation with Output (human feeling, damages, etc). It is automated in 1996.

In the past, seismic intensity was estimated from a compilation of human perception and the resultant casualties.
Since 1 April 1996, it has been measured automatically with seismic intensity meters and announced rapidly to the public and officials. There are about 600 JMA seismic intensity observation stations throughout Japan as of April 1996.

$$I_{MM}=1.85 \cdot I_{JMA}-2.04 \ (\sigma=0.315)$$

by Shabestari & Yamazaki (1998)

Accelerometers for JMA Intensity Determination

By the collaboration with municipalities and NIED, the number of Accelerographs used for Intensity determination has been increased.

Calculated Value	JMA Intensity (keisoku Shindo)
0~0.4	Intensity 0
0.5~1.4	Intensity 1
1.5~2.4	Intensity 2
2.5~3.4	Intensity 3
3.5~4.4	Intensity 4
4.5~4.9	Intensity 5 Lower
5.0~5.4	Intensity 5 Upper
5.5~5.9	Intensity 6 Lower
6.0~6.4	Intensity 6 Upper
6.5~	Intensity 7

JMA Seismic Intensity is now divided into 10 scales.

Because "intensity 5" or "intensity 6" didn't necessarily correspond to the same degree of damage, "intensity 5" and "intensity 6" have been divided into two scales : "intensity 5 Lower" and "intensity 5 Upper" and "intensity 6 Lower" and "intensity 6 Upper" respectively, since 1 October 1996. The intensity is now divided into 10 scales as a result.

7 scales to 10 scales, in 1996

Principle of Seismometer

Apparatus

	Detection	Record
Seismoscope	○	×
Seismograph	○	○

The first Seismoscope in the history

河北省唐山市地震記念館

張衡的地動儀（復元模型）

The first modern seismograph was made by Ewing, Gray and Milne, the British teachers invited by Japanese government.
They started observation of earthquakes in 1872.
Tokyo Meteorological Observatory started the routine observation in 1875.
The Seismological Society of Japan was established at that time due to the experience of the Yokohama Earthquake (1880)

http://neic.usgs.gov/neis/seismology/history_seis.html

The Early History of Seismometry (to 1900)
By James Dewey and Perry Byerly

Their survey is limited in literatures of Europe and Regions Influenced by Europe.
Until Medieval Period, Europe had been in the backwardness. The survey ought to be done in literatures of Arabs, Turkey, India, China and so on.

	L-4C 1.0 Hz SEISMOMETER	L-4A 2.0 Hz SEISMOMETER
TYPE	Moving dual coil, humbuck wound	Moving dual coil, humbuck wound
FREQUENCY	1.0 ± 0.05 Hz measured on 200 pound weight at 0.09 inches/second	2.0 ± 0.25 Hz measured on 200 pound weight at 0.09 inches/second
FREQUENCY CHANGE WITH TILT	Less than 0.05 Hz at 5° from vertical	less than 0.10 Hz at 10° from vertical
FREQUENCY CHANGE WITH EXCITATION	Less than 0.05 Hz from 0 to 0.09 inches/second	Less than 0.10 Hz from 0 to 0.18 inches/second
SUSPENDED MASS	1000 grams	500 grams
STANDARD COIL RESISTANCES	500, 2000, 5500	500, 2000, 5500
LEAKAGE TO CA		volts
TRANSDUCTION		
OPEN CIRCUIT D		
CURRENT DAMP		
COIL INDUCTAN	Lc in henries	Lc in henries
CASE TO COIL MOTION	PP 0.250 inches	PP 0.250 inches
ELECTRIC ANALOG OF CAPACITY	$Cc = \frac{73{,}500}{Rc}$ (microfarads)	$Cc = \frac{36{,}500}{Rc}$ (microfarads)
ELECTRIC ANALOG OF INDUCTANCE	$Lm = 0.345Rc$ (henries)	$Lm = 0.17Rc$ (henries)
CASE HEIGHT	5⅛ inches—13 cm	5⅛ inches—13 cm
CASE DIAMETER	3 inches—7.6 cm	3 inches—7.6 cm
TOTAL DENSITY	3.7 grams/cm^3	2.9 grams/cm^3
TOTAL WEIGHT	4¾ pounds—2.15 kilograms	3¾ pounds—1.7 kilograms
OPERATING TEMPERATURE	Range: −20° to 140°F or −29° to 60°C	Range: −20° to 140°F or −29° to 60°C

What devil is it?

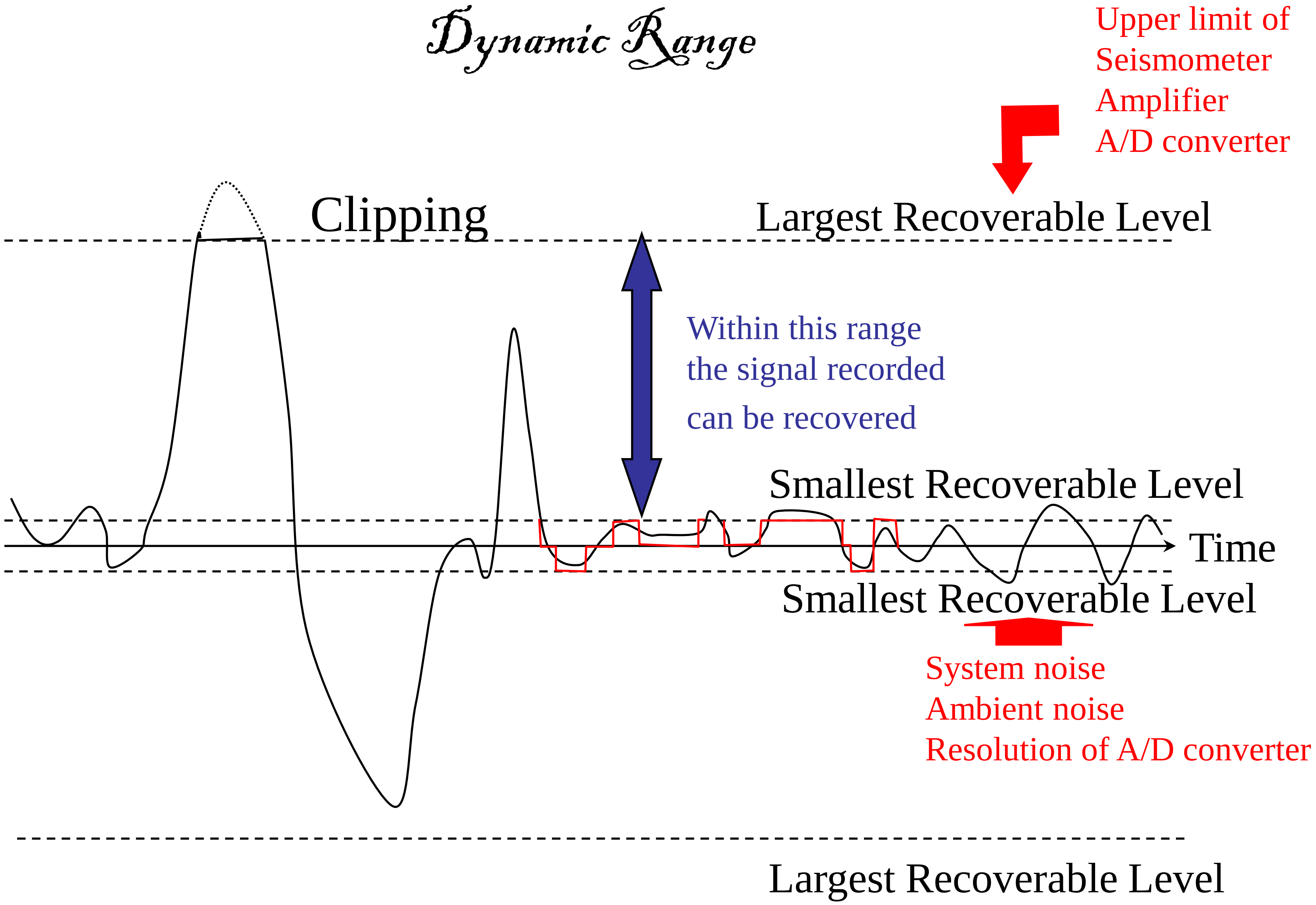
	L-4C 1.0 Hz SEISMOMETER			L-4A 2.0 Hz SEISMOMETER		
COIL RESISTANCE, OHMS	**500**	**2000**	**5500**	**500**	**2000**	**5500**
TRANSDUCTION, VOLTS/IN/SEC	2.12	4.23	7.02	2.12	4.23	7.02
COIL INDUCTANCE, HENRIES	0.55	2.20	6.05	0.55	2.20	6.05
ANALOG CAPACITANCE, MICROFARADS	147	36.8	13.4	73.0	18.3	6.64
ANALOG INDUCTANCE, HENRIES	173	690	1900	85.0	340	935
SHUNT FOR 0.70 DAMPING, OHM	810	3238	8905	810	3238	8905

Open Circuit Damping $(b_o) = 0.28$ Critical   Coil Current Damping $(b_c) = \frac{1.1\,Rc}{Rc + Rs}$ Total Damping $(b_t) = b_o + b_c$

Two Important Technical Terms for Seismographs

- **Dynamic Range**
- **Frequency Characteristics**

Dynamic Range



Dynamic Range

Definition

$$\frac{\textbf{The Largest recoverable Level}}{\textbf{The Smallest recoverable Level}}$$

Usually described by [db]

$$20\log_{10}(\text{ Ratio })\quad[\text{db}]$$

+6 db ～ 2 times
120 db ～ 1,000,000 times

Seismograph (high-magnification or sensitivity)

Instruments which can record the ground motion so small as teleseismic one or microtremor without distortion.

Strong Motion Seismograph

Instruments which can record the ground motion so strong as destructive one without clipping.

Wide Dynamic Range System

Observation systems or Observatories which can record from very small vibration to strong motion simultaneously without distortion.

Two Important Technical Terms for Seismographs

- **Dynamic Range**
- **<u>Frequency Characteristics</u>**

Most popular type of seismometer is composed of a pendulum, and there are some different types which show an importance in the seismology.
Particularly, the moving coil seismometer connected to an electric amplifier has obtained the majority due to its simplicity in operation.

The main target of this text is such type of seismograph.

For this purpose, a step-by-step explanation is preferable.

* Dynamics of Pendulum
* Mechanical Seismograph
* Moving Coil Type Electro-magnetic Seismometer
* Feed Back Seismometer

Simple Mass-Spring Pendulum

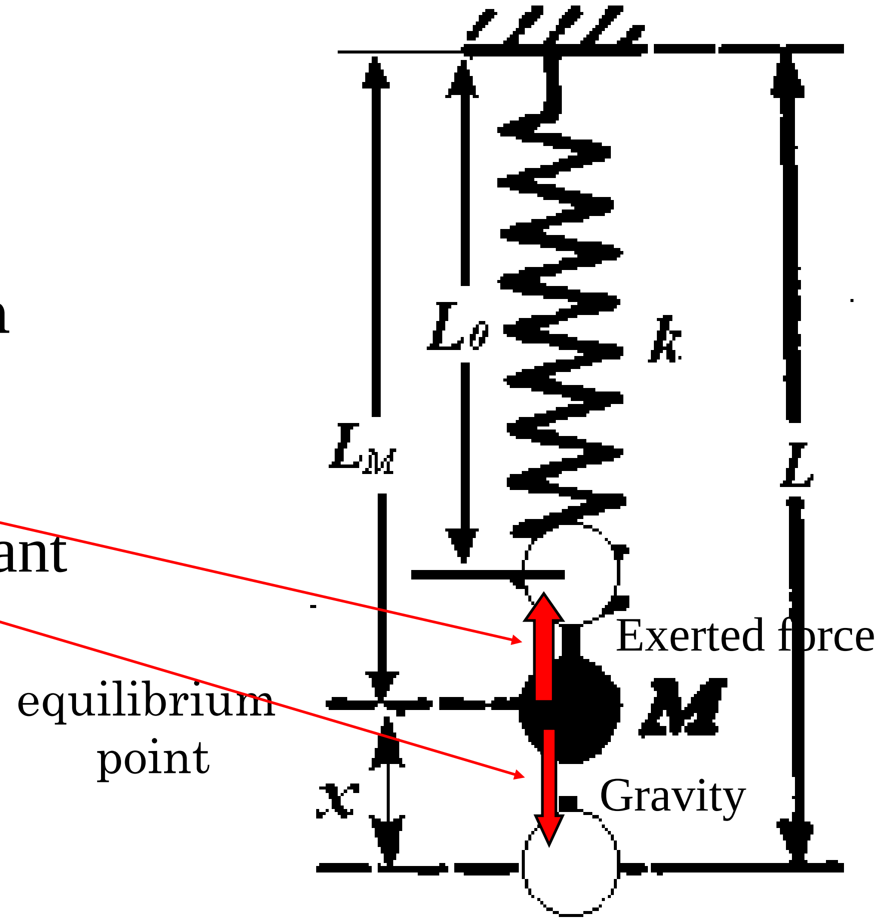
Stable situation

Equation of equilibrium

$Mg=k(L_M-L_0)$

This gives the spring constant

$k= Mg/(L_M-L_0)$



Simple Mass-Spring Pendulum

The mass is displaced a little.

Exerted force

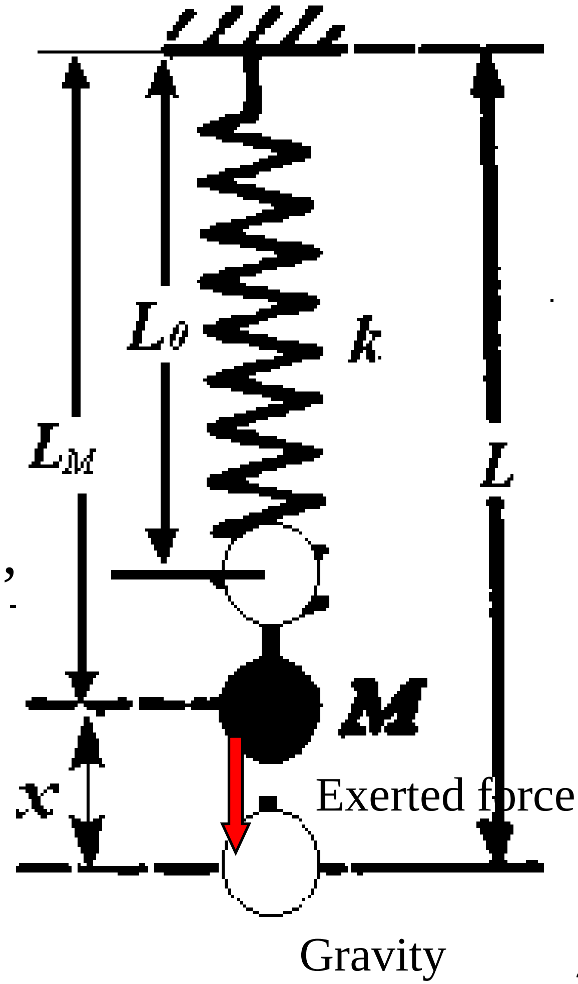
$$\left\{ k\left(L_M - L_0 \right) + kx \right\}$$

Equation of motion

$$M\frac{d^2x}{dt^2} = Mg - \left\{ k\left(L_M - L_0 \right) + kx \right\} ,$$

By using the relation for *k*

Eq. (1) $$M\frac{d^2x}{dt^2} = -kx \;\; or \;\; \frac{d^2x}{dt^2} = -\frac{k}{M}x \; .$$



To solve the second order differential equation

$$\frac{d^2x}{dt^2} = -\frac{k}{M}x$$

assume that the time dependence of x is sinusoidal with a constant x_0 and angular frequency ω_0,.

$$x = x_0 e^{i\omega_0 t}$$

Then,

$$-\omega_0^2 x = -\frac{k}{M}x$$

This require

$$\omega_0 = \pm\sqrt{\frac{k}{M}}$$

Natural Angular frequency of Pendulum

and

$$T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{M}{k}}$$

Natural Period of Pendulum

Simple Mass-Wire Pendulum

When the pendulum is displaced to a small angle θ ,the force of gravity *mgsin*θ with arm length *L* exerts a moment about O equal to-(*mg Lsin*θ*).* As the inertial moment of the mass is *mL²* , the equation of motion is

$$mL^2 \frac{d^2\theta}{d\,t^2} = -mgL\sin\theta\ .$$

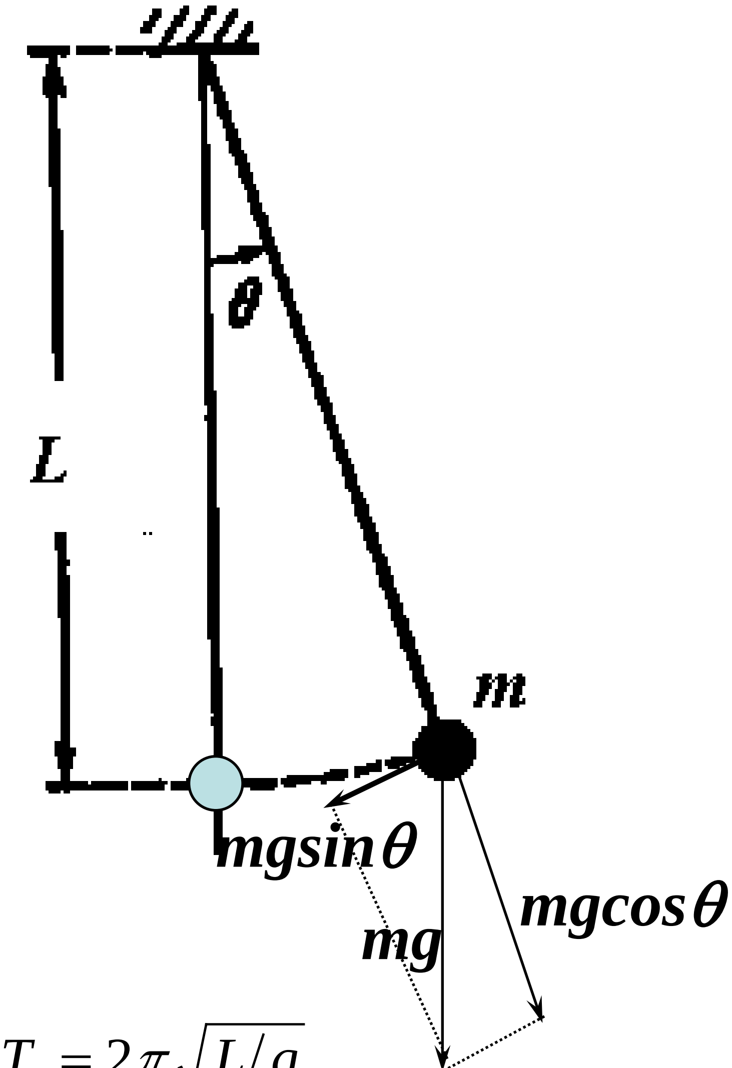
For values of θ small enough that θ is negligible compared to, then

$$\sin\theta = \theta - \frac{\theta^3}{6} + \frac{\theta^5}{120}\cdots \approx \theta\ .$$

The equation of motion

$$mL^2 \frac{d^2\theta}{d\,t^2} = -mgL\,\theta\ or\ \ \frac{d^2\theta}{d\,t^2} = -\frac{g}{L}\theta.$$

The natural period of this pendulum is $T_0 = 2\pi\sqrt{L/g}$

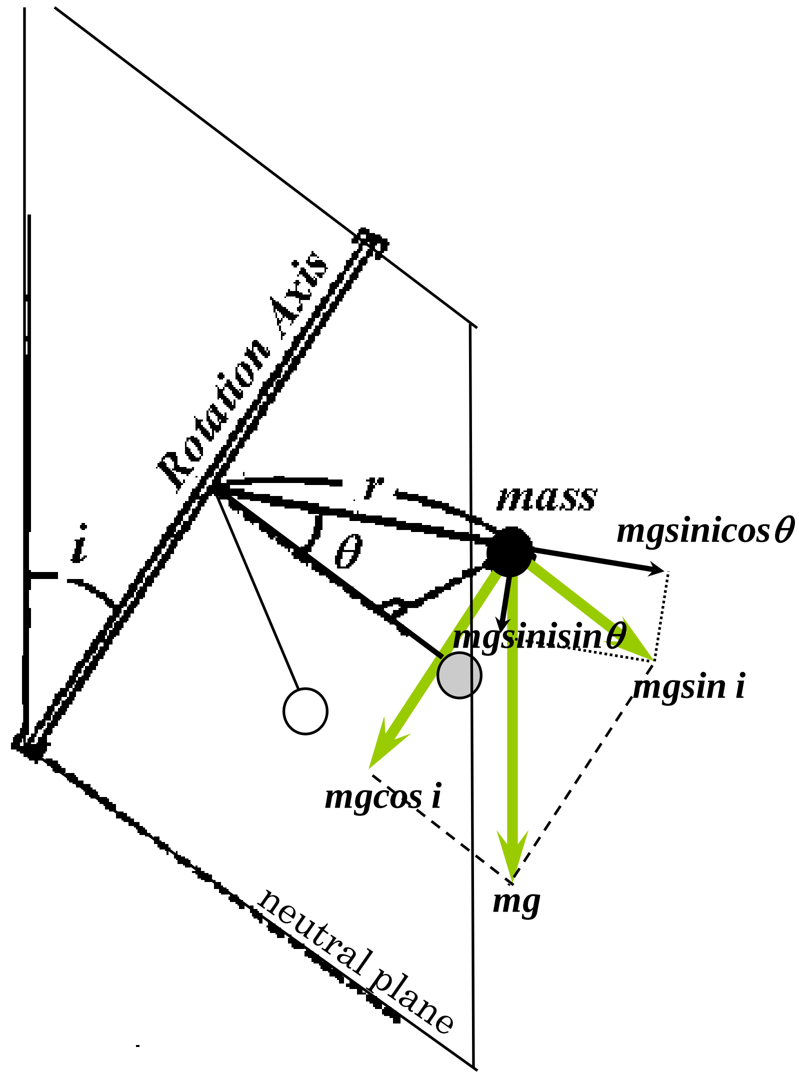


Dynamics of Pendulum

Horizontal Pendulum

Resolve the gravity force *mg* acting at the center of mass into components parallel to the rotation axis, *mgcosi*, and perpendicular to this direction, *mgsini*.

When the mass is in the plane determined by the vertical and the axis of rotation, called the neutral plane, this pendulum is in equilibrium.



Dynamics of Pendulum

Horizontal Pendulum

When the pendulum is displaced from the neutral plane through an angle θ about the axis of rotation, the latter component with arm length, *r*, gives the restoring moment -*mgr*sin*i*sinθ.

Thus, the equation of motion is

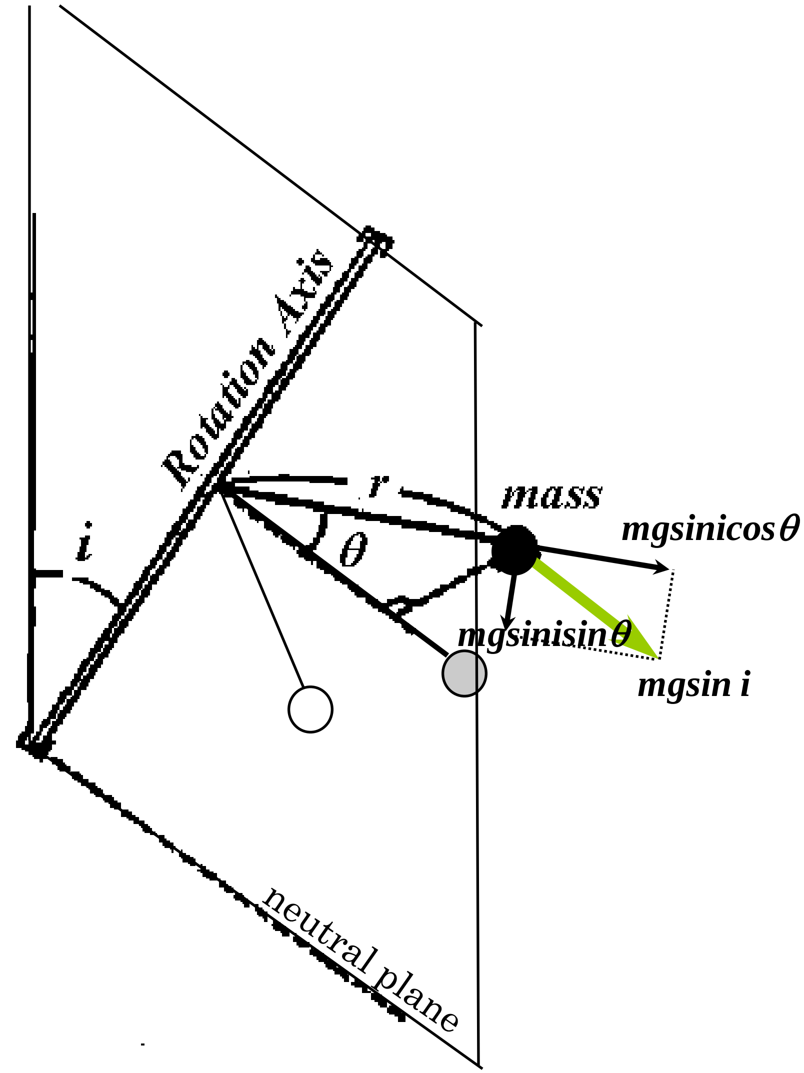
$$K\frac{d^2\theta}{d\,t^2} = -mg\sin i \cdot r\sin\theta \ .$$

Using the assumption of small θ, this is approximated in

$$K\frac{d^2\theta}{d\,t^2} = -mgr\sin i \cdot \theta \ .$$

The natural period is

$$T_0 = 2\pi\sqrt{\frac{K}{mgr\sin i}} \ .$$



Horizontal Pendulum

Hinge having finite stiffness

If the pivot at O is not perfectly flexible, but consists of a thin metal strip, an additional restoring moment is exerted by this hinge. Such strip is equivalent to another spring of which equivalent stiffness is k_H connected at the point two thirds of the strip length, l, from the pivot point. The equivalent stiffness is given by the flexural stiffness E_I.

$$k_H = 3E_I / l^3$$

The equation of motion is

$$K\frac{d^2\theta}{dt^2} = -mg\sin i \cdot r\sin\theta - k_H\left(\frac{2l}{3}\right)^2 \sin\theta \; .$$

Then,

$$K\frac{d^2\theta}{dt^2} = -\left(mgr\sin i + E_I\left(\frac{4}{3l}\right)\right)\theta \; .$$

Dynamics of Pendulum

Horizontal Pendulum

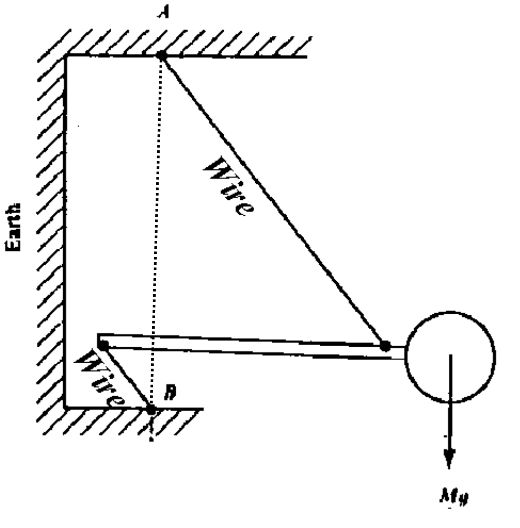
Hinge having finite stiffness

The natural period of the pendulum with hinge strip of finite stiffness is

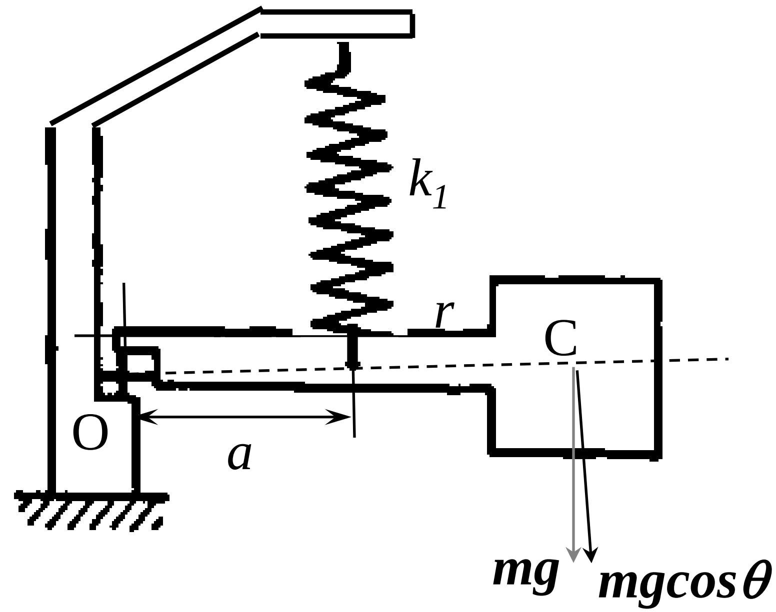
$$T_0 = 2\pi \sqrt{K \Big/ \left(mgr \sin i + E_I \left(\frac{4}{3l} \right) \right)}.$$

Even though the inclination angle, *i*, is very small or zero, the stiffness of such strip makes the natural period finite. If the strip is enough stiff, the inclination of axis can not give a considerable influence to the natural period.

Zero-stiffness hinge is achieved by Zöllner suspension. This can make the natural period infinite when the rotation axis becomes vertical. In such case, the system can not be in the equilibrium therefore unstable. The maximum stable period achieved by a standard instrument is around 30 seconds.



Spring Supported Hinged Bar



Suppose that the supporting spring is vertical for simplicity.
The gravity force acting at the center of mass, C, at distance *r* from the center of rotation, O, gives a moment which tends to pull down the pendulum. This moment is the product of the force component $mgcos\theta$ and arm length *r*.
The elongated spring gives a restoring moment, $k_1a(L-L_0)+k_1a^2sin\theta$.
Suppose the system is in equilibrium when the bar lays horizontally, then $mgr=k_1a(L-L_0)$.

The equation of motion with the assumption of small θ

$$K\frac{d^2\theta}{d\,t^2}=-k_1a^2\theta\ .$$

Spring Supported Hinged Bar

If the pivot consists of an elastic strip, the equation of motion is

$$K\frac{d^2\theta}{d\,t^2} = -(k_1 a^2 + E_I\,\frac{4}{3l})\theta\ .$$

The natural period of the pendulum with hinge strip of finite stiffness is

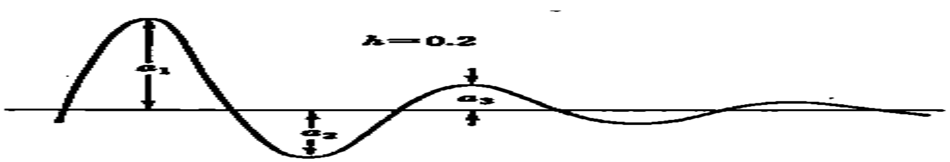
$$T_0 = 2\pi\sqrt{K\Big/(k_1 a^2 + E_I\,\frac{4}{3l})}\ .$$

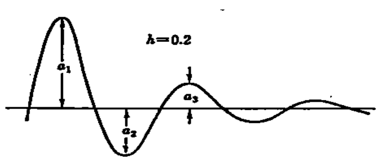
This shows the difficulty to realize a long natural period, i.e.,a big mass and very soft spring and strip are required. Therefore much effort has been paid in the history of seismometry to obtain a long period pendulum. Ewing type and La Coste type pendulum are examples of the results.

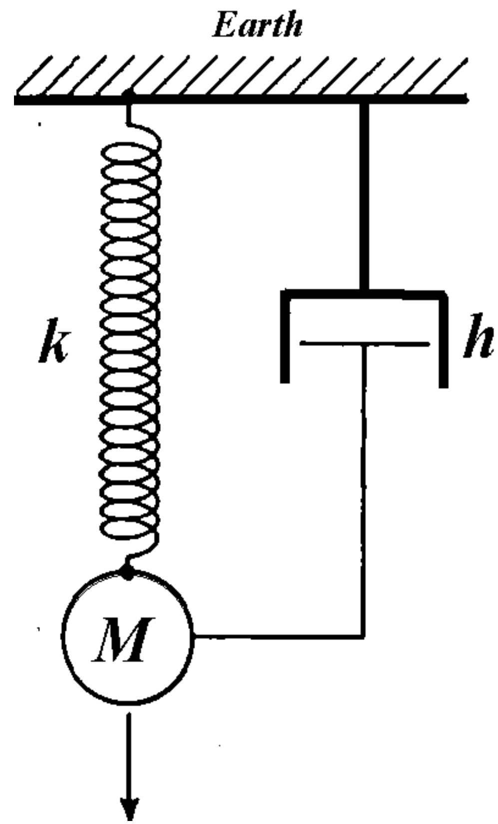
Dynamics of Pendulum

Damping

Damping is the phenomenon in that the oscillation slows down due to dissipation of the oscillation energy.







The portion of the force causing the **damping**, which is proportional to the mass velocity, is called the viscous damping. Its proportionality factor is the **coefficient of viscous damping**. This can be represented by a dashpot.

Pendulum suspended by a parallel arrangement of spring and dashpot representing the damping.

Dynamics of Pendulum

Damping

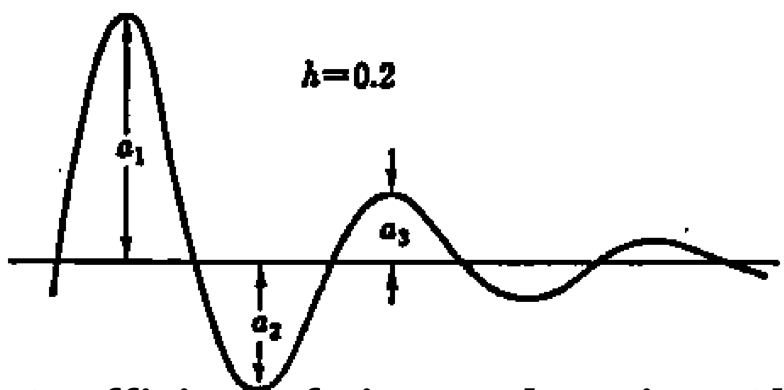
Measure of damping

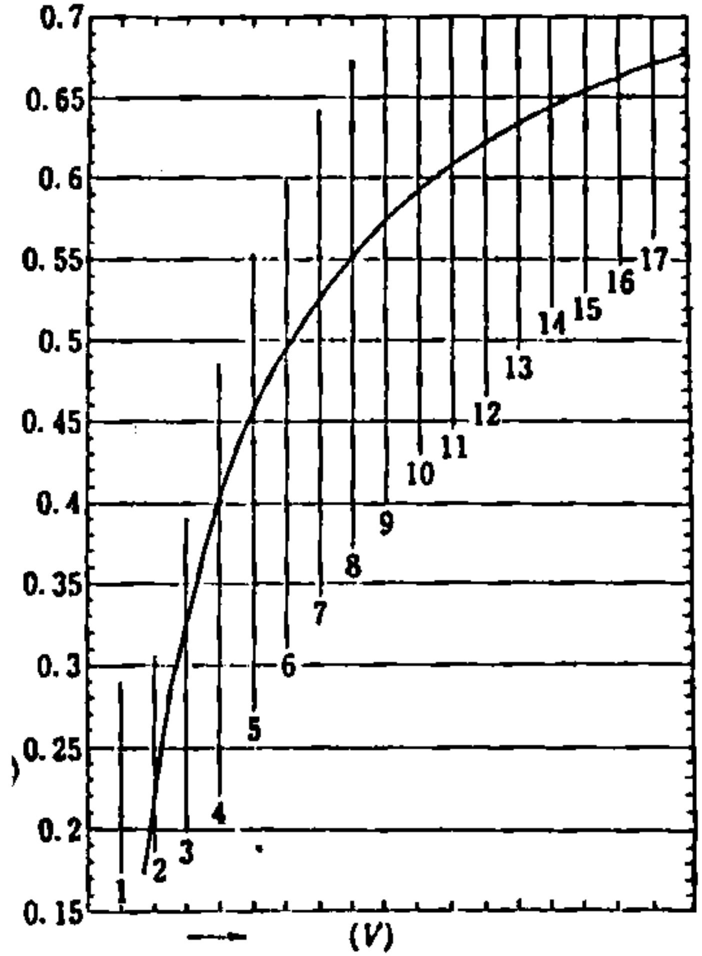
Damping Ratio

$$v = a_1 / a_2 = a_2 / a_3 = \cdots$$

Damping Factor

$$h = 1 \Big/ \sqrt{1 + \left(\pi / \log_e v \right)^2}$$



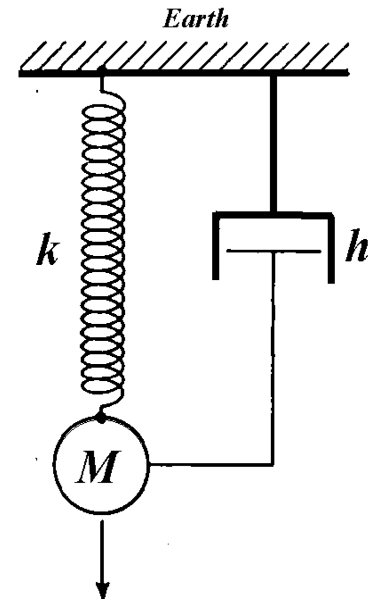


Coefficient of viscous damping=$2h\omega_0$

Dynamics of Pendulum

Damping

The equation of motion damped pendulum is



$$\frac{d^2x}{dt^2} = -\omega_0{}^2 x - 2h\omega_0 \frac{dx}{dt},$$

or

$$\frac{d^2x}{dt^2} + 2h\omega_0 \frac{dx}{dt} + \omega_0{}^2 x = 0.$$

Dynamics of Pendulum

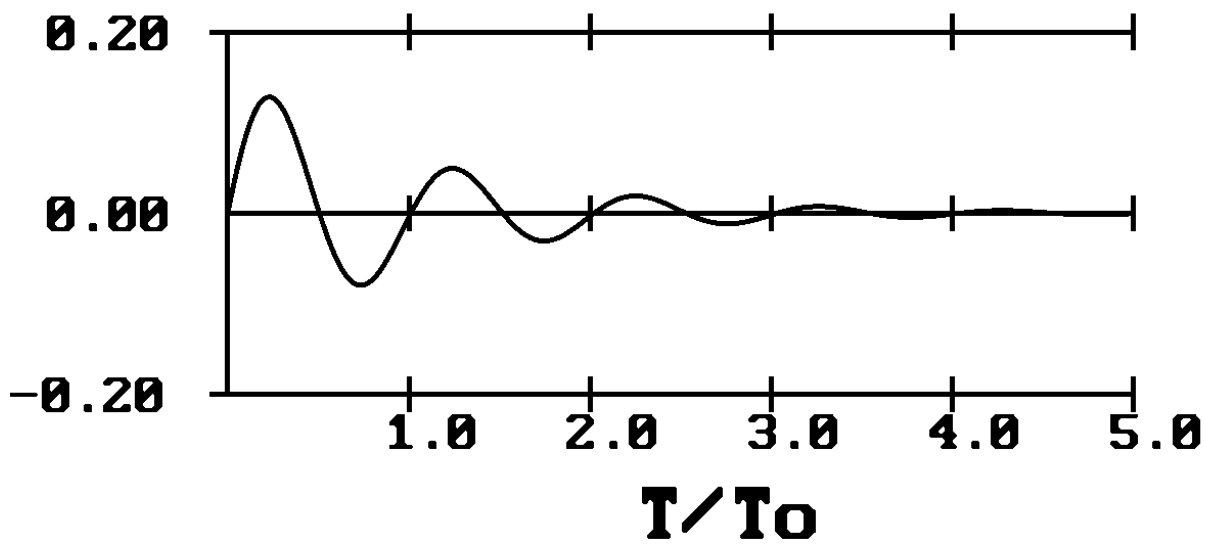
Damping

$$h < 1.0$$

Under damped

$$\begin{aligned} x(t) &= \exp(-h\omega_0 t)\cdot\left[A\exp\left(i\sqrt{1-h^2}\,\omega_0 t\right)+B\exp\left(-i\sqrt{1-h^2}\,\omega_0 t\right)\right] \\ &= C\exp(-h\omega_0 t)\cdot\sin\left(\sqrt{1-h^2}\,\omega_0 t+\phi\right) \end{aligned}$$

.

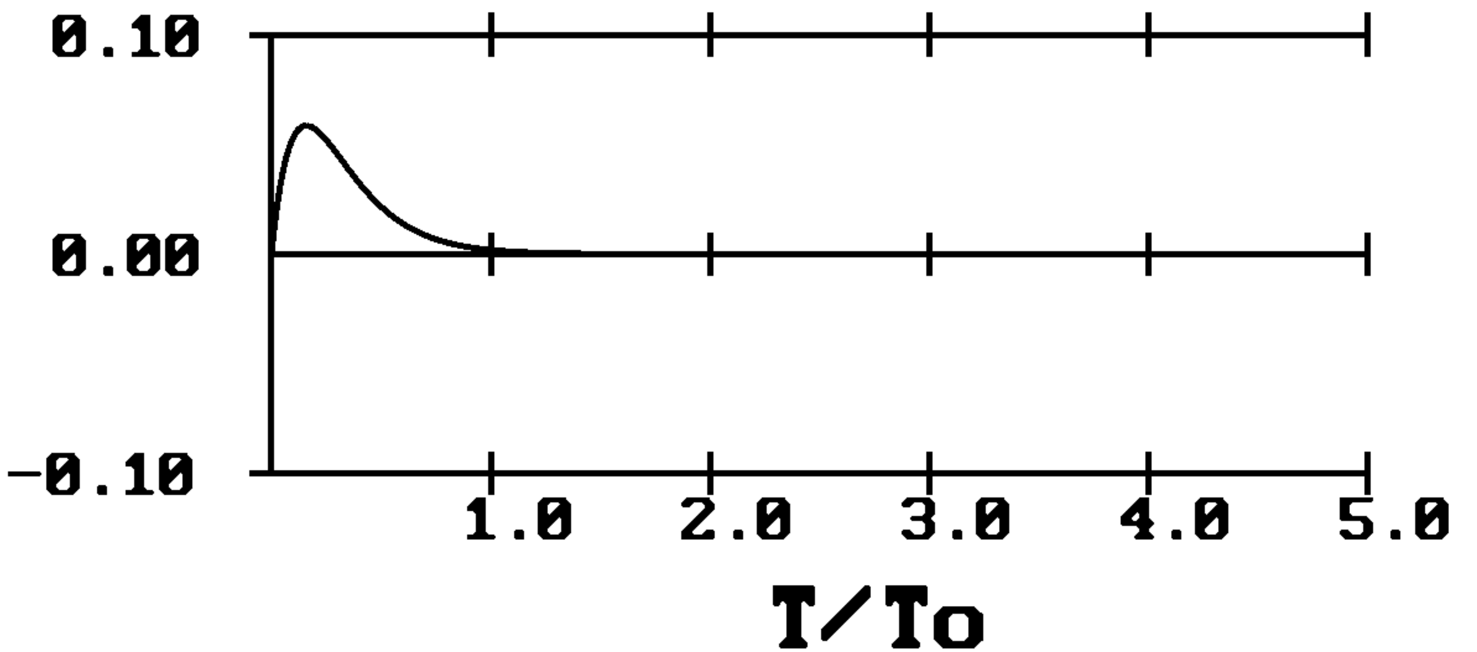


Damping

$$h = 1.0$$

Critically damped

$$x(t) = \exp(-\omega_0 t)[C_1 + C_2 t] \quad .$$

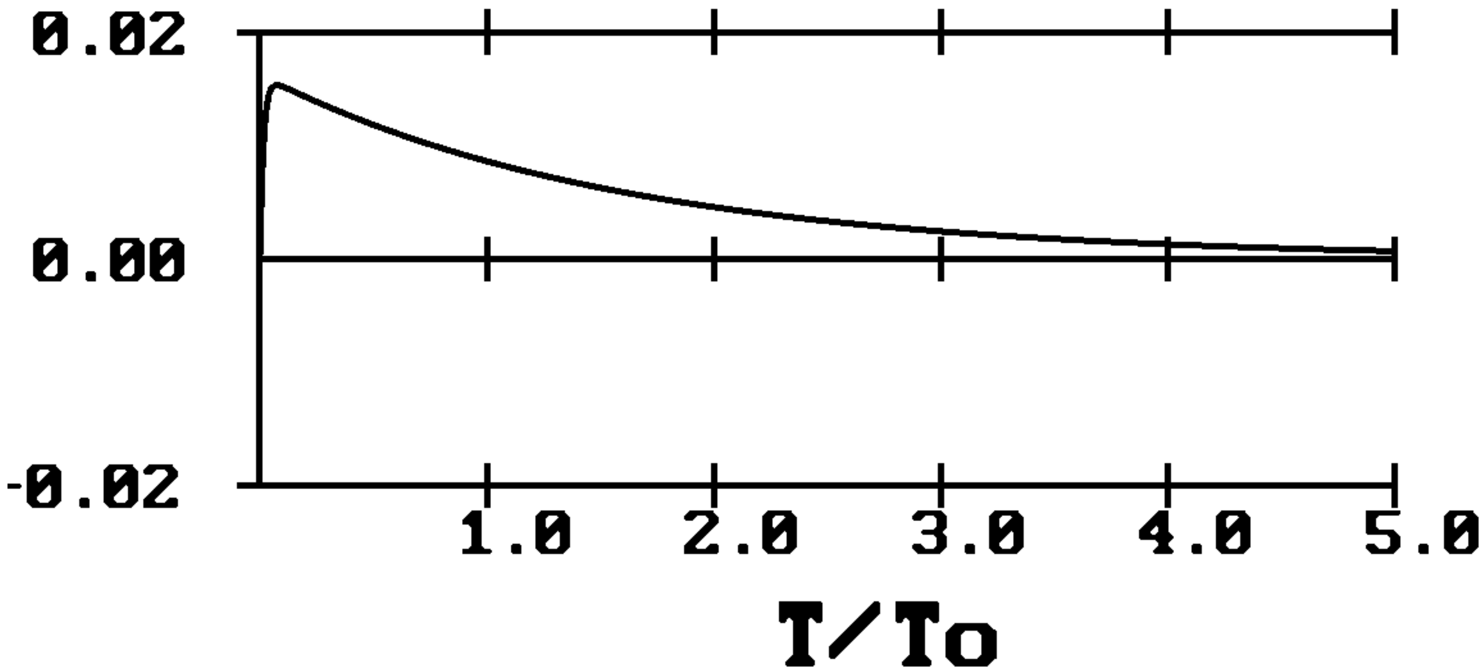


Damping

$$h > 1.0$$

Over damped

$$x(t) = \exp(-h\omega_0 t)\left[C_1 \cosh\left(\sqrt{h^2-1}\,\omega_0 t\right) + C_2 \sinh\left(\sqrt{h^2-1}\,\omega_0 t\right)\right] .$$



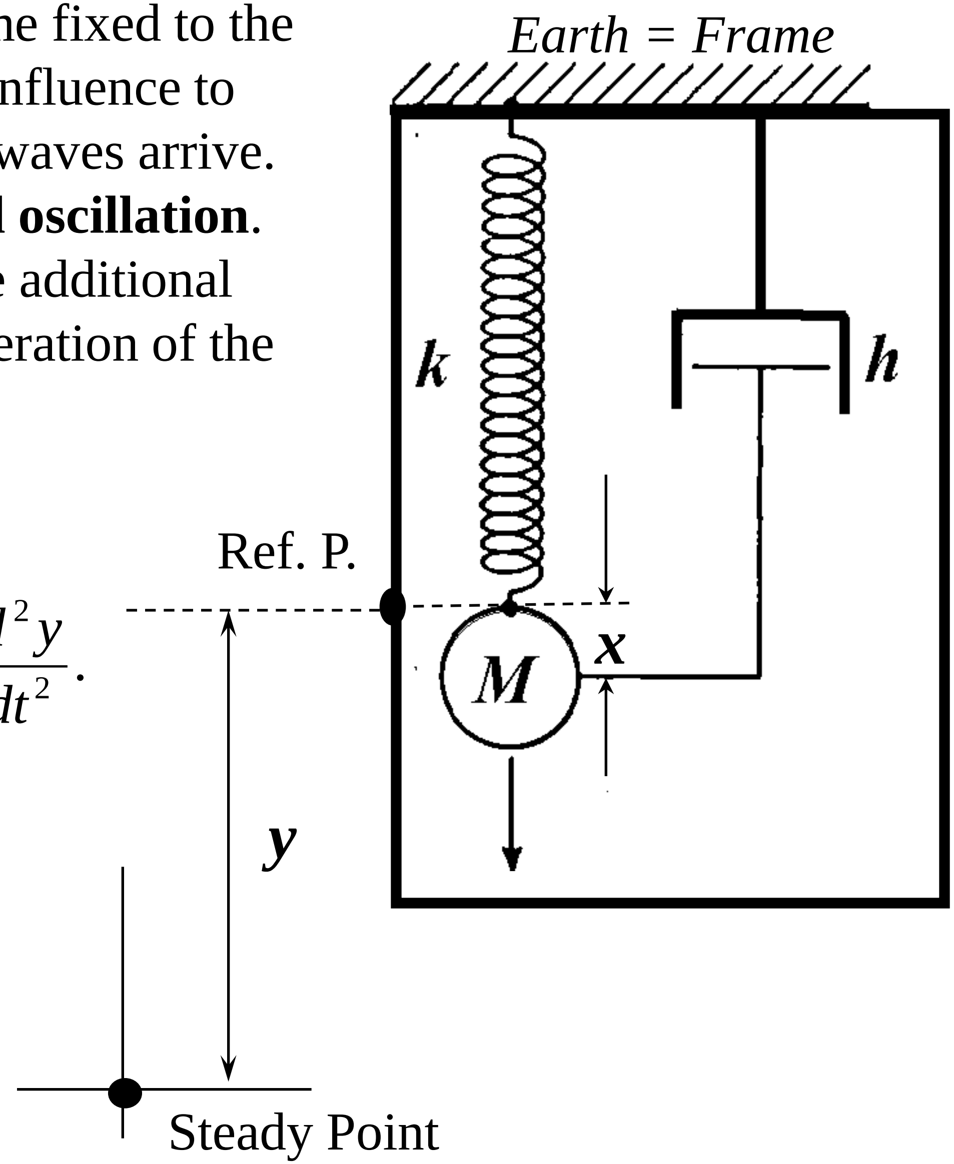
Dynamics of Pendulum

Forced Oscillation of a Damped Pendulum

Everything including the frame fixed to the ground moves and gives the influence to the pendulum, when seismic waves arrive. This situation is of the **forced oscillation**. Such influence is given as the additional inertial force due to the acceleration of the ground or the frame $-M\frac{d^2y}{dt^2}$

The equation of motion is

$$\frac{d^2x}{dt^2}+2h\omega_0\frac{dx}{dt}+\omega_0{}^2x=-\frac{d^2y}{dt^2}.$$



Steady Point and Response

If the mass of the pendulum can keep its position stable even in the vibrating circumstance, it would be possible to know the ground motion by measuring the relative motion of pendulum with respect to the frame.
Once upon a time, such steady or approximately steady pendulum mass had played very important role in the design and principle of seismometers.
Today, we think that the relation between the ground motion as the input and the mass motion relative to the frame as the output is important in the seismometry instead of such **steady point**.

Such relation is called the **response**. In the frequency domain, it is easy to obtain the response by calculating
the ratio of the relative mass motion to the ground displacement.

Output that we can measure. Input that we want to measure.

Dynamics of Pendulum

Forced Oscillation of a Damped Pendulum
Response in the Frequency Domain

To know the rough feature of the response at each frequency, we assume that the ground displacement, y, is a sinusoidal function with angular frequency, ω, ($y=y_m exp(i\omega t)$). The relative displacement of the mass, x, may also be a simple oscillation ($x=x_m exp(i\omega t)$). Then, the equation of motion gives

$$-\omega^2 x_m + 2ih\omega_0\omega x_m + \omega_0{}^2 x_m = \omega^2 y_m \quad .$$

ω_0 is the fixed value for each seismometer

Then,

$$-x_m \Big/ y_m = \frac{1}{1-2ih(\omega_0/\omega)-(\omega_0/\omega)^2}.$$

Or

$$\left|-x_m/y_m\right| = 1\Big/\sqrt{\left\{1-(\omega_0/\omega)^2\right\}^2 + 4h^2(\omega_0/\omega)^2},$$

Amplitude Response Characteristics

$$\arg(-x_m/y_m) = -\tan^{-1}\left(\frac{-2h(\omega_0/\omega)}{1-(\omega_0/\omega)^2}\right) + 2N\pi.$$

Phase Response Characteristics

Forced Oscillation of a Damped Pendulum

Low Frequency Range.

When the ground moves forth and back very slowly at a period much longer than the natural period of the pendulum, the mass follows exactly the movement of the ground. There is not any relative motion between the mass and the frame.

In other words, when the period of the ground vibration increases towards infinite, the amplitude or the relative movement (pendulum versus frame) tends towards zero.

Put $\left(\omega_0/\omega\right)$ is much larger than 1.

Then, $\left(\omega_0/\omega\right)^2$ is much smaller than $\left(\omega_0/\omega\right)^4$ and

$$\left|-x_m/y_m\right| = 1\Big/\sqrt{\left\{1-(\omega_0/\omega)^2\right\}^2 + 4h^2(\omega_0/\omega)^2},$$

$$\arg(-x_m/y_m) = -\tan^{-1}\left(\frac{-2h(\omega_0/\omega)}{1-(\omega_0/\omega)^2}\right) + 2N\pi.$$

$$\left|-x_m/y_m\right| = 1\Big/\sqrt{(\omega_0/\omega)^4 + 4h^2(\omega_0/\omega)^2},$$

$$\arg(-x_m/y_m) = -\tan^{-1}\left(\frac{2h(\omega_0/\omega)}{(\omega_0/\omega)^2}\right) + 2N\pi.$$

$$\left|-x_m/y_m\right| = 1\Big/\sqrt{(\omega_0/\omega)^4} = 1\big/(\omega_0/\omega)^2 = (\omega/\omega_0)^2,$$

$$\arg(-x_m/y_m) = -\tan^{-1}\left(\frac{2h}{(\omega_0/\omega)}\right) + 2N\pi.$$

$$\left|-x_m/y_m\right| \approx (\omega/\omega_0)^2,$$

$$\arg(-x_m/y_m) \approx -\tan^{-1}\left\{\frac{-2h}{-(\omega_0/\omega)}\right\} \approx -\tan^{-1}\left\{\frac{-2h}{-\infty}\right\} \approx \pi.$$

Namely,

$$(-x_m/y_m) \approx (i\omega/\omega_0)^2 \quad at \quad \omega \ll \omega_0 .$$

$$x_m \approx (i\omega)^2(-y_m)(1/\omega_0)^2 .$$

Dynamics of Pendulum

Forced Oscillation of a Damped Pendulum

$$x_m \approx (i\omega)^2(-y_m)(1/\omega_0)^2 . \quad x(t) \approx -\frac{1}{\omega_0{}^2} \cdot \frac{d^2}{dt^2} y(t).$$

This means that the response tends to zero as the frequency becomes smaller and is proportional to the ground acceleration.

Also this shows that the sensitivity of the pendulum to acceleration at the low frequency is proportional to the square of its natural period because $T_0=2\pi/\omega_0$.

A long natural period was achieved by a very small value of *k*, i.e., a very soft spring. Unfortunately, such soft and sensitive spring suffers a considerable influence of <u>temperature variation</u> and of <u>the Brownian motion of air molecule</u>. The natural period of such pendulum can not be stable. Thus, the sensitivity of the pendulum to acceleration at the low frequency neither can be stable. This makes difficult to recover the ground motion in the low frequency range.
Therefore, long period pendulum was the most important and difficult problem of instrumental seismology for many years.

Forced Oscillation of a Damped Pendulum

Intermediate frequency range.

If the period decreases, the amplitude increases and at the natural period the amplitude reaches its maximum value. This phenomenon is the **resonance** and the natural period is also called the **resonant period**.

The reciprocal of resonant period is the r**esonant frequency**. The natural angular frequency is 2π times of it.

Put $(\omega/\omega_0) \approx 1$ and $\left|1-(\omega_0/\omega)^2\right| \ll 2h(\omega_0/\omega)$ in the formula.

Then,

$$\left|-x_m/y_m\right| = 1\Big/\sqrt{\left\{1-(\omega_0/\omega)^2\right\}^2 + 4h^2(\omega_0/\omega)^2},$$

$$\arg(-x_m/y_m) = -\tan^{-1}\left(\frac{-2h(\omega_0/\omega)}{1-(\omega_0/\omega)^2}\right) + 2N\pi.$$

$$\left|-x_m/y_m\right| = 1\Big/\sqrt{4h^2(\omega_0/\omega)^2},$$

$$\arg(-x_m/y_m) = -\tan^{-1}\left(\frac{-2h(\omega_0/\omega)}{0}\right) + 2N\pi.$$

$$\left|-x_m/y_m\right| \approx \omega/2h\omega_0,$$

$$\arg(-x_m/y_m) \approx -\tan^{-1}\left\{\frac{-2h}{0}\right\} = \frac{\pi}{2}.$$

Dynamics of Pendulum

Forced Oscillation of a Damped Pendulum

$$\left(-x_m/y_m\right) \approx \left(i\omega/2h\omega_0\right) \quad at\,\omega \approx \omega_0 .$$

If h is much smaller than 1, the response has a big value near the natural frequency. This is known as the resonance.

The condition

$$\left|1-\left(\omega_0/\omega\right)^2\right| \langle\langle\ 2h\left(\omega_0/\omega\right)$$

limits the value of $\left(\omega_0/\omega\right)$ as

$$\sqrt{1+h^2} - h \langle\langle \left(\omega_0/\omega\right) \langle\langle \sqrt{1+h^2} + h$$

The greater value of h gives the wider frequency range in which the relative displacement is proportional to the velocity of the ground motion with the factor *1/2h*.

Dynamics of Pendulum

Forced Oscillation of a Damped Pendulum

High frequency range.

If we further decrease the period, namely, increase the frequency of the ground motion above the natural frequency, the pendulum's mass lags behind the ground vibrations and the amplitude again decreases. At the very high frequency, namely, short period of the ground motion, the mass does not move at all.

This means that the relative movement between the mass and the frame is equal to the ground motion itself.

Put (ω/ω_0) is much greater than 1 in the formula. Then,

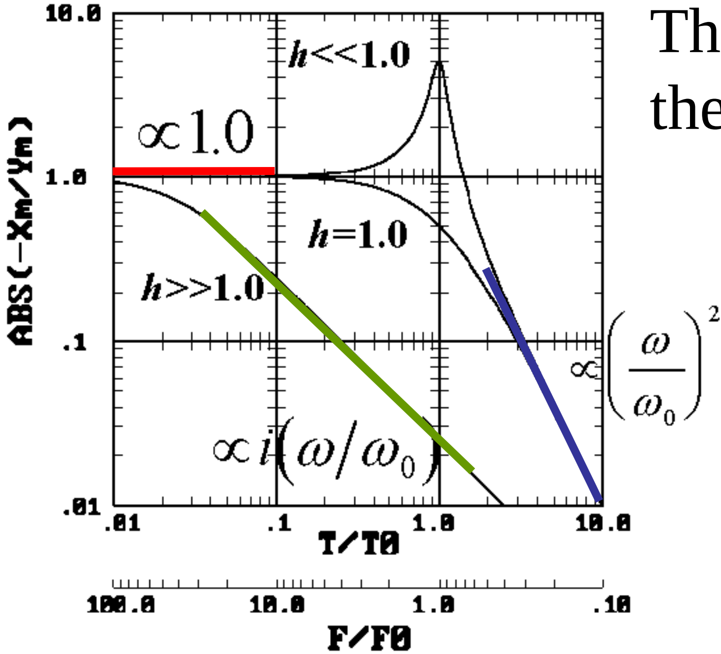
$$\left| -x_m/y_m \right| \approx 1 \text{ , } \arg\left(-x_m/y_m\right) \approx -\tan^{-1}\left\{\frac{-0}{1}\right\} = 0$$

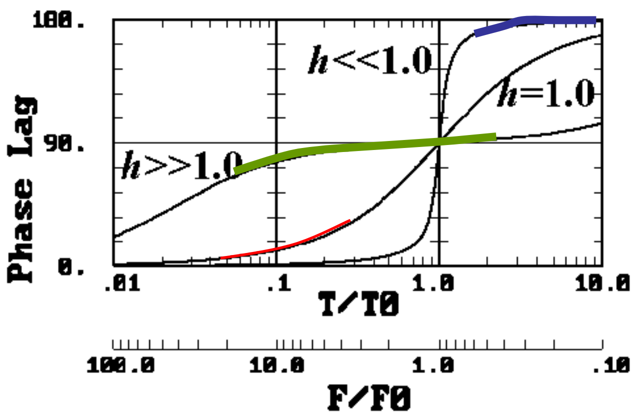
Namely,

$-x_m = y_m$.

The relative displacement is proportional to the ground displacement in this frequency range, or the mass of pendulum is steady in this frequency range.

Forced Oscillation of a Damped Pendulum



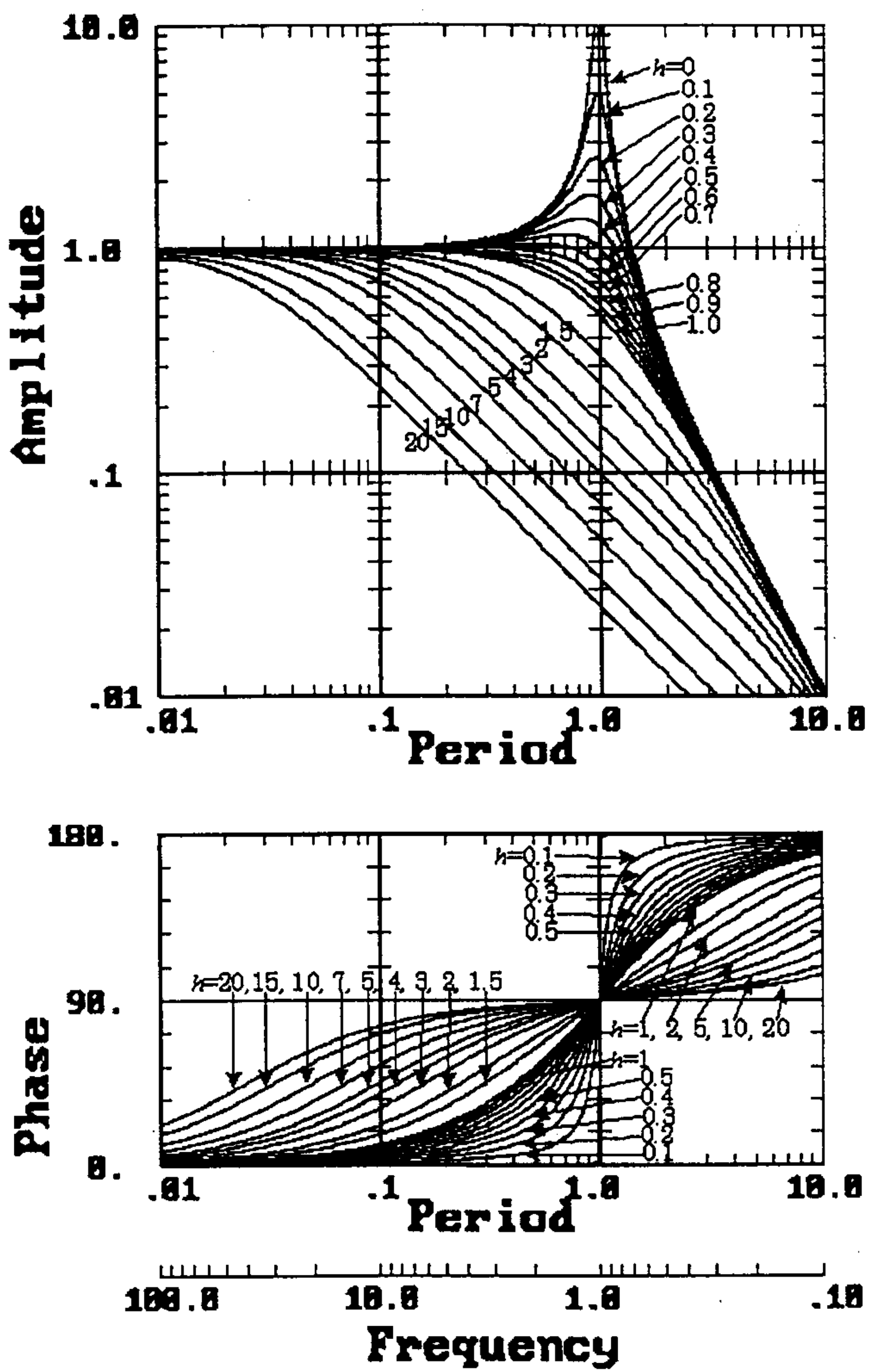


The Displacement of the mass relative to the frame (ground) is:

In Low Frequency Range.
Proportional to the ground acceleration, with the constant proportional to the squre of the natural period.

Intermediate frequency range.
Proportional to the velocity with the constant $1/2h$. Big value of h makes this frequency range wide.

In High Frequency Range.
Proportional to the ground displacement with the constant 1.



Amplitude
10.0
1.0
.1
.01
h=0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.5
2
3
4
5
7
10
15
20
.01
.1
1.0
10.0
Period
Phase
180.
90.
0.
h=0.1
0.2
0.3
0.4
0.5
h=20, 15, 10, 7, 5, 4, 3, 2, 1.5
h=1, 2, 5, 10, 20
h=1
0.5
0.4
0.3
0.2
0.1
.01
.1
1.0
10.0
Period
100.0
10.0
1.0
.10
Frequency

Summary

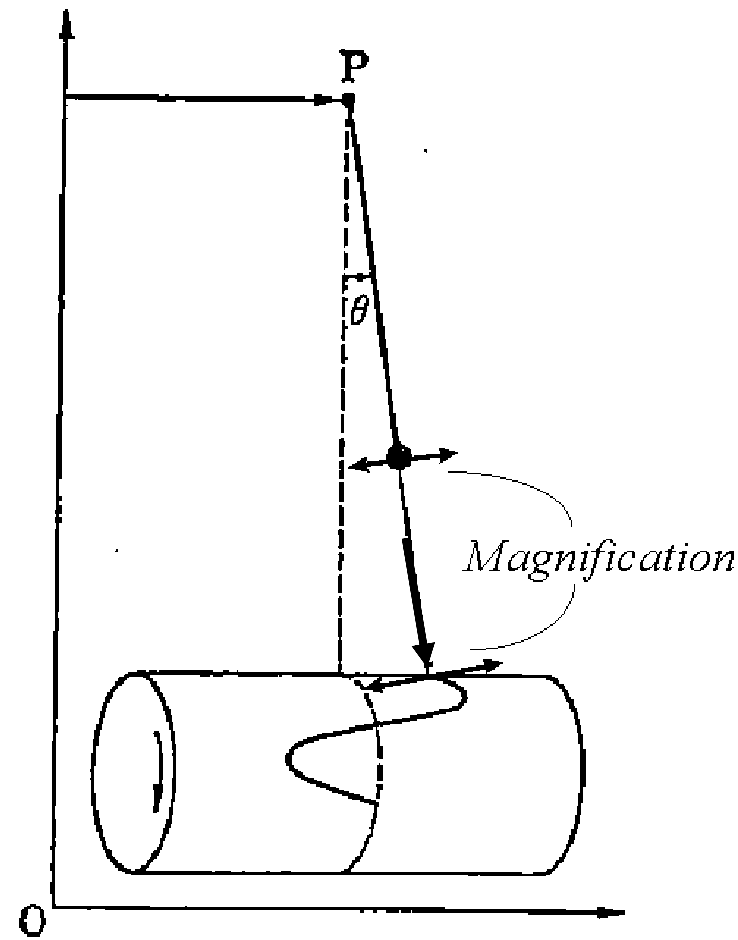
The dynamic behavior of simple pendulum described above is the core of the principle of seismometer.

+ The natural angular frequency, ω_0, and
+ the damping constant, h,

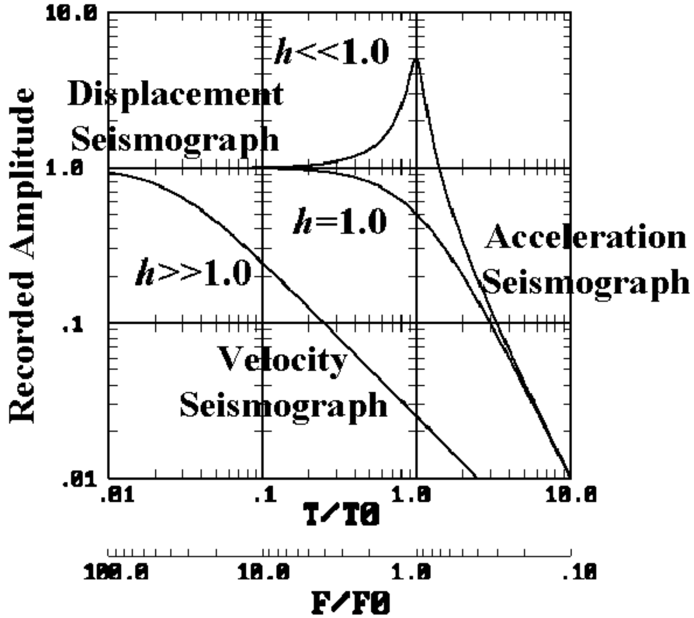
are two important parameters presented.

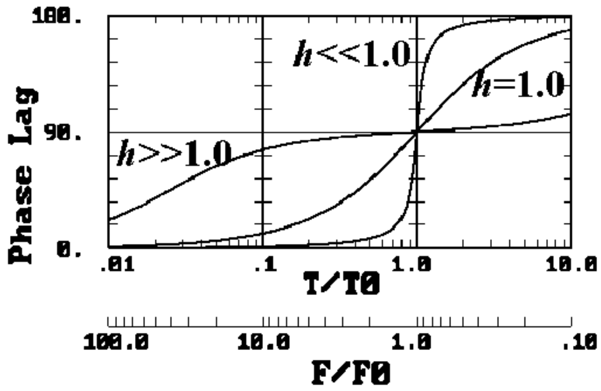
Mechanical Seismograph

In extremely simple words, the mechanical seismograph records the ground motion on the paper fixed to the frame with a pen connected to the pendulum mass. The displacement of the pen is proportional to that of the mass and the proportionality constant is called the **static magnification**, which defines the characteristics of a mechanical seismograph with the natural period and the damping constant of the pendulum.



Mechanical Seismograph





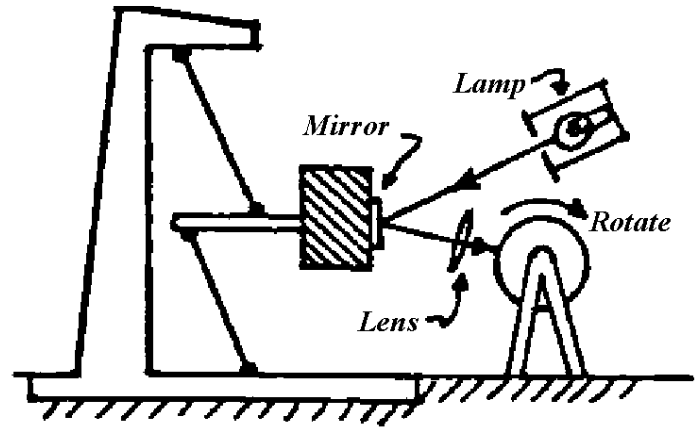
Mechanical Seismograph is arranged as

In Low Frequency Range.
Accerelation Seismograph.
For $h \leq 1.0$

Intermediate frequency range.
Velocity Seismograph.
For $h >> 1.0$
.

In High Frequency Range.
Displacement Seismograph.
For $h \approx 0.6$

Optical Seismograph

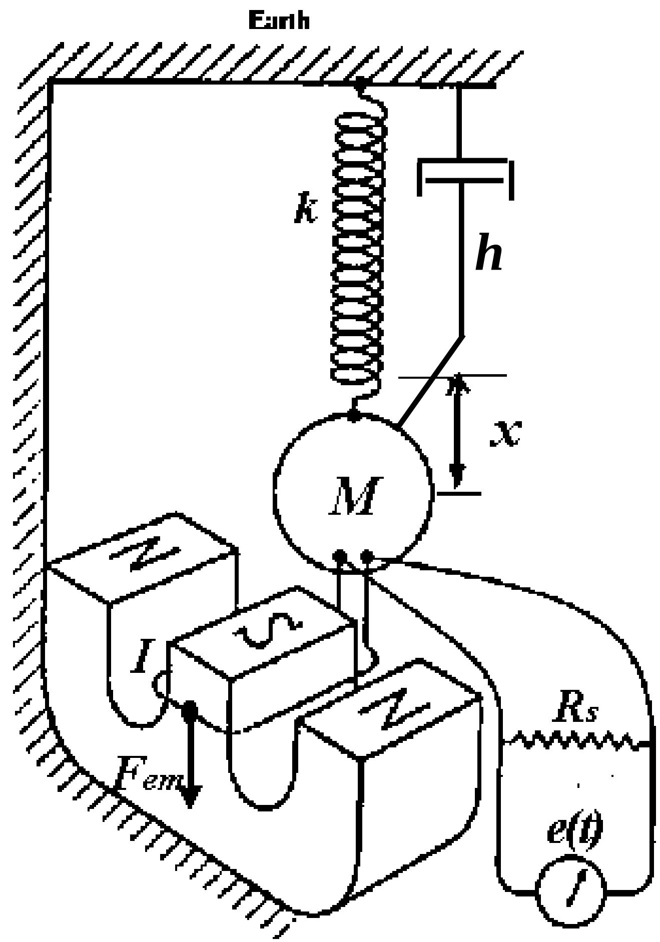


The adjustment of the damping depends on the mechanical damper which make use of the viscosity of silicon oil, air and so on. Also, a great effort had been paid to minimize the friction between the pen and recording paper, which does act as solid damper.

Optical lever which magnifies the mass motion in place of mechanical one, could give a solution for the friction of pen. This is called the **optical seismograph** or mechanical seismograph with optical recording system.

Electro-Magnetic Seismograph

The **moving coil type seismometer** is the most popular transducer for seismometry . This consists of a permanent magnet and a coil of wire of which resistance is R_0. The former is fixed to the frame and to the ground, the latter is firmly connected to the inertial mass of the pendulum. The relative motion between the two parts produces the electromotive force E_{em} in the coil.



$$E_{em} = G \cdot \frac{dx}{dt}$$

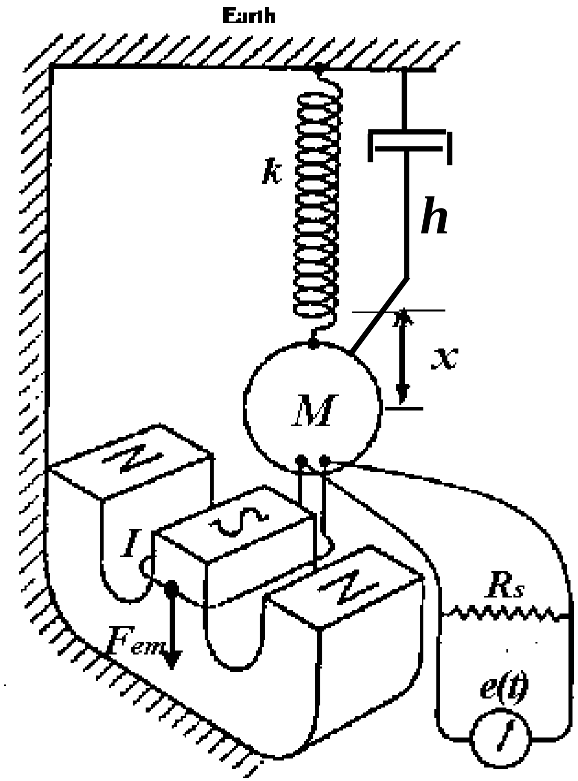
The proportionality factor, G, is called the **electro-dynamical constant**, which is given by the product of the flux density of the magnetic field and the length of the coil wire in it.

related to coil resistance

Electro-Magnetic Seismograph

When a resistance, R_s, so called the **shunt resistance**, is connected across the two ends of the coil, the electromotive force induced by the mass motion is proportional to the relative velocity of the mass. Thus, the current which flows in the coil, I, is proportional to the relative velocity of mass and inversely proportional to the sum of the shunt resistance and that of the coil itself

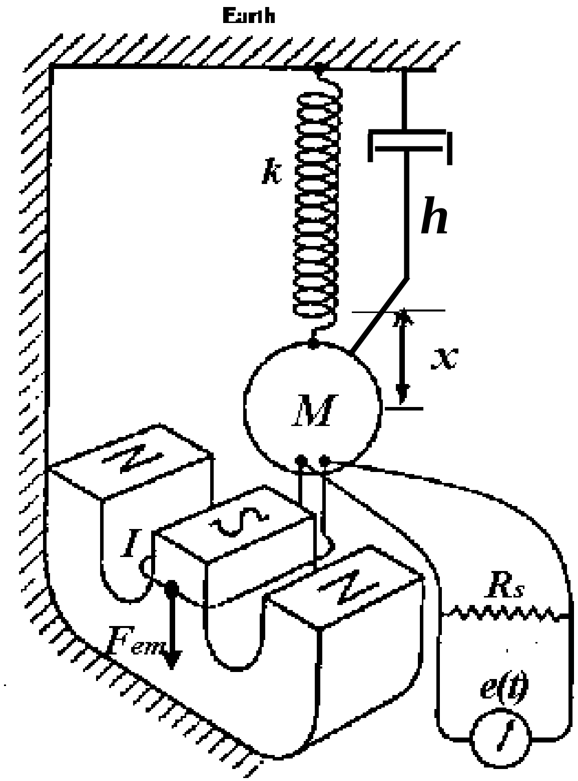
$$I = \frac{E_{em}}{R_0 + R_s} = \frac{G}{R_0 + R_s} \cdot \frac{dx}{dt}$$



Electro-Magnetic Seismograph

That current induced in the coil simultaneously produces the electromagnetic force, F_{em}, which is the electro-dynamical constant times of the current itself, *i.e.*, proportional to the mass velocity

$$F_{em} = GI = \frac{G^2}{R_0 + R_s} \cdot \frac{dx}{dt}$$



This force acts as an additional viscous damper (**electromagnetic damper**) as shown in the following.

$$\frac{d^2x}{dt^2} + 2h\omega_0 \frac{dx}{dt} + \omega_0{}^2 x = -\frac{d^2y}{dt^2} - \frac{1}{M} F_{em}.$$

Or

$$\frac{d^2x}{dt^2} + 2\omega_0\left(h + \frac{G^2}{2M\omega_0(R_0 + R_s)}\right)\frac{dx}{dt} + \omega_0{}^2 x = -\frac{d^2y}{dt^2}.$$

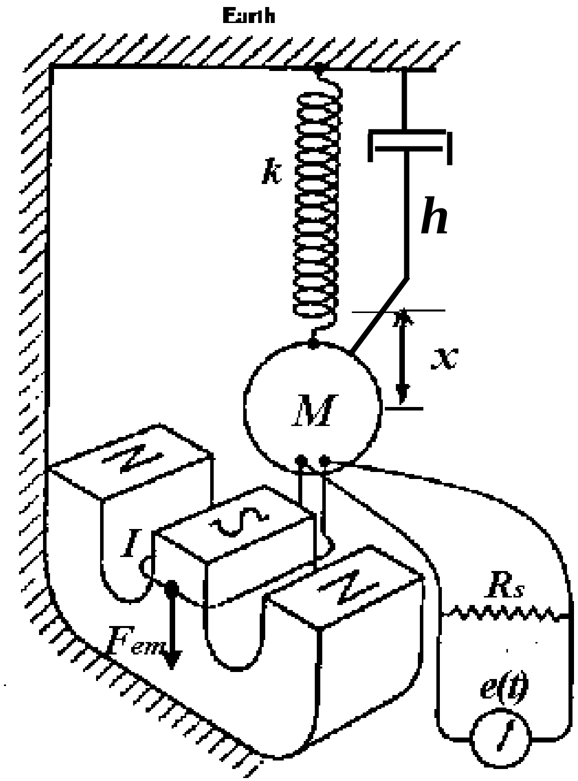
Electro-Magnetic Seismograph

The effect of F_{em} is represented by the **electromagnetic damping constant**, h_e.

$$h_e = \frac{G^2}{2M\omega_0\left(R_0 + R_s\right)}$$

Here after, we use the notation, h_m, for the damping constant in case of R_s is infinite (**mechanical damping constant**).
The total damping constant for the entire system, h, is the sum of the mechanical damping constant, h_m and the electro-magnetic damping constant, h_e.

$$h = h_m + h_e$$



Electro-Magnetic Seismograph

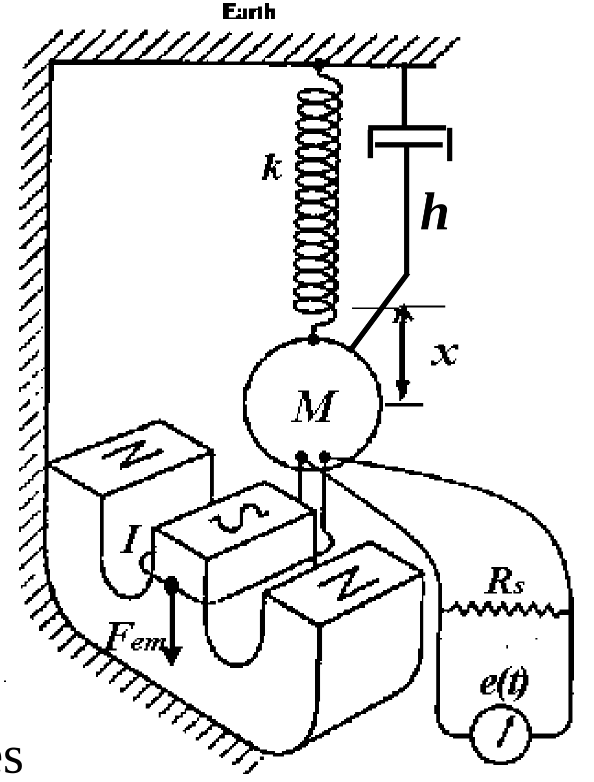
The potential difference across the shunt resistance,

$e(t)=IRs$

, i.e., the output voltage of this seismometer, is proportional to the relative velocity of the mass. This is because

$$e(t) = IR_s = \frac{GR_s}{R_0 + R_s} \cdot \frac{dx}{dt}$$

The shunt resistance controls also magnification.

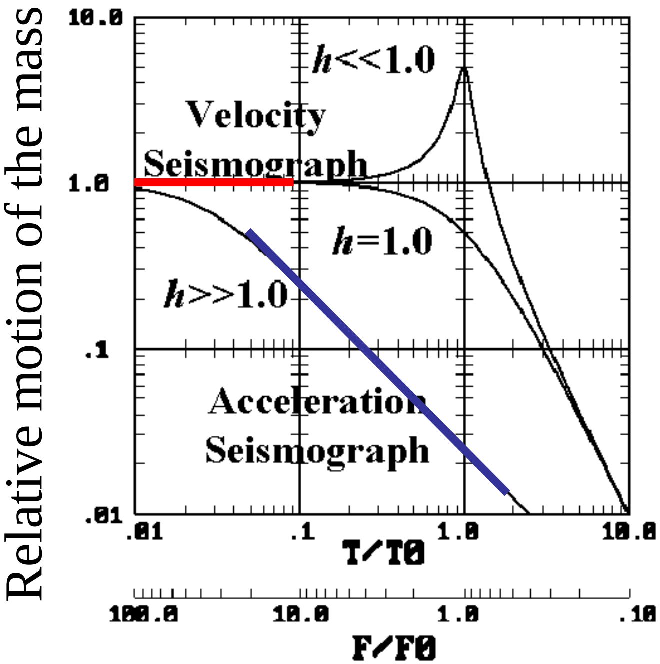


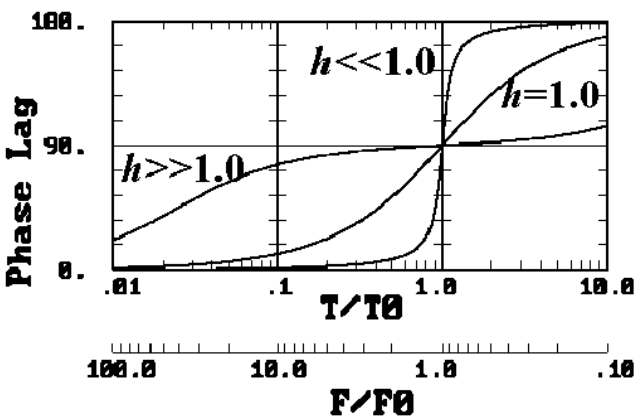
In the open circuit case, the shunt resistance becomes infinite and

$$e(t) = G\frac{dx}{dt}$$

Electro-Magnetic Seismograph

Electro-Magnetic Seismograph is arranged as





In Low Frequency Range.

Non

Intermediate frequency range.

Accerelation Seismograph.

For $h >> 1.0$

.

In High Frequency Range.

Velocity Seismograph.

For $h \approx 0.6$

Summary

The electro-dynamical constant, G,
the value of the coil resistance, R_0,
the shunt resistance, R_s, and
the electro-magnetic damping constant, h_e,
are introduced.

G plays a similar role for the moving coil type seismometer as the static magnification for mechanical ones.
R_s is the important tool to adjust the electromagnetic damping constant, h_e.
R_0 is also required to calculate this constant.

		L-4C 1.0 Hz SEISMOMETER	L-4A 2.0 Hz SEISMOMETER
	TYPE	Moving dual coil, humbuck wound	Moving dual coil, humbuck wound
$1/T_0$	FREQUENCY	1.0 ± 0.05 Hz measured on 200 pound weight at 0.09 inches/second	2.0 ± 0.25 Hz measured on 200 pound weight at 0.09 inches/second
	FREQUENCY CHANGE WITH TILT	Less than 0.05 Hz at 5° from vertical	less than 0.10 Hz at 10° from vertical
	FREQUENCY CHANGE WITH EXCITATION	Less than 0.05 Hz from 0 to 0.09 inches/second	Less than 0.10 Hz from 0 to 0.18 inches/second
M	SUSPENDED MASS	1000 grams	500 grams
R_0	STANDARD COIL RESISTANCES	500, 2000, 5500 R_c	500, 2000, 5500
	LEAKAGE TO CASE	100 megohm minimum at 500 volts	100 megohm minimum at 500 volts
	TRANSDUCTION POWER	0947 $\sqrt{Rc}$	.0947 $\sqrt{Rc}$
h_m	OPEN CIRCUIT DAMPING	(bo) = 0.28 critical	(bo) = 0.28 critical
h_c	CURRENT DAMPING	$(bc) = \frac{1.1\ Rc}{Rs + Rc}$	$(bc) = \frac{1.1\ Rc}{Rs + Rc}$
	COIL INDUCTANCE	Lc = 0.0011 Rc Lc in henries	Lc = 0.0011 Rc Lc in henries
	CASE TO COIL MOTION	PP 0.250 inches	PP 0.250 inches
	ELECTRIC ANALOG OF CAPACITY	$Cc = \frac{73{,}500}{Rc}$ (microfarads)	$Cc = \frac{36{,}500}{Rc}$ (microfarads)
	ELECTRIC ANALOG OF INDUCTANCE	Lm = 0.345Rc (henries)	Lm = 0.17Rc (henries)
	CASE HEIGHT	5⅛ inches—13 cm.	5⅛ inches—13 cm.
	CASE DIAMETER	3 inches—7.6 cm.	3 inches—7.6 cm.
	TOTAL DENSITY	3.7 grams/cm^3	2.9 grams/cm^3
	TOTAL WEIGHT	4¾ pounds—2.15 kilograms	3¾ pounds—1.7 kilograms
	OPERATING TEMPERATURE	Range: −20° to 140°F or −29° to 60°C	Range: −20° to 140°F or −29° to 60°C

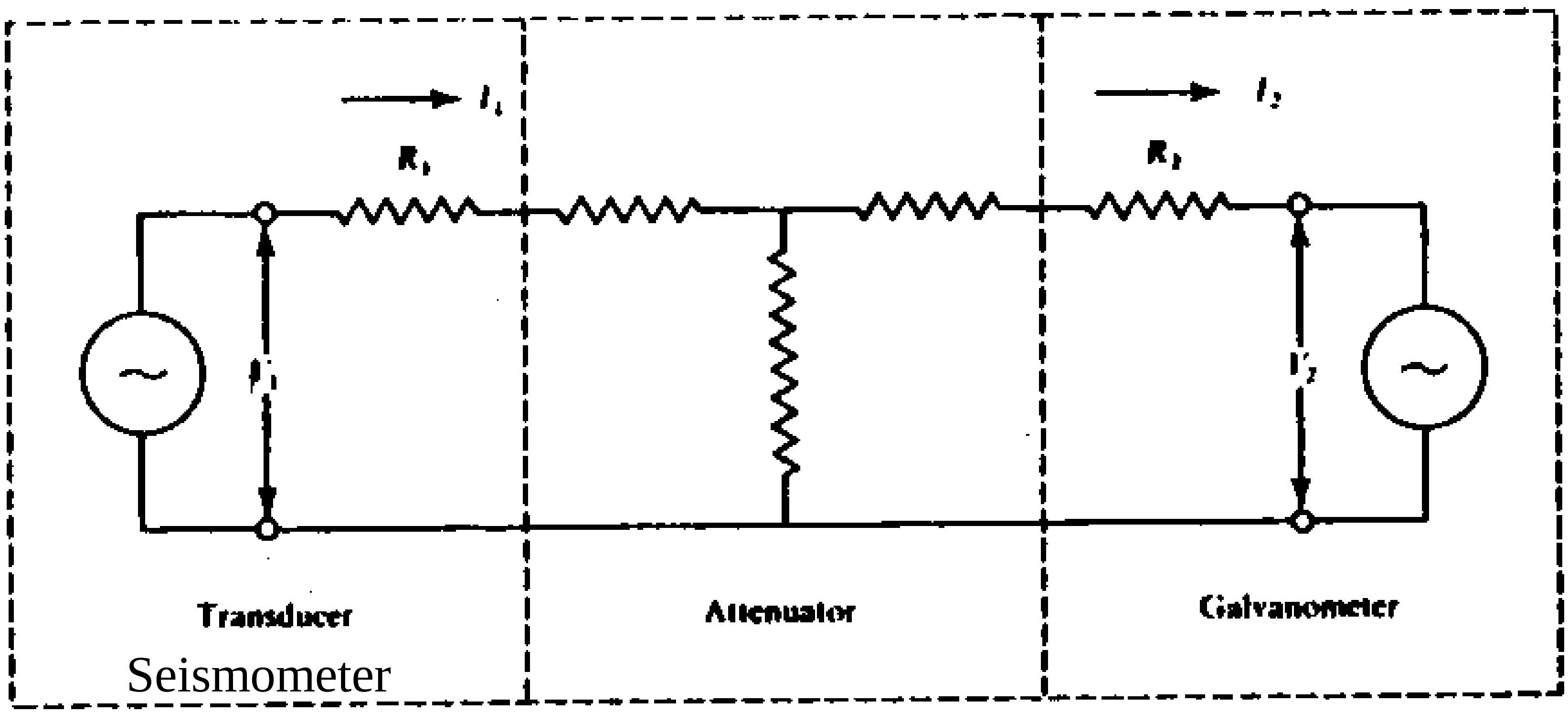
		L-4C 1.0 Hz SEISMOMETER			L-4A 2.0 Hz SEISMOMETER		
R_0	**COIL RESISTANCE, OHMS**	**500**	**2000**	**5500**	**500**	**2000**	**5500**
G	TRANSDUCTION, VOLTS/IN/SEC	2.12	4.23	7.02	2.12	4.23	7.02
	COIL INDUCTANCE, HENRIES	0.55	2.20	6.05	0.55	2.20	6.05
	ANALOG CAPACITANCE, MICROFARADS	147	36.8	13.4	73.0	18.3	6.64
	ANALOG INDUCTANCE, HENRIES	173	690	1900	85.0	340	935
R_s	SHUNT FOR 0.70 DAMPING, OHM	810	3238	8905	810	3238	8905

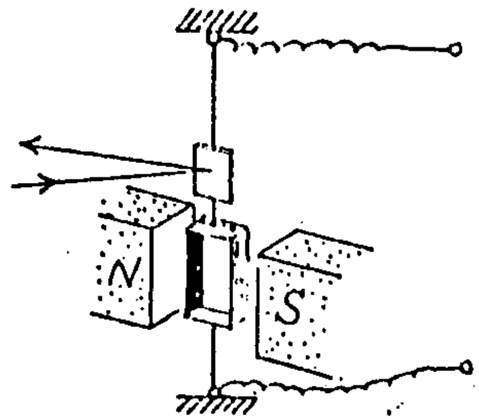
Open Circuit Damping $(b_o) = 0.28$ Critical   Coil Current Damping $(b_c) = \frac{1.1\ Rc}{Rc + Rs}$ Total Damping $(b_t) = b_o + b_c$

Electro-Magnetic Seismograph

Annex-1

Once upon a time, moving coil type seismometer connected directly or via an **attenuater circuit** to the **galvanometer** had been widely used in seismometry. For such seismometer-galvanometer systems, the relative movement between the mass and the ground becomes more complicated because galvanometers itself also have oscillatory characteristics of their own.

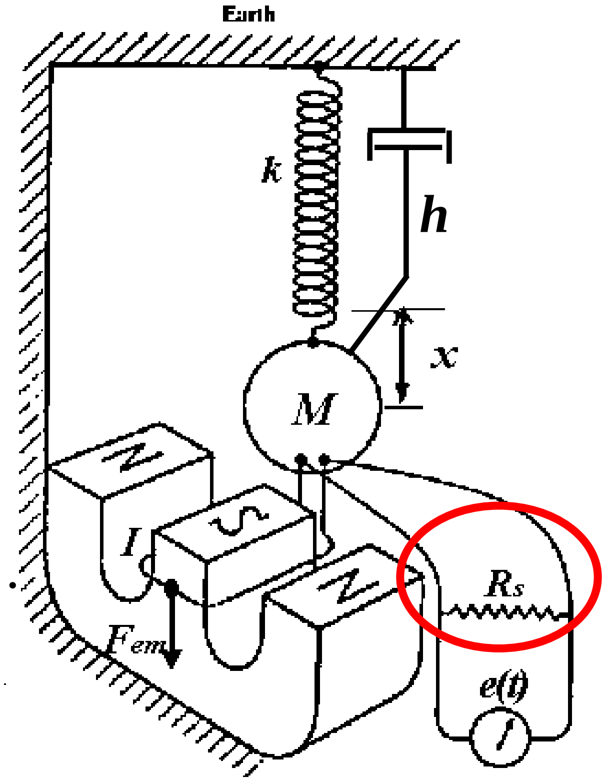




A seismometer-galvanometer circuit operates as a feedback system so that not only the seismometer but also the galvanometer gives influence to the entire appearance of the seismogram. Seismologists in old days arranged the frequency characteristics of their seismographs nearly optimum for many particular purposes by a proper choice of the combination of the seismometer's constants and the galvanometer's ones. Today, we can achieve easily the optimum frequency characteristics by designing electric filter circuit or, if necessary, by applying digital filter to recorded seismic signals. The explanation for such seismographs is separated from the main description (See Appendix-2)

Feed Back Seismometer

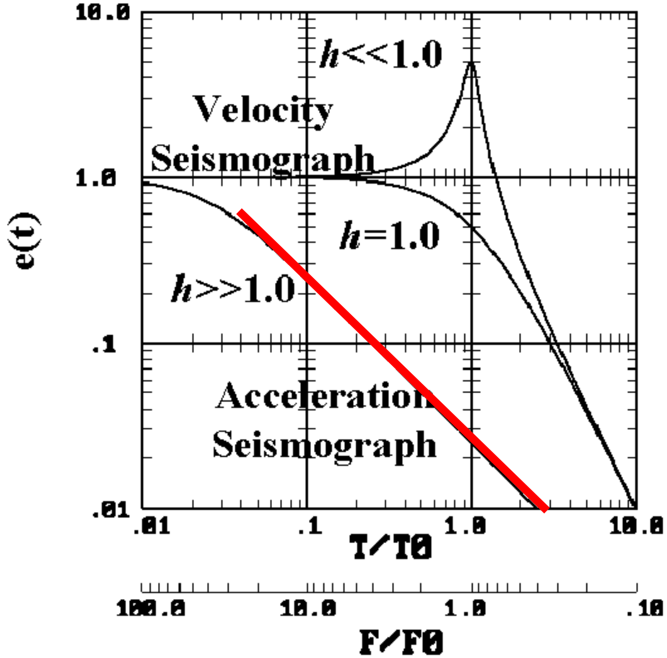
The simplest feed back circuit is **Shunt Resistance**. Feed back force is produced by the transducer's coil itself. This force is proportional to the velocity of the pendulum mass and gives additional damping force.

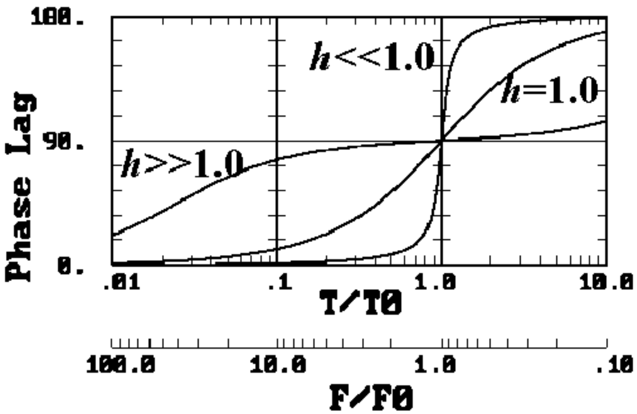


$$\frac{d^2x}{dt^2} + 2\omega_0\left(h + \underbrace{\boxed{\frac{G^2}{2M\omega_0\left(R_0 + R_s\right)}}}_{h_e}\right)\frac{dx}{dt} + \omega_0{}^2 x = -\frac{d^2y}{dt^2}.$$

$$e(t) = IR_s = \frac{GR_s}{R_0 + R_s}\cdot\frac{dx}{dt}$$

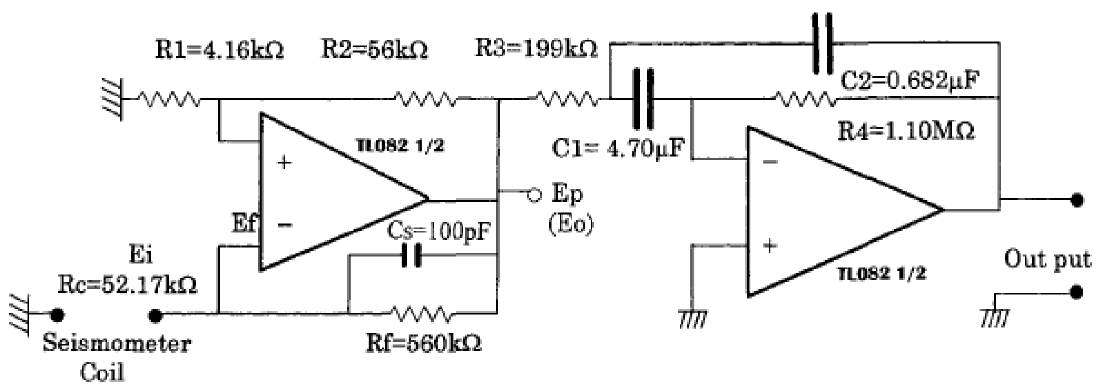
The maximum value of the damping factor is given by the minimum value of R_s. A big value can not be obtained by only one resistor.
Note $R_s=0$ gives $h_e=G^2/2M\omega_0R_0$ that can become just 3~10.
A possible way to make h_e bigger is to use the **negative shunt resistance**. Namely, to attach a electronic circuit that make the voltage difference to the terminals of seismometer with **the opposite sense** compared with that caused by the relative motion of pendulum.





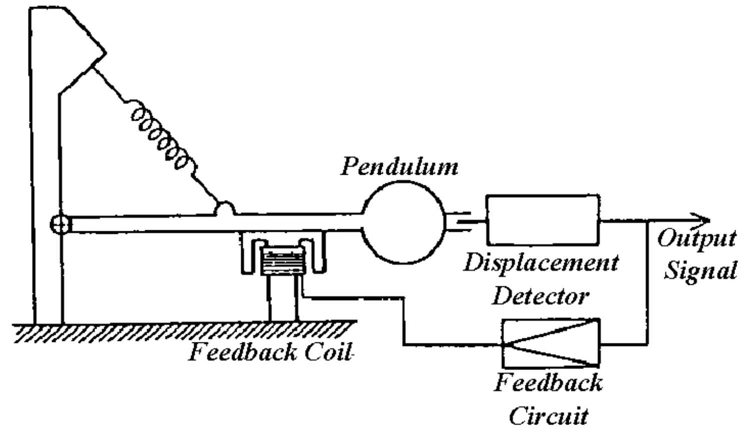
Example:Moriya et al. 1998:
A specially designed electronic circuit can make this additional damping factor as big as 80 or more. Then, the output voltage imbalance becomes proportional to the ground acceleration over a wide frequency range from DC. MTV1C (f_0=1.0Hz) obtained new natural frequemcy (f_0=0.19Hz) by the circuit shown below.

The same principle is used by LE3D/5s manufactured by Lennartz Inc. The seismometer actually L22D of Mark Product (f_0=2.0Hz) is used and obtained the new natural frequency (f_0=0.19Hz).



A circuit diagram to obtain a negative resistance and secondary natural frequency.

Feed Back Seismometer

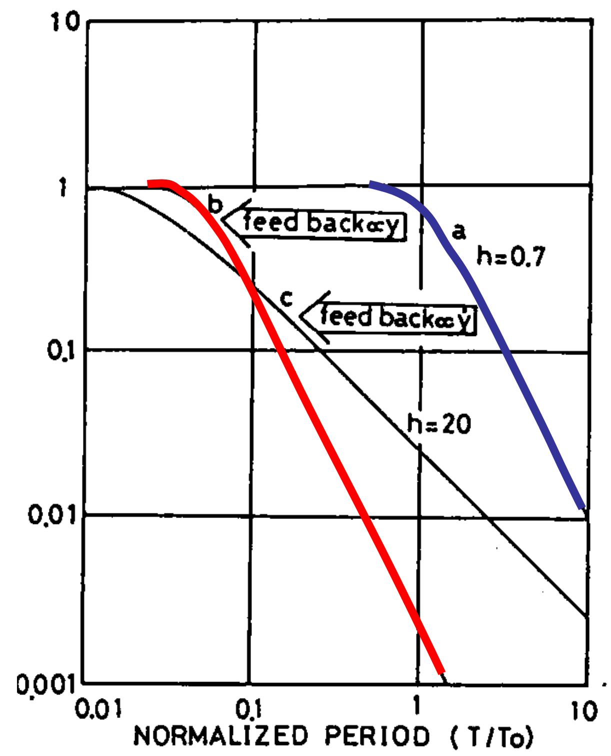


The feedback seismometer has two transducers. One is the sensor to detect the mass movement, another is the **feedback coil** to give the negative **feedback force**, , to the pendulum. The output of the former detector is divided in two. One directly becomes the output signal of the seismometer, another is sent to the feedback circuit. This amplifies, arranges and sends the feedback signal to the feedback coil which gives the feedback force to the pendulum. The signal is arranged for that this feedback force is proportional to displacement, velocity or acceleration of the pendulum mass depending on the purpose.

$$\frac{d^2x}{dt^2} + 2h\omega_0 \frac{dx}{dt} + \omega_0{}^2 x = -\frac{d^2y}{dt^2} - f(x) .$$

Feed Back Seismometer

If the feedback force is proportional to the mass displacement ($f(x) = k_d^2 x$), this force acts as additional restoring force. The equation of motion is



$$\frac{d^2x}{dt^2} + 2h\omega_0 \frac{dx}{dt} + (\omega_0{}^2 + k_d^2 / M)x = -\frac{d^2y}{dt^2}$$

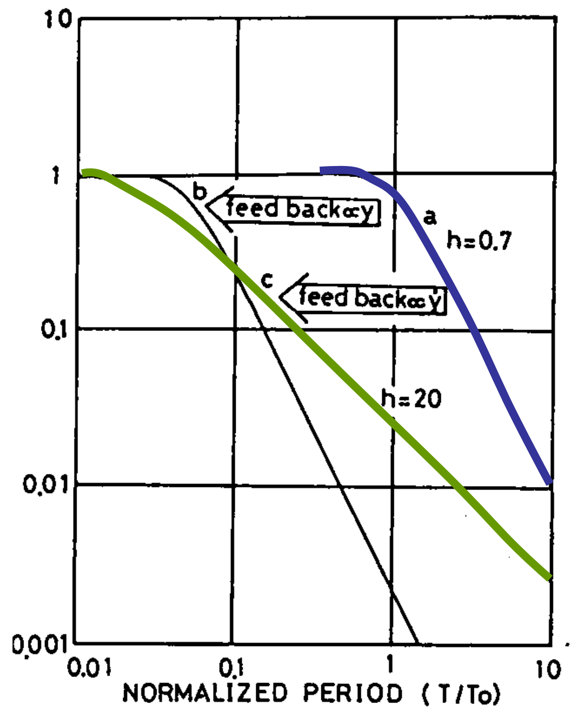
This means that the natural period of the pendulum apparently becomes shorter. The displacement of the mass is proportional to the ground acceleration in the frequency range lower than the modified resonant frequency (curve b). Since the natural period is much shorten, the response flat to acceleration is performed in a wide band from DC to the shorten natural period. Thus, this seismometer theoretically does not have longer period side limit for observation, although the practical limit is given by ambient noise or system noise. This type is called also the **force balanced type**. Most of strong motion seismographs commonly in use belong to this type.

Feed Back Seismometer

If the feedback force is proportional to the mass velocity ($f(x) = k_v^2 \frac{dx}{dt}$), this force acts as an additional viscous damping.

$$\frac{d^2x}{dt^2} + (2h\omega_0 + k_v^2 / M)\frac{dx}{dt} + \omega_0{}^2 x = -\frac{d^2y}{dt^2} \ .$$

This means that the damping constant apparently becomes big. Thus, the displacement of the mass is proportional to the ground velocity in a wide frequency range (curve c).

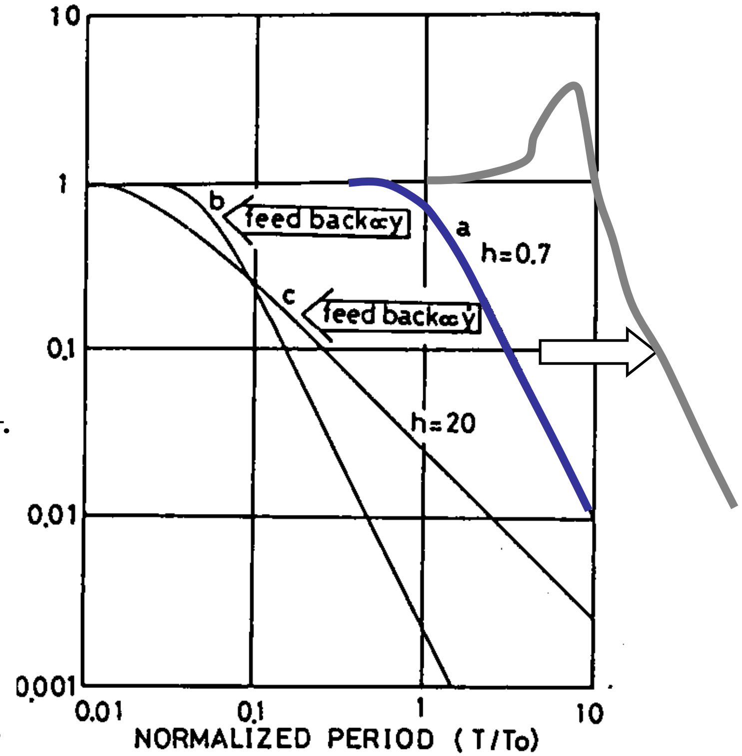


Feed Back Seismometer

If the feedback force is proportional to the ground acceleration ($f(x) = k_a^2 \dfrac{d^2x}{dt^2}$), this force acts as an additional inertial force.

$$\left(1+k_a^2/M\right)\frac{d^2x}{dt^2}+2h\omega_0\frac{dx}{dt}+\omega_0{}^2x=-\frac{d^2y}{dt^2}.$$

$$\frac{d^2x}{dt^2}+2\frac{h}{\left(1+k_a^2/M\right)}\omega_0\frac{dx}{dt}+\frac{\omega_0{}^2}{\left(1+k_a^2/M\right)}x=-\frac{1}{\left(1+k_a^2/M\right)}\frac{d^2y}{dt^2}.$$



This means that the natural period apparently becomes longer. Choosing an appropriate damping, the displacement of the mass is proportional to the ground displacement in the range higher than the modified natural frequency.

Feed Back Seismometer

Benefits of Force Balanced type seismometer:

The instrumentation for seismology has required highly sensitive at long period and stable seismometer for many years. The equation of motion

$$M\frac{d^2x}{dt^2} = -kx \quad or \quad \frac{d^2x}{dt^2} = -\omega_0{}^2 x,\ where\ \omega_0{}^2 = k/M$$

suggests that bigger *M*, the heavier mass, and smaller *k*, which means softer spring can compose longer period pendulum. A huge mass suspended by a very soft spring seems to be a contradictive requirements. Moreover, pendulum consisting of very soft spring is unstable or so sensitive as to be affected by the temperature variation and by the Brownian motion of air molecules.

Feedback type seismometer gives a solution for this difficulties. For example,

$$\frac{d^2x}{dt^2} + 2h\omega_0\frac{dx}{dt} + (\omega_0{}^2 + k_d^2 / M)x = -\frac{d^2y}{dt^2}$$

means that the natural frequency of a displacement feedback seismometer is determined by .

$$2\pi \Big/ \sqrt{\omega_0^2 + k_d^2 / M}$$

Feed Back Seismometer

If k_d^2 is sufficiently bigger than $\omega_0^2 M$, the apparent natural period is controlled almost only by k_d^2. As k_d^2 is determined only by the feedback circuit, its characteristics alone control the apparent natural period. Today, the highly developed electronics can guarantee the stability for the characteristics of the circuit and therefore for the natural period and the sensitivity of such seismometer in long period range. Once the sensitivity is stabilized sufficiently, it is very easy to convert the output voltage variation to displacement, velocity and acceleration of the ground motion by data processing or by other electric circuits.

Feedback system of force balanced type can stabilize the sensitivity of seismometer in the frequency range lower than the modified natural period. Therefore it makes possible to observe ground motion from modified natural frequency to such long period accerelation as DC.

A BROADBAND SEISMOMETER IS ACHIEVED.

Feed Back Seismometer

The negative feedback force makes the amplitude of the mass motion small. Thus, the linearity of the seismometer is bettered and a **wide dynamic range** is achieved. Moreover, a huge pendulum mass is not necessary. Smaller mass is more controllable as shown by *M* at the denominator.

The feed back seismometer makes possible to achieve small and very sensitive seismometers which covers a wide dynamic range and a broad frequency band. New types of strong motion seismographs and of long period seismographs employ the feed back system.

It seems that this type will dominate over the seismic observation in near future.

This type is getting dominate over the seismic observation today.

Pole-Zero Representation

Example: Moving Coil Type Seismometer

The equation of motion for pendulum's displacement of a seismometer relative to the ground *x(t)* induced by the ground motion *y(t)* is given by

$$\frac{d^2x}{dt^2} + 2h\omega_0 \frac{dx}{dt} + \omega_0{}^2 x = -\frac{d^2y}{dt^2},$$

where ω_0 denotes the natural frequency of the pendulum, h the damping factor. The response in the frequency domain or the transfer function is given by

$$T(i\omega) = -x_m \Big/ y_m = \frac{1}{1 - 2ih(\omega_0/\omega) - (\omega_0/\omega)^2} = \frac{(i\omega)^2}{(i\omega)^2 + 2h\omega_0(i\omega) + \omega_0^2}.$$

The substitution of $i\omega$ with s gives the transfer function in s-domain.

$$T(i\omega) = \frac{(i\omega)^2}{(i\omega)^2 + 2h\omega_0(i\omega) + \omega_0^2}.$$

$$\Downarrow$$

$$T(s) = \frac{s^2}{s^2 + 2h\omega_0 s + \omega_0^2}.$$

Example: Moving Coil Type Seismometer

Transfer function in s-domain is

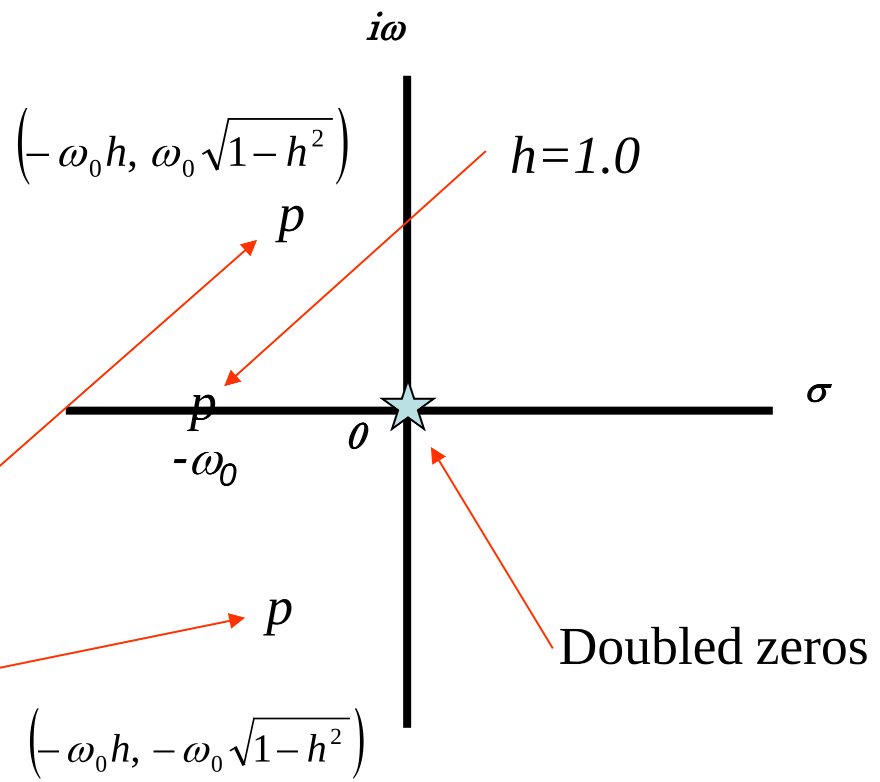
$$T(s) = \frac{s^2}{s^2 + 2h\omega_0 s + \omega_0^2}.$$

Zeros: $s^2 = 0,$

$$zeros = (0.,0.),\ \ (0.,0.).$$

Poles: $s^2 + 2h\omega_0 s + \omega_0^2 = 0,$

$$poles = \omega_0\left(-h \pm \sqrt{h^2 - 1}\right)$$



H<1.0

Example: Filter equivalent to a Simple Moving Coil Type Seismometer-2

In previous example, the recursive filter that gives the relative displacement of pendulum -x_m for ground displacement y_m is given. Usually, the data obtained by digital recorder are given in Digit and a constant is given for conversion to Volts. The potential difference, the output from seismometer is given as follows.

$$e_m = \frac{G_0 R_s}{R_0 + R_s}(i\omega)x_m,$$

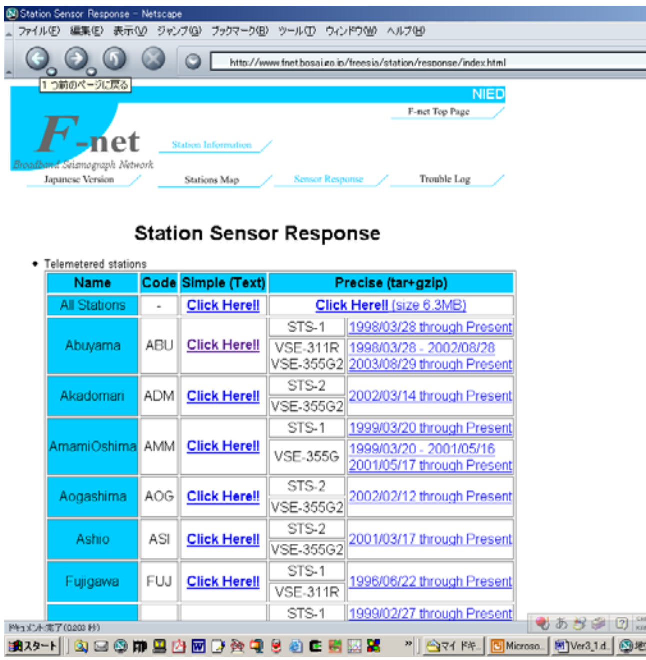
where ($i\omega$) shows the effect of differentiation due to moving coil type transducer, R_0 the coil resistance, R_s the shunt resistance, G_0 the product of the sensitivity of seismometer to the conversion constant of digital recorder. Therefore, the system response is

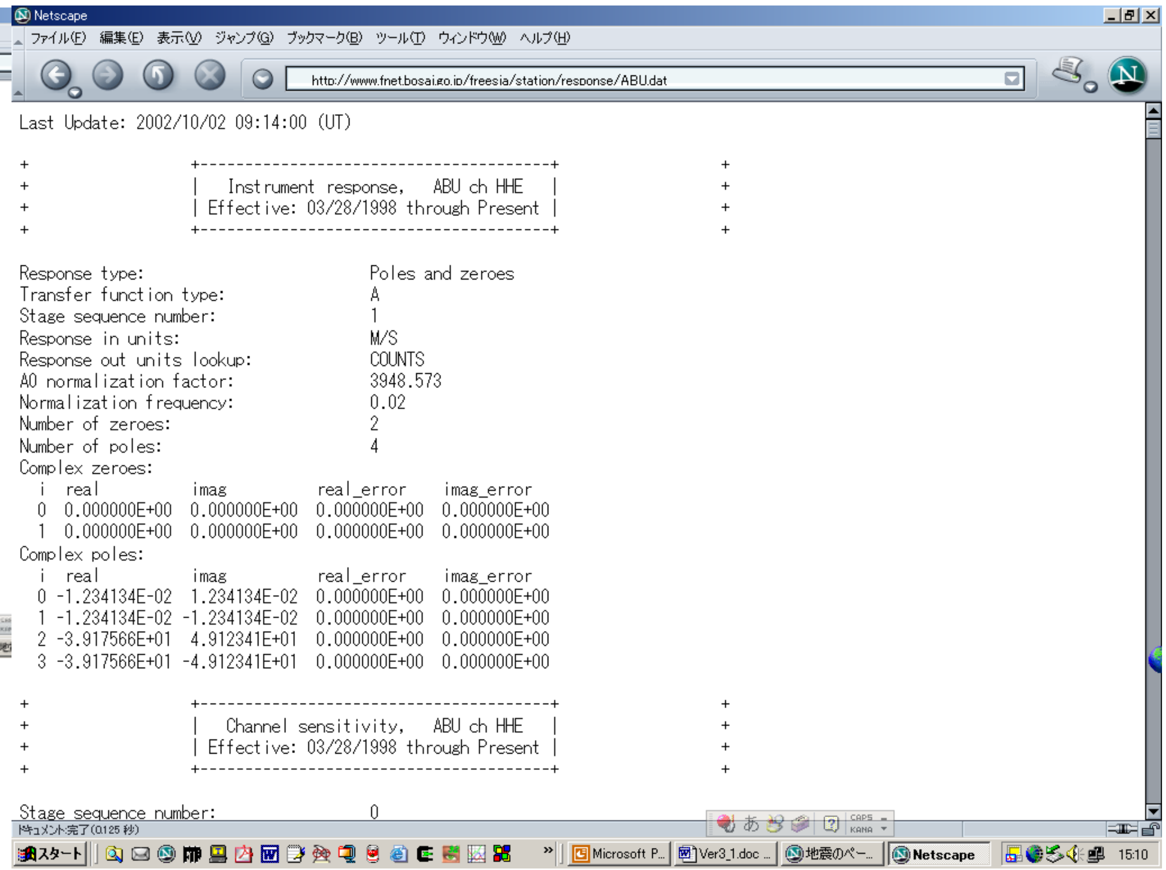
$$e_m / y_m = \frac{G_0 R_s}{R_0 + R_s} \cdot \frac{(i\omega)}{1 - 2ih(\omega_0/\omega) - (\omega_0/\omega)^2}.$$

This has an equivalent digital filter. The corresponding transfer function in the s-domain is

$$T(s) = G \cdot \frac{s^3}{s^2 + 2h\omega_0 s + \omega_0^2}, \; G = \frac{G_0 R_s}{R_0 + R_s}. \qquad (33.1)$$

Today, many seismic observation organizations open their data in public via Internet for that any researcher can make use of them. Some of these organizations provide the information of instrumental characteristics with Pole-Zero representation. Then, it may be useful to show the way to reconstruct the transfer function in the frequency domain from given value of poles and zeros.





Example: F-NET, NIED

Exercise: Reconstruction of transfer function from given poles and zeros.

Suppose that the following data are given for an observation system. Actually, these are of STS-2 feedback type seismometer.

Normalization factor: A_0=5.42787E+07
Normalization frequency: 0.02 (Hz)
Complex **zeroes(Rad/sec)**:

i	real part	imaginary part	Index
0	0.000000E+00	0.000000E+00	X_0
1	0.000000E+00	0.000000E+00	X_1

Complex **poles(Rad/sec)**:

i	real part	imaginary part	Index
0	-1.247510E+02	-4.171480E+02	x_0
1	-1.247510E+02	4.171480E+02	x_1
2	-4.873870E-02	-1.552120E-02	x_2
3	-4.873870E-02	1.552120E-02	x_3
4	-2.513300E+02	0.000000E+00	x_4

Sensitivity: G_0=6.291456E+08 (digit/(M/Sec))
Frequency of sensitivity: 0.02 (HZ)
The normalized amplitude spectra of the transfer function are given as follows

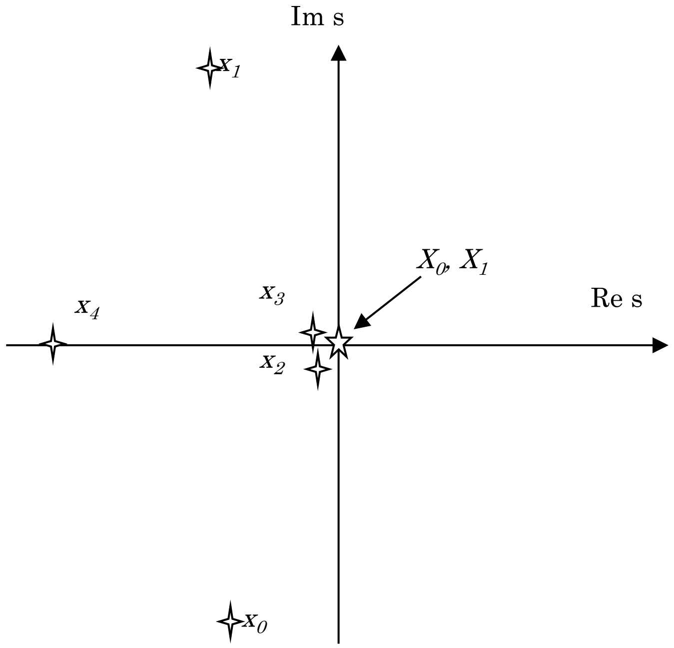


Fig. 45.2 Configuration of poles and zeros for the example.

The normalized amplitude spectra of the transfer function are given as follows.

$$\left|\tilde{T}(\omega)\right| = \frac{|s - X_0| \cdot |s - X_1|}{\prod_{m=0}^{4} |s - x_m|} = \frac{\omega^2}{\prod_{m=0}^{4} \sqrt{(\mathrm{Re}(x_m))^2 + (\omega - \mathrm{Im}(x_m))^2}}$$

Normalizing factor A_0 is the reciprocal of this value at f=0.02 (Hz). $A_0 \left|\tilde{T}(\omega)\right| = 1.0$ at f=0.02 Hz. The reconstructed transfer function is given as follows.

$$|T(\omega)| = G_0 A_0 \left|\tilde{T}(s)\right| = \frac{G_0 A_0 \omega^2}{\prod_{m=0}^{4} \sqrt{(\mathrm{Re}(x_m))^2 + (\omega - \mathrm{Im}(x_m))^2}}$$

For phase spectra,

$$Arg(T(\omega)) = \sum_{l=0}^{1} \frac{\pi}{2} - \sum_{m=0}^{4} \left\{ \tan^{-1}\left(\frac{\omega - \mathrm{Im}(x_m)}{0 - \mathrm{Re}(x_m)} \right) \right\}$$

The first term corresponds to two zeros at the origin. Calculation of these spectra can be achieved even with handy calculator.

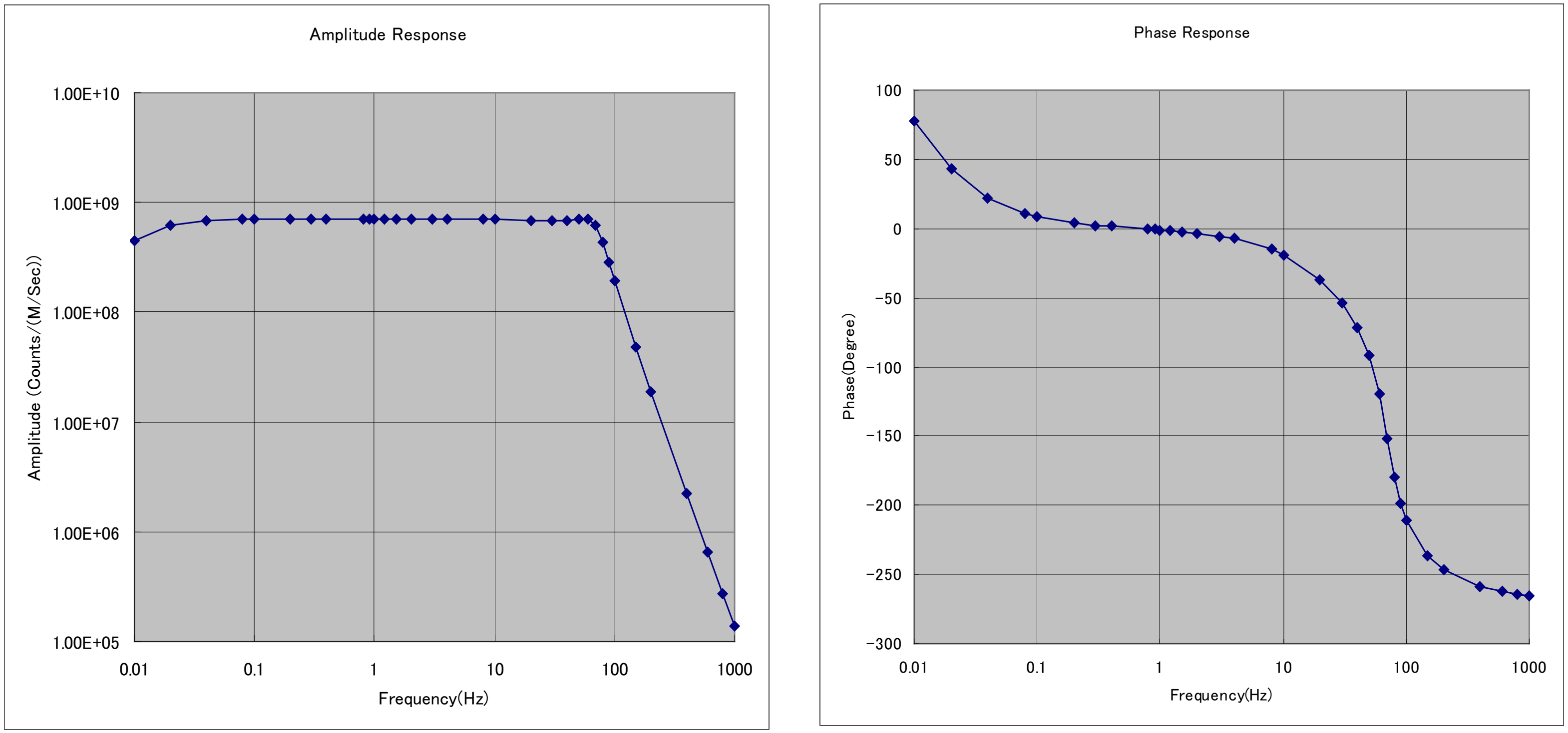


Fig. 45.2a Amplitude (Top) and Phase (Bottom) response of STS-2 feed back type seismometer. These are calculated by the formulae with poles and zeros data given in the page 53. Calculation has been done easily by MicroSoft Excel.

Exercise: CMG40T

Normalizing Factor Ao= -0.314

Normalizing Frequency 1.0(Hz)

Complex Zeros(1/sec)

i	Real Part	Imaginary Part
0	0.0E+00	0.00E+00
1	0.0E+00	0.00E+00
2	1.59E+02	0.00E+00

Poles & zeros divided by 2π

Complex Poles(1/sec)

i	Real Part	Imaginary Part
0	-2.356E-2	2.356E-2
1	-2.356E-2	-2.356E-2
2	-5.00E+01	0.00E+00

Sensitivity Go=804 (V/m/s)

$$|T(\omega)| = G_0 \left| A_0 \tilde{T}(s) \right| = G_0 |A_0| \frac{\prod_{l=0}^{2} \sqrt{(\mathrm{Re}(x_l))^2 + (f - \mathrm{Im}(x_l))^2}}{\prod_{m=0}^{2} \sqrt{(\mathrm{Re}(x_m))^2 + (f - \mathrm{Im}(x_m))^2}}$$

$$Arg(T(\omega)) = \sum_{l=0}^{2} \left\{ \tan^{-1}\left(\frac{f - \mathrm{Im}(x_l)}{0 - \mathrm{Re}(x_l)} \right) \right\} - \sum_{m=0}^{2} \left\{ \tan^{-1}\left(\frac{f - \mathrm{Im}(x_m)}{0 - \mathrm{Re}(x_m)} \right) \right\} + Arg(A_o)$$

Exercise: CMG40T

Normalizing Factor Ao= -0.314

Normalizing Frequency 1.0(Hz)

Complex Zeros(rad/sec)

i	Real Part	Imaginary Part
0	0.0E+00	0.00E+00
1	0.0E+00	0.00E+00
2	9.99E+02	0.00E+00

Complex Poles(rad/sec)

i	Real Part	Imaginary Part
0	-1.480E-1	1.480E-1
1	-1.480E-1	-1.480E-1
2	-3.14E+02	0.00E+00

Sensitivity Go=804 (V/m/s)

$$\left|T(\omega)\right| = G_0\left|A_0\tilde{T}(s)\right| = G_0\left|A_0\right|\frac{\prod_{l=0}^{2}\sqrt{(\mathrm{Re}(x_l))^2+(\omega-\mathrm{Im}(x_l))^2}}{\prod_{m=0}^{2}\sqrt{(\mathrm{Re}(x_m))^2+(\omega-\mathrm{Im}(x_m))^2}}$$

$$Arg(T(\omega)) = \sum_{l=0}^{2}\left\{\tan^{-1}\left(\frac{\omega-\mathrm{Im}(x_l)}{0-\mathrm{Re}(x_l)}\right)\right\} - \sum_{m=0}^{2}\left\{\tan^{-1}\left(\frac{\omega-\mathrm{Im}(x_m)}{0-\mathrm{Re}(x_m)}\right)\right\} + Arg(A_o)$$

3.4.5. Analogue Filters and Their Transfer Function.

There are several electronic filters that are commonly used for seismometry. Their transfer functions are introduced in the following. It is matter of course that each circuit has its equivalent digital filter.

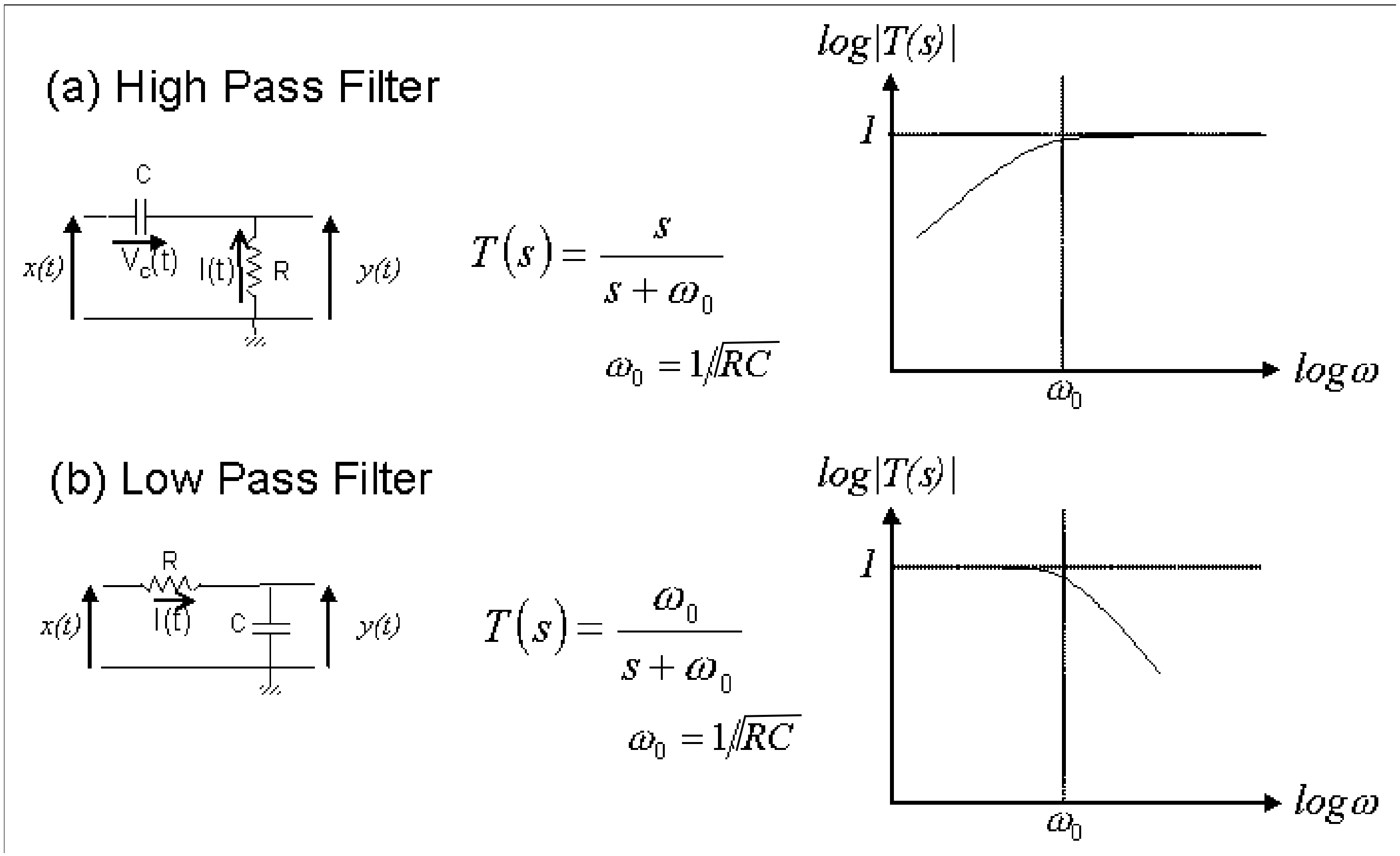


Fig.46 *Top* RC high pass filter. Its circuit, transfer function and the response characteristics. *Bottom* RC low pass filter.

(2) **Active filter**.

Today, many electronic circuits with known characteristics are widely used for filtering. Some of those characteristics have their proper names. The requirements for the filters may be flat amplitude characteristics in the pass band, sharp cut off, flat delay characteristics in the pass band. The last one means linear phase characteristics, because delay is defined as phase differentiated by the angular frequency.

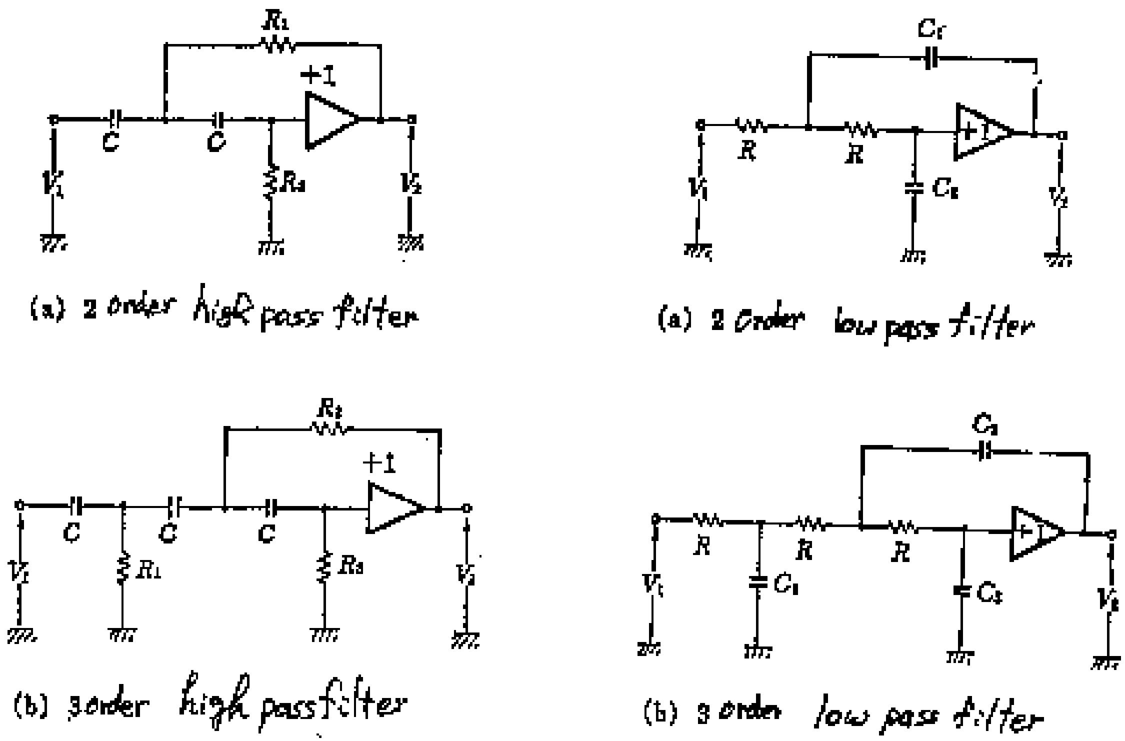


Fig.47 Schematic circuit for active filter composed of OP amp. and corresponding transfer functions.

Butterworth filter has its amplitude characteristics

$$\left|T(s)\right| = \frac{1}{\sqrt{1+\left(\omega/\omega_c\right)^{2n}}},$$

where ω_c is the cut off angular frequency. *n* denotes the order of the filter and the slope of amplitude characteristics in the stop frequency band. Namely, *n*-order filter has the decay *20n* db/oct.. The transfer function itself is given for low pass filter

$$T(s) = \Pi \frac{\Omega_0^2}{\left(s/\omega_c\right)^2 + \left(\Omega_0/Q\right)\left(s/\omega_c\right) + \Omega_0^2}, \ for\ even\, n,$$

$$T(s) = \frac{\Omega_a}{\left(s/\omega_c\right) + \Omega_a} \Pi \frac{\Omega_0^2}{\left(s/\omega_c\right)^2 + \left(\Omega_0/Q\right)\left(s/\omega_c\right) + \Omega_0^2}, \ for\ odd\ n,$$

and for high pass filter

$$T(s) = \Pi \frac{\Omega_0^2}{\left(\omega_c/s\right)^2 + \left(\Omega_0/Q\right)\left(\omega_c/s\right) + \Omega_0^2}, \ for\ even\, n,$$

$$T(s) = \frac{\Omega_a}{\left(\omega_c/s\right) + \Omega_a} \Pi \frac{\Omega_0^2}{\left(\omega_c/s\right)^2 + \left(\Omega_0/Q\right)\left(\omega_c/s\right) + \Omega_0^2}, \ for\ odd\ n,$$

where ω_c is the cut off angular frequency and other coefficients are given in the following table.

Table 10 Coefficients for *Butterworth* filter

n	Q	Ω_0	Ω_a	$C1$	$C2$	$C3$
2	0.707107	1.000000	---	1.4142	0.7071	---
3	1.000000	1.000000	1.000000	1.3926	3.5468	0.2025
4	0.541196	1.000000	---	1.0824	0.9239	---
	1.306536	1.000000	---	2.6131	0.3827	---
5	0.618034	1.000000	1.000000	1.3541	1.7529	0.4213
	1.618034	1.000000	---	3.2361	0.3090	---

The value of C_n ($n=1, 2, 3$) in Table 10 corresponds to the actual value of capacitor C_a ($a=1, 2, 3$) in Fig.47 (*right*) but normalized one, i.e., $C_a=C_n/\omega_c R$, where R is the resistance selected in advance. For high pass filter (Fig.47 *left*), R_n ($n=1, 2, 3$) are calculated by $1/C_n$ ($n=1, 2, 3$) in Table 10. The value of the actual resistor R_a ($a=1, 2, 3$) in Fig.47 (*left*) is given by $R_a=R_n/\omega_c C$, where C is the value of the capacitance selected in advance.

Butterworth filter has plane amplitude characteristics in the pass frequency band and is used for shaping of the spectra. The delay characteristics of this filter, however, are not flat even in the pass band. These have a peak of delay around the cut off frequency. This means that the shape of the signal around the cut off frequency in the time domain is considerably distorted by filtering operation.

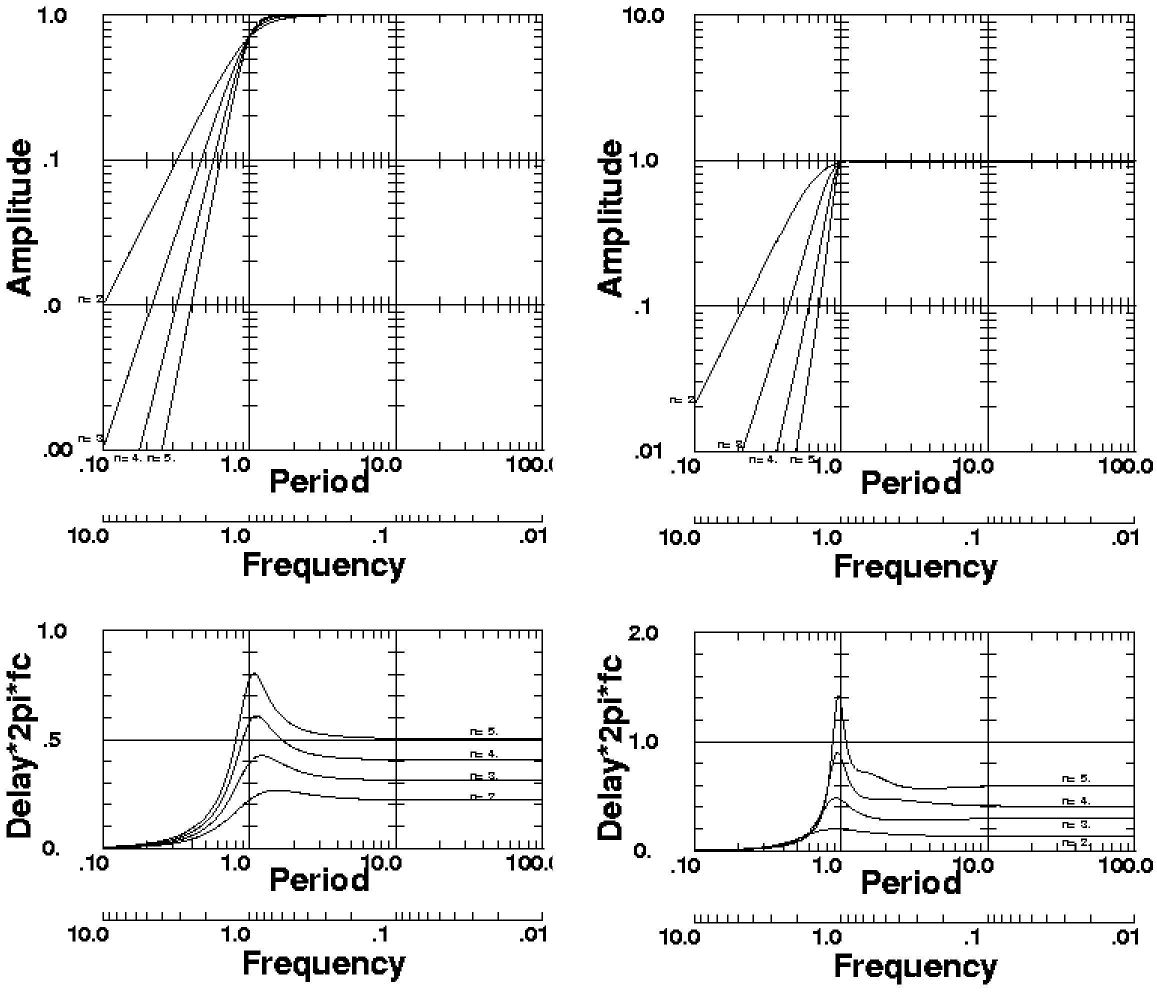


Fig.48 Amplitude and delay of (a) Butterworth filter and (b) Chevyshev filter in the frequency domain. Both are plotted for the case of low pass filter

Chevyshev filter : If we allow the ripple in the pass band, we can perform a sharp cut off. *Chevyshev* filter belongs to this category. The transfer function is given for low pass filter

$$T(s)=\frac{1}{\sqrt{10^{Rp/10}}}\Pi\frac{\Omega_0^2}{(s/\omega_c)^2+(\Omega_0/Q)(s/\omega_c)+\Omega_0^2},\ for\ even\ n,$$

$$T(s)=\frac{\Omega_a}{(s/\omega_c)+\Omega_a}\Pi\frac{\Omega_0^2}{(s/\omega_c)^2+(\Omega_0/Q)(s/\omega_c)+\Omega_0^2},\ for\ odd\ n,$$

and for high pass filter

$$T(s)=\frac{1}{\sqrt{10^{Rp/10}}}\Pi\frac{\Omega_0^2}{(\omega_c/s)^2+(\Omega_0/Q)(\omega_c/s)+\Omega_0^2},\ for\ even\ n,$$

$$T(s)=\frac{\Omega_a}{(\omega_c/s)+\Omega_a}\Pi\frac{\Omega_0^2}{(\omega_c/s)^2+(\Omega_0/Q)(\omega_c/s)+\Omega_0^2},\ for\ odd\ n,$$

The coefficients are given in the following table.

Table 11 Coefficients for Chevyshev filter with ripple of 0.25db in the pass band

n	*Q*	$\Omega 0$	Ωa	*C1*	*C2*	*C3*
2	0.809254	1.453972	---	1.1132	0.4249	---
3	1.508026	1.156992	0.767223	1.6110	6.8272	0.0885
4	0.657249	0.674422	---	1.9491	1.1280	---
	2.536110	1.077939	---	4.7055	0.1829	---
5	1.035932	0.732405	0.436951	2.6625	5.0919	0.3147
	3.875683	1.046630	---	7.4060	0.1233	---

The delay characteristics of this filter are not flat in the pass band. Then, the filtering operation causes a considerable distortion of waveform in the time domain.

Bessel filter: can perform flat delay characteristics in the pass band. However, the cut off is not sharp. The amplitude characteristics are also flat in the pass band. The transfer function is given by the same formulas as those of **Butterworth** filter. The coefficients are given in the following table.

Table 12 Coefficients for Bessel filter

n	*Q*	$\Omega 0$	Ωa	*C1*	*C2*	*C3*
2	0.577350	1.732051	---	0.6667	0.5000	---
3	0.691047	2.541547	2.322165	0.5647	0.8136	0.1451
4	0.521935	3.023265	---	0.3453	0.3169	---
	0.805538	3.389366	---	0.4753	0.1831	---
5	0.563536	3.777894	3.646739	0.3601	0.4171	0.1280
	0.916479	4.261031	---	0.4302	0.1280	---

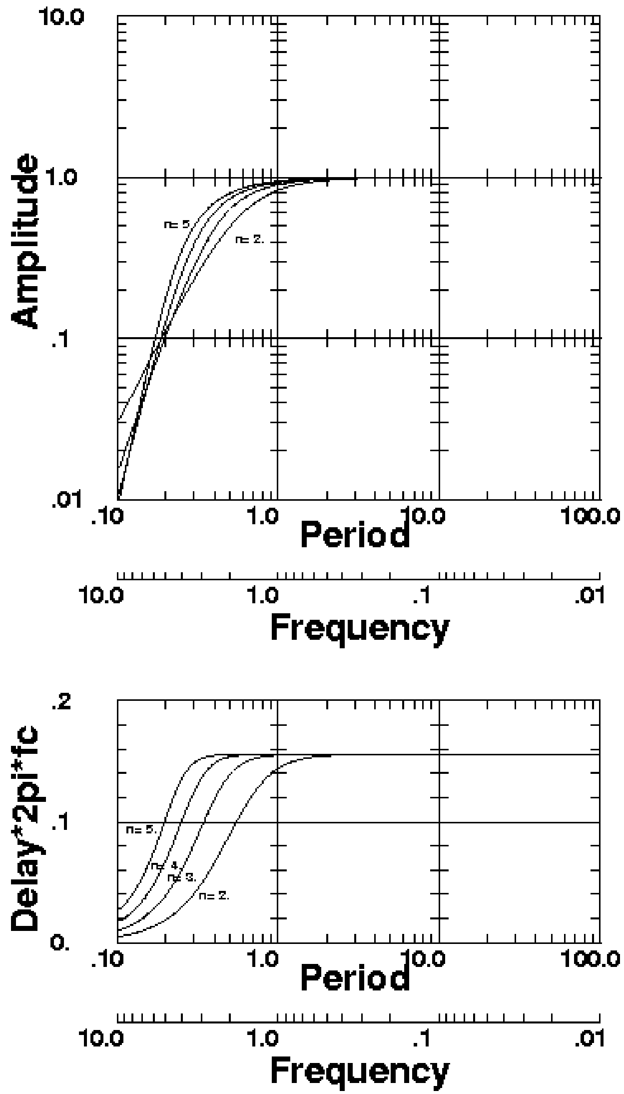


Fig. 48 (continue) (c) Amplitude and delay of the Bessel filter in the frequency domain plotted for low pass filtered case.

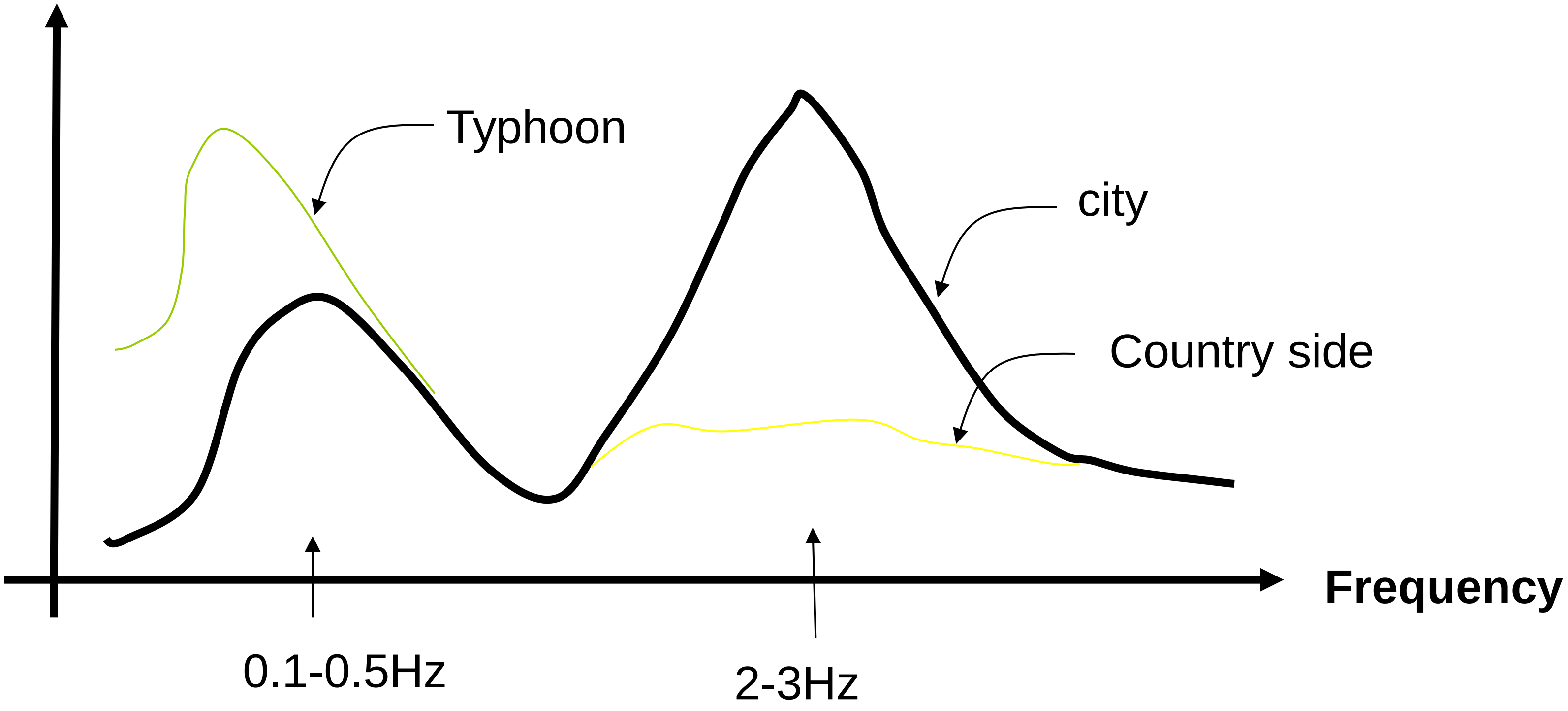
Back Ground Noise

Microtremor: Background Noise of Ground Motion

Short period microtremor caused by human activities such as traffic and industry. This reflects the shallow underground structure of sediment.

Long period microtremor caused by oceanic waves. This reflects the deep sedimentary structure.

Spectra of Ground Noise



Peterson’s Noise Model

U.S. DEPARTMENT OF INTERIOR GEOLOGICAL SURVEY

OBSERVATIONS AND MODELING OF SEISMIC BACKGROUND NOISE
Jon Peterson
Open-File Report 93-322

1st step: Select the “good” stations in the world that belong to IRIS, IDA, DWWSSN, CDSN, SRO, TERRA, RSTN.

2nd step: Calculate Station Noise Spectra (Power Spectral Density)

How much of the lower envelope of the overlay in Figure 2 is a true representation of Earth background noise, and how much of it is shaped by instrument noise?

3rd step: Define high and low noise envelope

For Selection of Seismometer, the followings have to be taken into account.

- Dynamic Range
  - Clipping Level
  - Noise Level
- Frequency (Period) Coverage
- Price

Seismometers listed in IASPEI NMSOP Volume II)

CMG-3T (Guralp System Ltd.)
http://www.guralp.net/products/weak/#3T

Episensor ES-T (Kinemetrics/OYO)
http://www.kinemetrics.com/product_Content.asp?newsid=111

Le-3d (Lennartz Electronic)
http://www.lennartz-electronic.de/Pages/Seismology/Seismometers/Seismometers.html

PMD
http://www.eentec.com/

S-13 (Geotech Instruments LLC)
http://www.geoinstr.com/s-13.htm

Trillium (Nanometrics Inc.)
http://www.nanometrics.ca/products/trillium/trillium_1_new.htm

L4-3D (Mark Products)
http://www.geoinstruments.com.au/main.htm

STS-1 VBB & STS-2 (Streckeisen AG)

Relation of Digital world to Analogue world

Z-transform: $$z = e^{i\omega_k \Delta t}.$$

3.4.3. Z-transform

Discrete Fourier Transform

$$X_k = TC_k = \Delta t \sum_{m=1}^{N} x_m e^{-im\omega_k \Delta t},$$

$$x_m = \frac{1}{N\Delta t} \sum_{k=1}^{N} X_k e^{im\omega_k \Delta t}, \qquad (14)$$

$$where \ \ \omega_k = \frac{2\pi k}{N\Delta t}.$$

This X_k has a certain physical meaning. Amplitude and Phase of sinusoidal function.
Let's change Eq. (14) slightly in the following way.

$$\tilde{X}_k = \sum_{m=1}^{N} x_m e^{-im\omega_k \Delta t},$$

$$x_m = \frac{1}{N} \sum_{k=1}^{N} \tilde{X}_k e^{im\omega_k \Delta t}.$$

This $\tilde{X}_k$ gives an abstract quantity in the transformed domain.

A new variable is introduced as

$$z = e^{i\omega_k \Delta t}. \qquad (15)$$

Thus,

$$Z(x_m) = \tilde{X}_k = \sum_{m=1}^{N} x_m z^{-m},$$

$$x_m = \frac{1}{N}\sum_{k=1}^{N} \tilde{X}_k z^k. \qquad (16)$$

This new integral transform for discrete system is called *z-transform*. The definition Eq. (15) can be extended to relate the discrete z-transform to continuous Laplace transform with $s=\sigma+i\omega$.

$$z = e^{s\Delta t}. \qquad (17)$$

The product with z means the time shift of Δt toward the future (delay) , whereas that with z^{-1} toward the past (advance) .

3.4.4. Filter Operator in the Z-domain

Suppose *x(t)* denotes the input time series, *X(ω)* its Fourier spectrum, *y(t)* filtered output, *Y(ω)* its spectrum and *F(ω)* the spectrum of the applied filter. Therefore,

$$Y(\omega) = F(\omega)X(\omega). \qquad (18)$$

Suppose the filter spectra can be written, *e. g.*, in the following form for the facility of discussion.

$$F(z(\omega)) = \frac{a_2 z^{-2} + a_1 z^{-1} + a_0}{b_2 z^{-2} + b_1 z^{-1} + b_0}, \qquad (19)$$

Thus,

$$\left[b_2 z^{-2} + b_1 z^{-1} + b_0\right]Y(\omega) = \left[a_2 z^{-2} + a_1 z^{-2} + a_0\right]X(\omega).$$

The inverse Fourier transform of both members gives

$$b_0 y(t) + b_1 y(t - \Delta t) + b_2 y(t - 2\Delta t) = a_0 x(t) + a_1 x(t - \Delta t) + a_2 x(t - 2\Delta t),$$

Why?
Because,

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} z^{-n} Y(\omega) e^{i\omega t} d\omega$$

$z = e^{s\Delta t}$.

s ➔ $i\omega$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega) e^{i\omega(t-n\Delta t)} d\omega$$

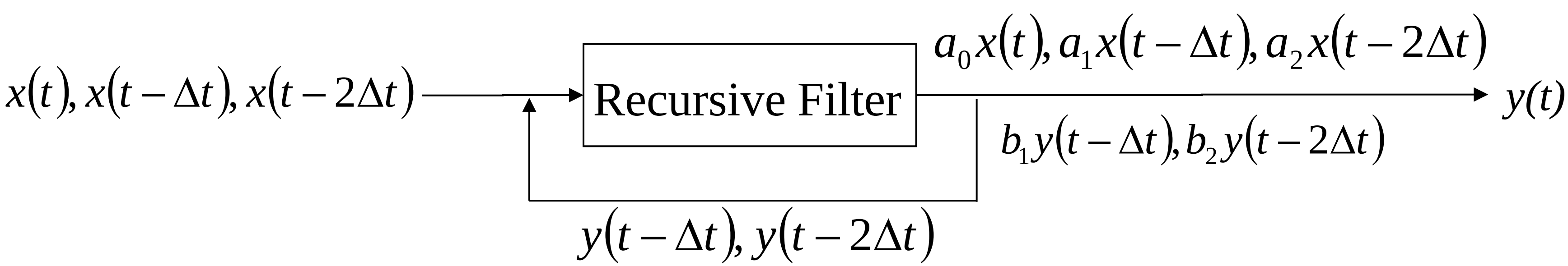
$$= y(t - n\Delta t) \quad .$$

This gives the recursive form as follows.

$$b_0 y(t) + b_1 y(t-\Delta t) + b_2 y(t-2\Delta t) = a_0 x(t) + a_1 x(t-\Delta t) + a_2 x(t-2\Delta t),$$

$$b_0 y(t) = a_0 x(t) + a_1 x(t-\Delta t) + a_2 x(t-2\Delta t) - b_1 y(t-\Delta t) - b_2 y(t-2\Delta t),$$

$$y(t) = \frac{1}{b_0}\left\{a_0 x(t) + a_1 x(t-\Delta t) + a_2 x(t-2\Delta t) - b_1 y(t-\Delta t) - b_2 y(t-2\Delta t)\right\},$$



Then, the filtered output can be calculated rapidly with defined value of coefficients, few precedent data of the input time series, and few precedent data of the output. For Eq. (19),

$$\left.\begin{aligned} y_0 &= \frac{a_0}{b_0} x_0 \\ y_1 &= \frac{1}{b_0}\left(a_0 x_1 + a_1 x_0 - b_1 y_0\right) \\ y_j &= \frac{1}{b_0}\left(a_0 x_j + a_1 x_{j-1} + a_2 x_{j-2} - b_1 y_{j-1} - b_2 y_{j-2}\right),\ j \geq 2 \end{aligned}\right\} . \qquad (20)$$

This shows an example for a recursive filter operated in the time domain.

Note, this recursive filter has its own amplitude and phase characteristics. Then, this gives phase lag. The causality, however, is always kept.

To compensate the phase lag, namely, to apply a zero-phase filter, inverse the time axis and apply the same filter in such way as

$$\left.\begin{aligned} y_N &= \frac{a_0}{b_0} x_{N-1} \\ y_{N-2} &= \frac{1}{b_0}\left(a_0 x_{N-2} + a_1 x_{N-1} - b_1 y_{N-1}\right) \\ y_{N-j} &= \frac{1}{b_0}\left(a_0 x_{N-j} + a_1 x_{N-j+1} + a_2 x_{N-j+2} - b_1 y_{N-j+1} - b_2 y_{N-j+2}\right),\ j \geq 3 \end{aligned}\right\} \cdot \quad (21)$$

It is of course that we can employ more coefficients a_l and b_m if necessary. The general form of Eq. (19) may be

$$F(z) = \frac{a_L z^{-L} + a_{L-1} z^{-(L-1)} + \cdots + a_2 z^{-2} + a_1 z^{-1} + a_0}{b_M z^{-M} + b_{M-1} z^{-(M-1)} + \cdots + b_2 z^{-2} + b_1 z^{-1} + b_0}, z = e^{i\omega\Delta t}. \quad (22)$$

The corresponding recursive filter may be

$$\begin{aligned} y_j = \frac{1}{b_0}\Big\{ & a_0 x_j + a_1 x_{j-1} + a_2 x_{j-2} + \cdots + a_{L-1} x_{j-L+1} + a_L x_{j-L} \\ & - (b_1 y_{j-1} + b_2 y_{j-2} + \cdots + b_{M-1} y_{j-M+1} + b_M y_{j-M}) \Big\}. \end{aligned} \quad (23)$$

Pay attention to the direct coincidence of the coefficients of filter wavelet in the time domain shown in Eq. (20) and Eq. (21) to the coefficients used in the transfer function in the *Z*-domain that is shown in Eq. (19). This shows that the analysis of transfer function in *Z*-domain gives the value of coefficients for recursive filtering in the time domain.

Transfer function given in the frequency domain and that given in *s*-domain are analogue functions, whereas that represented by a recursive filter is applied to discrete time series in computer. *Z*-transform acts as an interpreter at the border between two worlds different each other, one continuous world, another digital one.

It may be easy to find appropriate values for the coefficients of the recursive filter that is equivalent to a given transfer function, if we know a way to convert the transfer function given in s-domain to that in Z-domain.

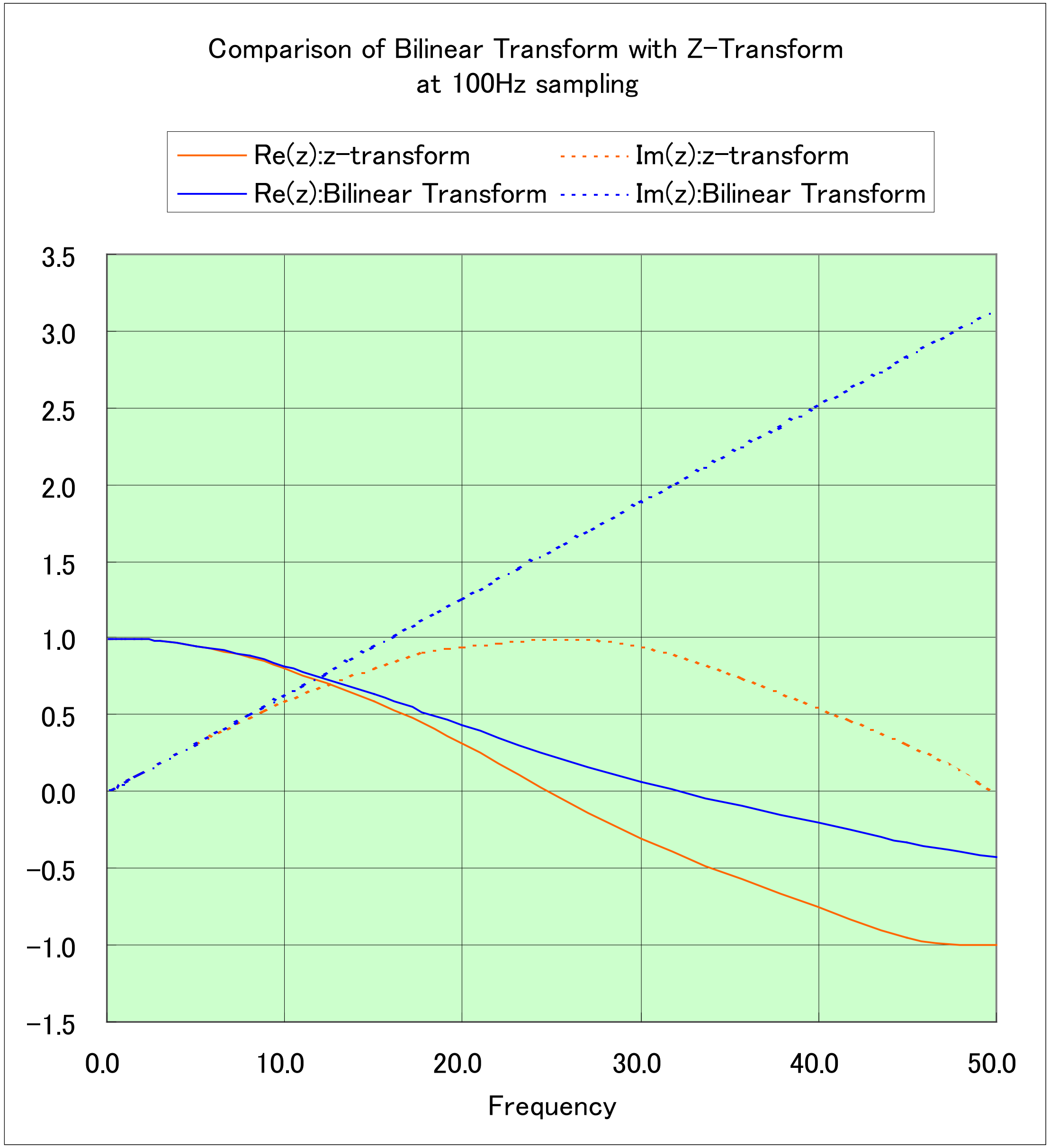
The relation $z = e^{s\Delta t}$ or $s = \dfrac{1}{\Delta t}\ln(z)$, however, makes the conversion complicated.

An approximation of Z-transform

$$s = \frac{1}{\Delta t}\ln(z),\ z = e^{s\Delta t}$$

is introduced, i. e., so called bi-linear transform.

$$s = \frac{2}{\Delta t}\cdot\frac{1-z^{-1}}{1+z^{-1}},\ \ z = \frac{1+\dfrac{\Delta t\cdot s}{2}}{1-\dfrac{\Delta t\cdot s}{2}}. \tag{25}$$



s=iω

z-transform

$$z = e^{s\Delta t}$$

Bi-linear transform

$$z = \frac{1 + \dfrac{\Delta t \cdot s}{2}}{1 - \dfrac{\Delta t \cdot s}{2}}.$$

Fig.44a Example for comparison of bi-linear transform with z-transform for Δt=0.01 sec. The difference is negligible at frequencies less than 10Hz.

The discrete angular frequency ω_k also mapped onto the continuous one.

For $z=exp(i\omega_k\Delta t)$, the bilinear transform gives

$$s = \frac{1}{\Delta t}\cdot\frac{1-\left(e^{i\omega_k\Delta t}\right)^{-1}}{1+\left(e^{i\omega_k\Delta t}\right)^{-1}} = \frac{2}{\Delta t}\cdot\frac{1-e^{-i\omega_k\Delta t}}{1+e^{-i\omega_k\Delta t}}$$

$$= \frac{2i}{\Delta t}\cdot\tan\left(\omega_k\Delta t/2\right).$$

Compare it with $s=i\omega$ *for* $\sigma=0$.

$$\omega = \frac{2}{\Delta t}\cdot\tan\left(\omega_k\Delta t/2\right). \qquad (26)$$

Or

$$\omega_k = \frac{2}{\Delta t}\cdot\tan^{-1}\left(\omega\Delta t/2\right).$$

This is mapping of the continuous angular frequency ω onto to the discrete one ω_k (Fig. 44).

$$\omega_k = \frac{2}{\Delta t} \cdot \tan^{-1}(\omega \Delta t/2).$$

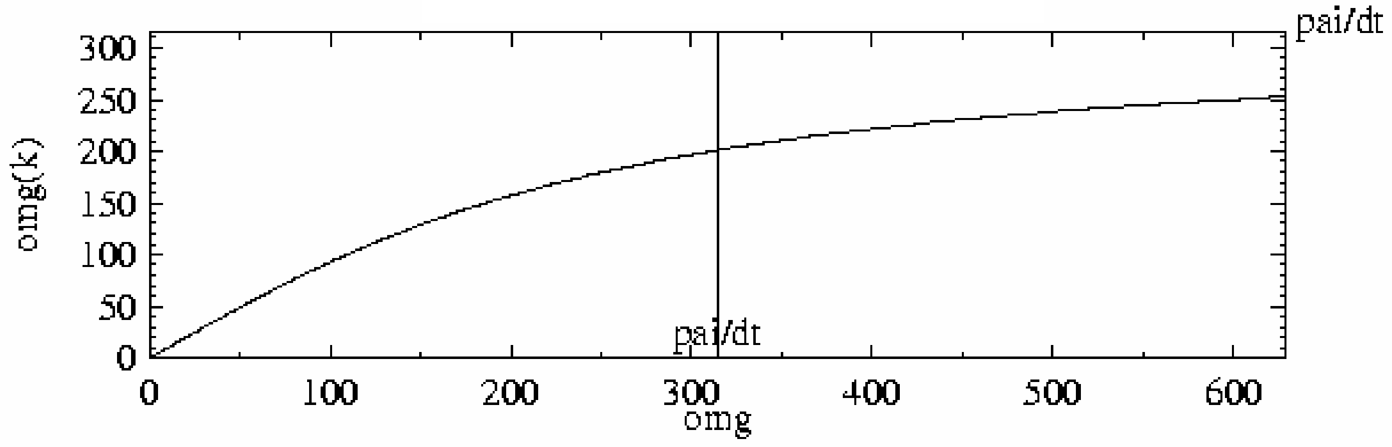
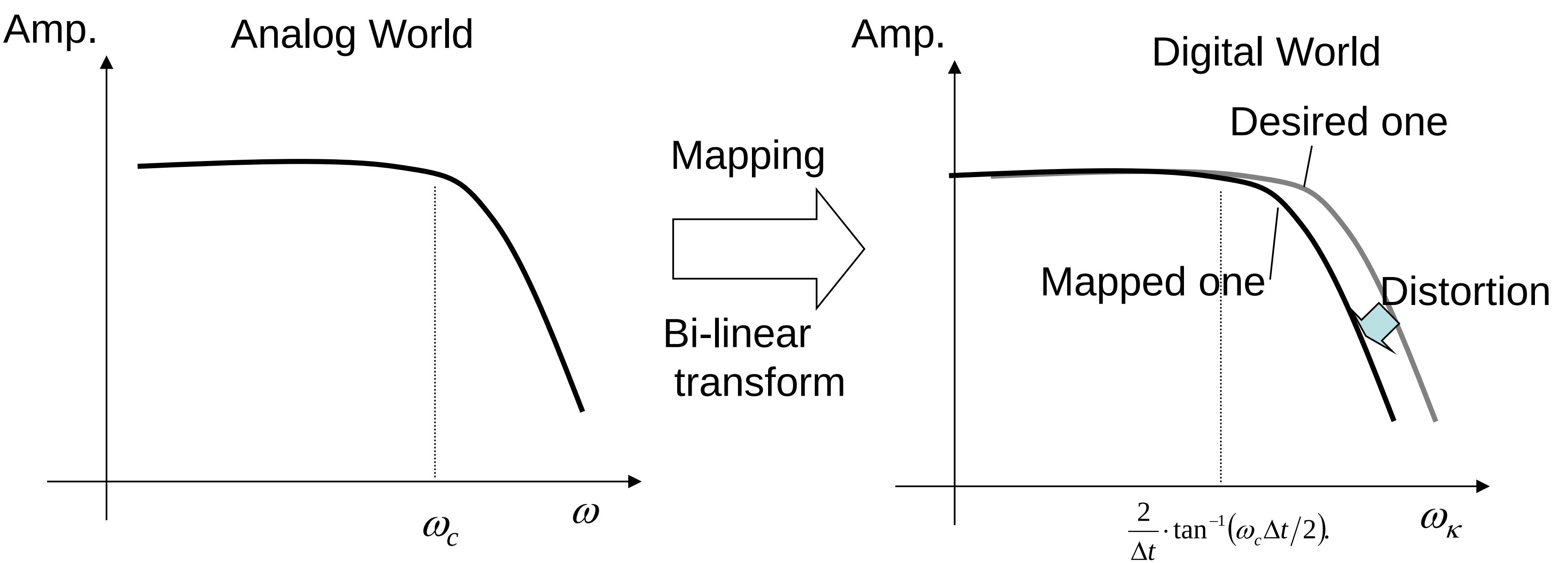


Fig. 44 Mapping of the continuous angular frequency ω onto the discrete frequency ω_k. This is an example of $\Delta t=0.01$, $N=1024$. The Nyquist angular frequency is $\pi/\Delta t$.

The bi-linear transform distorts the frequency axis in non-linear way shown in Fig. 44.
For example, a low pass filter with a cutoff frequency of ω_c in the continuous transfer function will have a corner frequency

$$\frac{2}{\Delta t}\cdot \tan^{-1}\left(\omega_c \Delta t/2\right).$$

in the discrete function which is lower than desired.



To compensate this distortion, a warped angular frequency,

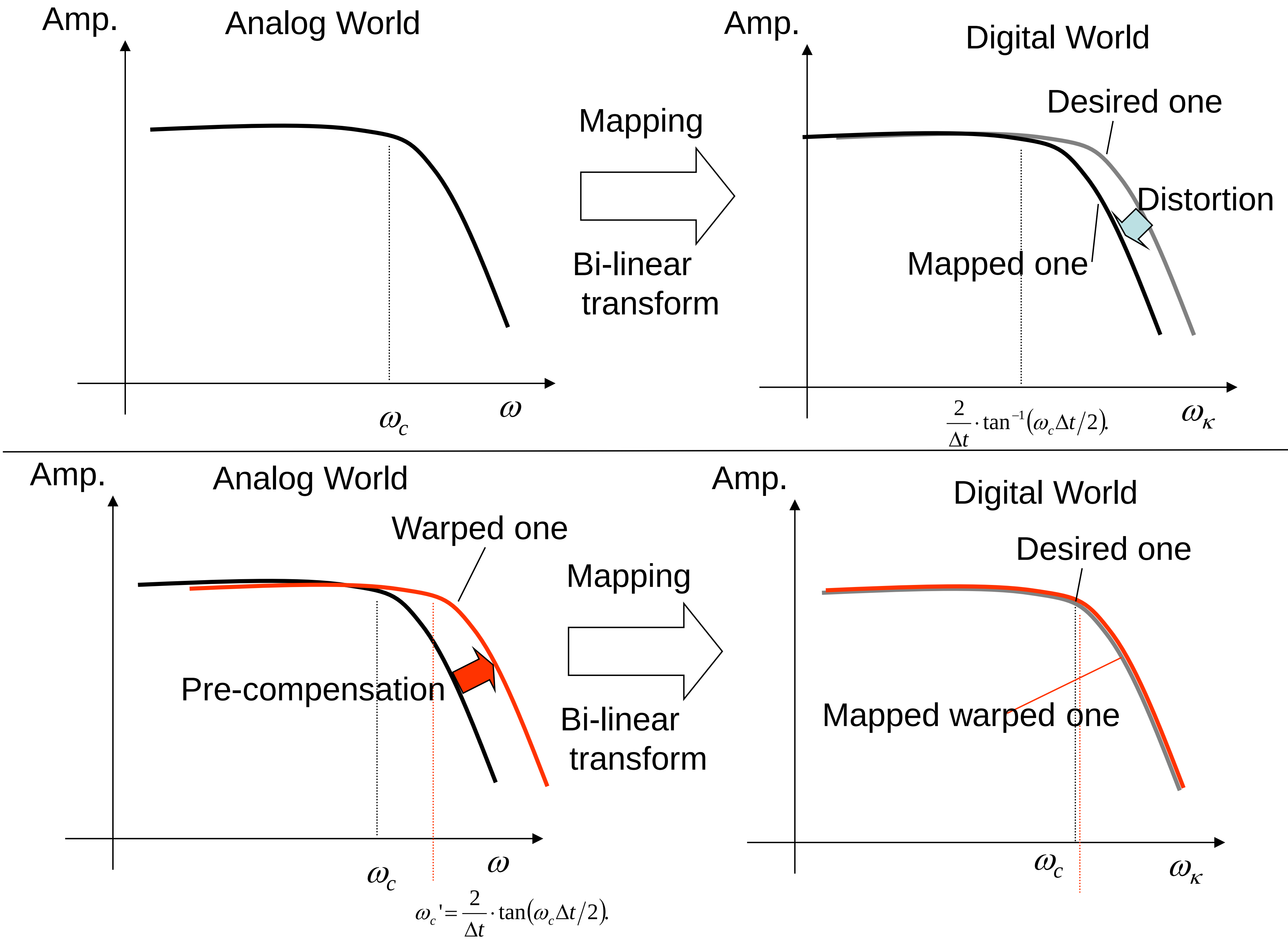
$$\omega_c' = \frac{2}{\Delta t} \cdot \tan\left(\omega_c \Delta t / 2\right), \qquad (27)$$

is introduced.
Critical angular frequencies of the continuous transfer function are converted to the corresponding warped angular frequencies at first, then the bi-linear transform using the warped ones is applied to obtain the equivalent discrete transfer function.

Eq. (27) shows that ω' tends to ω for small value of

$$\frac{\omega \Delta t}{2} = \frac{\pi f}{f_{Sampling}} = 2\pi \cdot \frac{f}{f_{Nyquist}}$$

This means that consideration on the warped frequency is not necessary for the frequency much smaller than the Nyquist frequency. The natural frequency of seismometer is usually much smaller than the Nyquist frequency, whereas the anti-aliasing filter has its cut off frequency comparable with the Nyquist one.



Amp.
Analog World
ω_c
ω
Mapping
Bi-linear transform
Amp.
Digital World
Desired one
Distortion
Mapped one
$\frac{2}{\Delta t}\cdot\tan^{-1}(\omega_c\Delta t/2)$.
ω_κ
Amp.
Analog World
Warped one
Pre-compensation
ω_c
ω
$\omega_c' = \frac{2}{\Delta t}\cdot\tan(\omega_c\Delta t/2)$.
Mapping
Bi-linear transform
Amp.
Digital World
Desired one
Mapped warped one
ω_c
ω_κ

Example: Moving Coil Type Seismometer

Transfer function in s-domain is

$$T(s) = \frac{s^2}{s^2 + 2h\omega_0 s + \omega_0^2}.$$

Let's introduce the warped angular frequency for ω_0

$$\omega_0' = \frac{2}{\Delta t} \cdot \tan(\omega_0 \Delta t / 2),$$

Then,

$$T(s) = \frac{s^2}{s^2 + 2h\omega'_0\, s + \omega'^2_0}. \qquad (31)$$

Applying the bilinear transformation to Eq. (31) gives the simulated transfer function in the z-domain,

$$T(s)=\frac{\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right)^2}{\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right)^2+2h\tan\left(\dfrac{\omega_0\Delta t}{2}\right)\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right)+\tan^2\left(\dfrac{\omega_0\Delta t}{2}\right)}$$

$$=\frac{\left(1-z^{-1}\right)^2}{\left(1-z^{-1}\right)^2+2h\tan\left(\dfrac{\omega_0\Delta t}{2}\right)\left(1-z^{-1}\right)\left(1+z^{-1}\right)+\tan^2\left(\dfrac{\omega_0\Delta t}{2}\right)\left(1+z^{-1}\right)^2}$$

$$=\frac{a_0+a_1z^{-1}+a_2z^{-2}}{b_0+b_1z^{-1}+b_2z^{-2}},$$

where

$$a_0=1.0, a_1=-2.0, a_2=1.0,$$

$$b_0=1+2h\tan\left(\omega_0\Delta t/2\right)+\tan^2\left(\omega_0\Delta t/2\right),$$

$$b_1=-2+2\tan^2\left(\omega_0\Delta t/2\right),$$

$$b_2=1-2h\tan\left(\omega_0\Delta t/2\right)+\tan^2\left(\omega_0\Delta t/2\right).$$

These coefficients of the recursive filter give approximately equivalent discrete transfer function.

Exercise: **Filter Equivalent to a Simple Moving Coil Type Seismometer-1.**

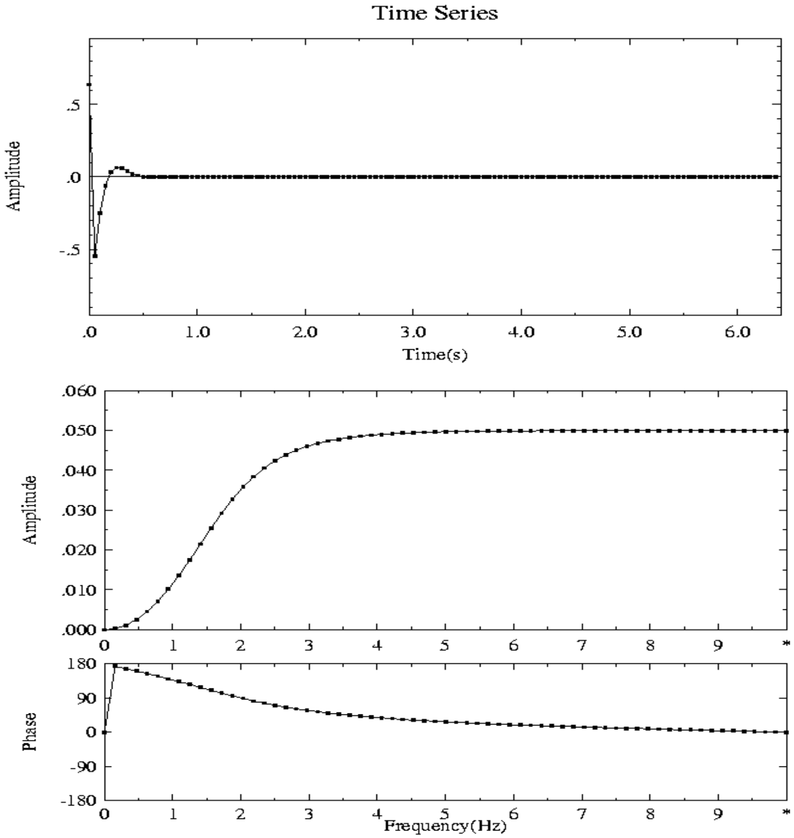


Fig.45.1 Filtered time series and its Fourier spectra obtained by *DSEISM.EXE*. These are equivalent to the impulse response of the relative motion of pendulum mass of a seismometer with the natural period $T_0=0.5$ sec, the damping factor $h=0.71$, $\Delta t=0.05$ and the gain $G_0=1.0$. The phase at zero frequency should converge to 180 degree. It, however, is forced to be zero to avoid the numerical zero-divide.

Example: Filter equivalent to a Simple Moving Coil Type Seismometer-2

In previous example, the recursive filter that gives the relative displacement of pendulum $-x_m$ for ground displacement y_m is given. Usually, the data obtained by digital recorder are given in Digit and a constant is given for conversion to Volts. The potential difference, the output from seismometer is given as follows.

$$e_m = \frac{G_0 R_s}{R_0 + R_s} (i\omega) x_m ,$$

where $(i\omega)$ shows the effect of differentiation due to moving coil type transducer, R_0 the coil resistance, R_s the shunt resistance, G_0 the product of the sensitivity of seismometer to the conversion constant of digital recorder. Therefore, the system response is

$$e_m / y_m = \frac{G_0 R_s}{R_0 + R_s} \cdot \frac{(i\omega)}{1 - 2ih(\omega_0/\omega) - (\omega_0/\omega)^2} .$$

This has an equivalent digital filter. The corresponding transfer function in the s-domain is

$$T(s) = G \cdot \frac{s^3}{s^2 + 2h\omega_0 s + \omega_0^2}, \quad G = \frac{G_0 R_s}{R_0 + R_s}. \qquad (33.1)$$

The solutions of the equation, i.e., the denominator is equal to zero, are

$$s = \omega_0 \left(-h \pm \sqrt{h^2 - 1} \right)$$

By using the warping frequency, the transfer function is given approximately as follows.

$$T(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}, \qquad (33.2)$$

where,

$$a_0 = G, a_1 = -3G, a_2 = 3G, a_3 = -G,.$$

$$b_0 = (\Delta t/2) \cdot \{1 + 2h \tan(\omega_0 \Delta t/2) + \tan^2(\omega_0 \Delta t/2)\},$$

$$b_1 = (\Delta t/2) \cdot \{-1 + 2h \tan(\omega_0 \Delta t/2) + 3\tan^2(\omega_0 \Delta t/2)\},$$

$$b_2 = (\Delta t/2) \cdot \{-1 - 2h \tan(\omega_0 \Delta t/2) + 3\tan^2(\omega_0 \Delta t/2)\},$$

$$b_3 = (\Delta t/2) \cdot \{1 - 2h \tan(\omega_0 \Delta t/2) + \tan^2(\omega_0 \Delta t/2)\}.$$

Exercise: **Design of a recursive filter equivalent to a seismometer.**

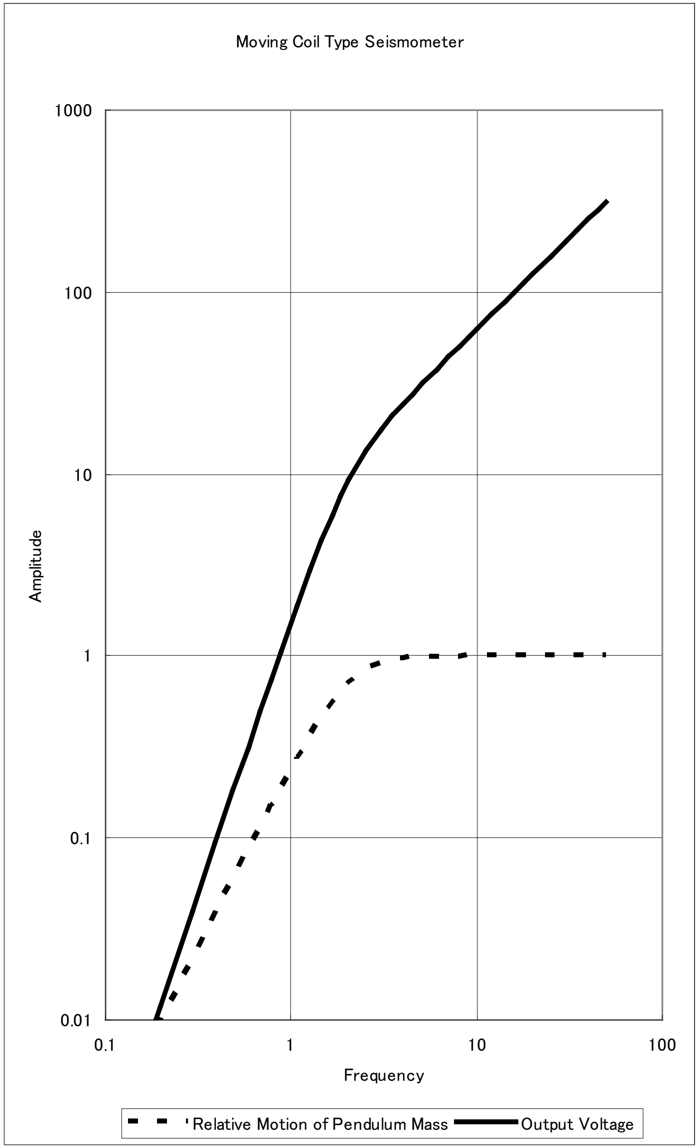


Fig. 45-2 Response of the recursive filters. *Dashed line*: Filter equivalent to the relative motion of pendulum mass of a simple moving coil type seismometer, against ground displacement. *Solid line*: Filter equivalent to the voltage change between two output terminals of a simple moving coil type seismometer, against ground displacement. The parameters used are T_0=0.5, h=0.71.

3.5. Deconvolution or Inverse Filtering

Remember that recorded signal is given by the convolution of the instrumental characteristics to the ground motion in the time domain and is given by the product of them in the frequency domain. The instrumental characteristics are the same as the response of instruments to an impulsive signal (Fig.51 *top*).

$$
\begin{aligned}
r(t) &= f(t) * g(t), \\
R(\omega) &= F(\omega) G(\omega).
\end{aligned}
\tag{36}
$$

where $r(t)$, $f(t)$ and $g(t)$ denote the recorded signals, the instrumental response and the ground motion in the time domain, $R(\omega)$, $F(\omega)$ and $G(\omega)$ these in the frequency domain.

Deconvolution in the seismology, usually, is the process to remove the effect of the instrumental characteristics from the observed data and to recover the true ground motion. Mathematically, this is the reverse process of convolution. Deconvolution in the time domain corresponds to the quotient in the frequency domain.

In comparison with the complexity for deconvolution in the time domain, the frequency domain operation is composed of only three steps. These are to apply the FFT to the instrumental response and the recorded signal, to divide of the recorded signal spectra by the instrumental response spectra and to apply the inverse FFT to the quotient (Fig.51 *middle*).

$$
G(\omega) = \{F(\omega)\}^{-1} \cdot R(\omega). \tag{37}
$$

Keep it in mind

The information once lost in the observation or processing never can be recovered by any technique, even by very sophisticated and efficient ones.

This is because we can not avoid noises that are recorded simultaneously with the signal or that invade into the record during the processing. Once the signals weaken and become smaller than the noise level, any recovering process just amplifies these noises. Such amplified noise can be dominant in the recovered ground motion.

The signals just weaken a little during the recording or the processing can be strengthen or recovered by the deconvolution technique.

Usually, the true ground motion has band limited feature at far field and low passed feature at near field, whereas the ground noise present at every frequency.
Let's handle only far filed ground motion just for having a simple example.
Then, $G(\omega)$ is band limited. The instrumental response $F(\omega)$ is also band limited,
because seismometer is a low cut filter and used with a high cut anti-alias filter.

The recovering operator in the frequency domain $\{F(\omega)\}-1$ has big amplitude at the frequency outside of this limited frequency band.

The recorded signal $R(\omega)$ is almost band limited but it has a little energy at outside of the frequency band of $G(\omega)$, *i.e.*, the contribution of the noise. By applying the inverse filter, *i.e.*, the recovering operator $\{F(\omega)\}-1$, this small contribution of the noise will be amplified much and contaminates the recovered ground motion transformed into the time domain by the inverse FFT .

This shows that we have to select the frequency range in that the signal is sufficiently larger than the noise in order to prevent the instability at applying the inverse filter. We can not recover the ground motion at outside of this frequency band.

The recommendable way, however, is to handle the data only within the pass band of the instrumental response. Usually, this is given by the natural frequency of the seismometer and the cut off frequency of the anti-alias filter. It is possible but not easy to use the information at outside of this range. Even within this frequency range, the instability problem mentioned above can take place due to smaller frequency band of $G(\omega)$. Thus, it is also recommendable to observe the shape of $R(\omega)$ and select the useful frequency band before starting the data processing.

Example: Inverse Filter for a Simple Moving Coil Type Seismometer

The transfer function that gives the inverse response of the previous example is given as follows.

$$T(s) = G^{-1} \cdot \frac{s^2 + 2h\omega_0 s + \omega_0^2}{s^3}, \; G = \frac{G_0 R_s}{R_0 + R_s}. \qquad (36)$$

Fig. 52 shows the amplitude response of this transfer function.
The solutions of the equation, i.e., the numerator is equal to zero, are . This gives the zero position at

$\left(-\omega_0 h,\ \omega_0\sqrt{1-h^2}\right)$, $\left(-\omega_0 h,\ -\omega_0\sqrt{1-h^2}\right)$, for h<1.0, under damped case,

$(-\omega_0,\ 0.)$ doubled for h=1.0, critically damped case,

$\left(-\omega_0\left(h-\sqrt{h^2-1}\right), 0.\right)$, $\left(-\omega_0\left(h+\sqrt{h^2-1}\right), 0.\right)$, for h>1.0, over damped case.

The denominator, however, gives a tripled pole at (0., 0.).

By using the warped frequency, the transfer function is given approximately as follows.

$$T(s) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}},$$

where,

$$a_0 = (\Delta t/2) \cdot \{1 + 2h \tan(\omega_0 \Delta t/2) + \tan^2(\omega_0 \Delta t/2)\},$$
$$a_1 = (\Delta t/2) \cdot \{-1 + 2h \tan(\omega_0 \Delta t/2) + 3\tan^2(\omega_0 \Delta t/2)\},$$
$$a_2 = (\Delta t/2) \cdot \{-1 - 2h \tan(\omega_0 \Delta t/2) + 3\tan^2(\omega_0 \Delta t/2)\},$$
$$a_3 = (\Delta t/2) \cdot \{1 - 2h \tan(\omega_0 \Delta t/2) + \tan^2(\omega_0 \Delta t/2)\}$$
$$b_0 = G, b_1 = -3G, b_2 = 3G, b_3 = -G.$$

As simple moving coil type seismometer is a low cut filter, its inverse filter makes the component grow in the frequency range lower than the natural frequency of the seismometer. This causes instability of the output from the filter, because the signal to noise ratio is small at this frequency range.

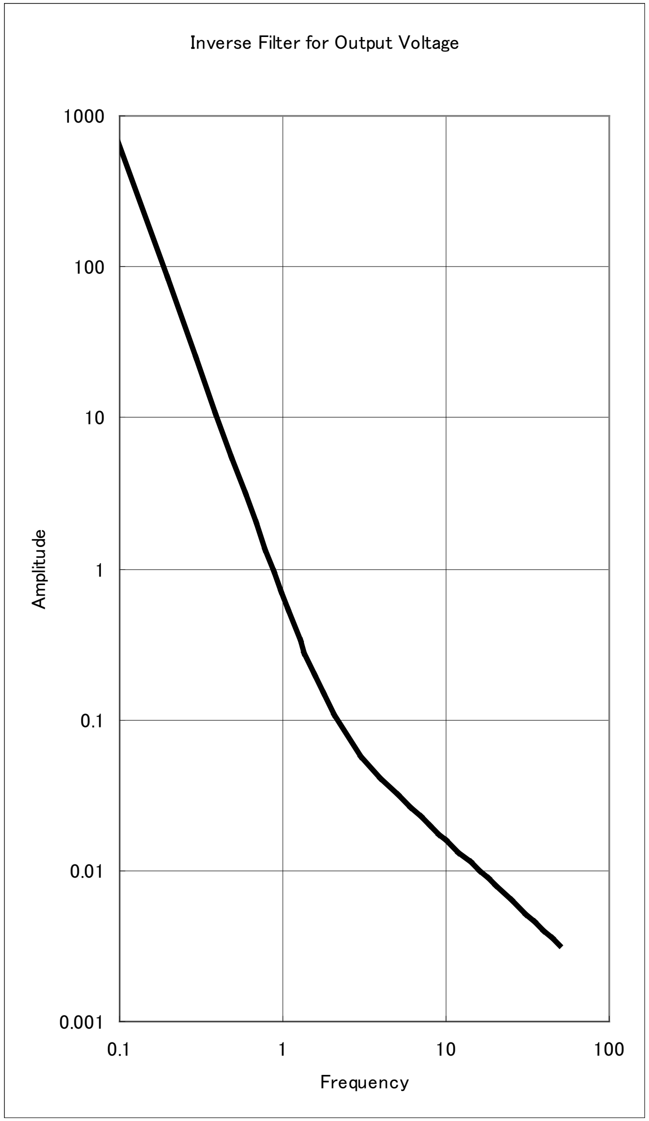


Fig. 52 Amplitude response of the inverse filter for the voltage change between two output terminals of a simple moving coil type seismometer that is calculated by Eq. (36). The parameters used are T_0=0.5, h=0.71. Theoretically, this filter can convert the output voltage change to the ground displacement. However, this amplifies much the low frequency components as shown clearly in this figure.

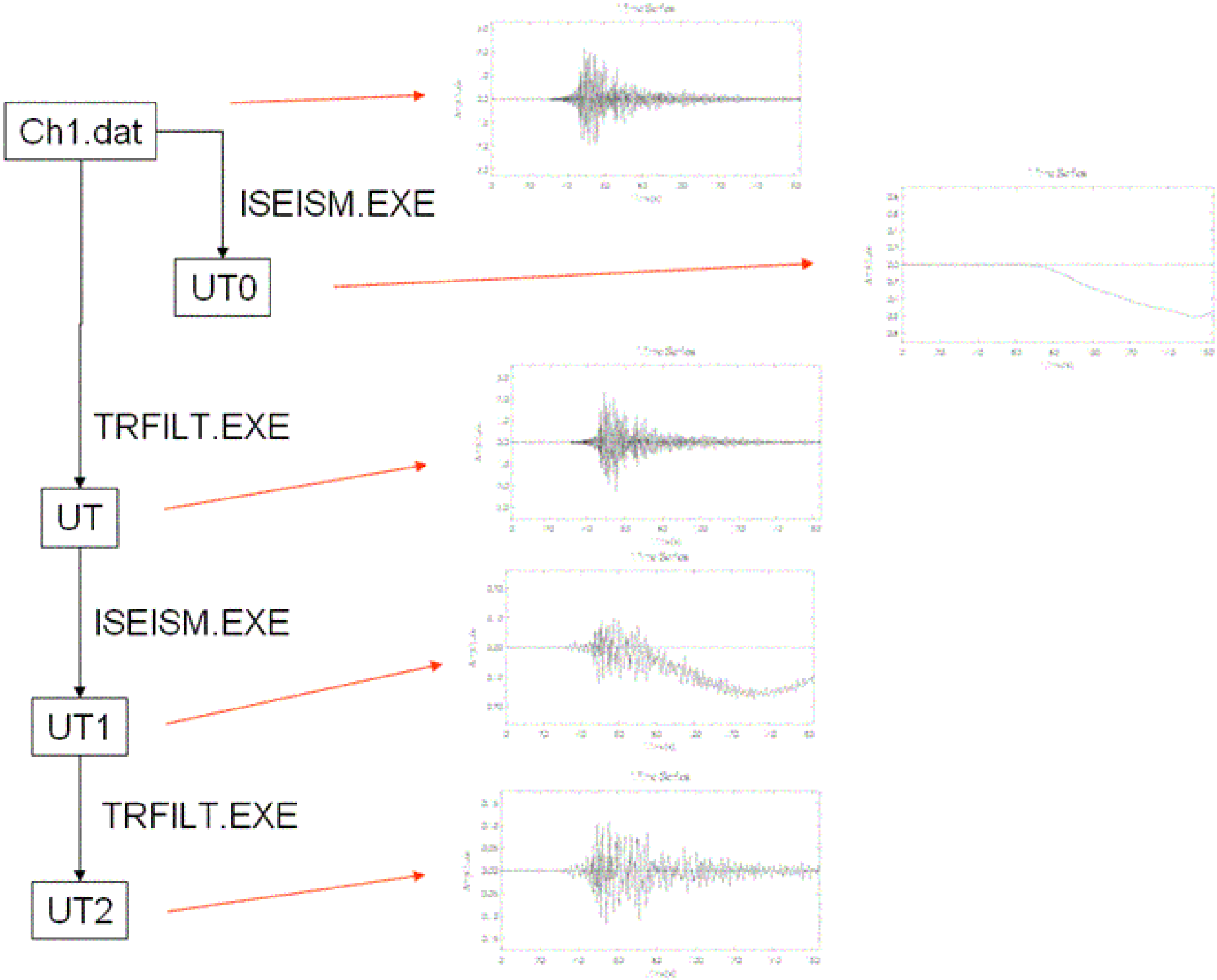


Fig. 53.2 Procedure and output of the example for a real seismogram.

Example: Filter for Conversion of Seismogram obtained by a Simple Moving Coil Type Seismometer to that of Longer Period Seismometer.

A simple way to avoid the problem mentioned above is to convert the system response with a longer period seismometer.

Suppose that the damping constant and the natural angular frequency of the seismometer used in observation are h_0, ω_0, then its response is

$$e_m / y_m = G_0 \cdot \frac{(i\omega)}{1-2ih_0(\omega_0/\omega)-(\omega_0/\omega)^2}.$$

Let's convert it with the response of seismometer, of which damping constant and natural angular frequency are h_1, ω_1 .

$$e_m / y_m = G_1 \cdot \frac{(i\omega)}{1-2ih_1(\omega_1/\omega)-(\omega_1/\omega)^2}.$$

The conversing function in the frequency domain is

$$G^{-1} \cdot \frac{1-2ih_0(\omega_0/\omega)-(\omega_0/\omega)^2}{1-2ih_1(\omega_1/\omega)-(\omega_1/\omega)^2}, \; G = G_0/G_1 .$$

The transfer function in the *s*-domain is given as follows.

$$T(s) = G^{-1} \cdot \frac{s^2 + 2h_0\omega_0 s + \omega_0^2}{s^2 + 2h_1\omega_1 s + \omega_1^2}. \tag{37}$$

Then, using the warped frequency gives

$$T(s) = G^{-1} \cdot \frac{(1-z^{-1})^2 + 2h_0 \tan\left(\frac{\omega_0 \Delta t}{2}\right)(1-z^{-1})(1+z^{-1}) + \tan^2\left(\frac{\omega_0 \Delta t}{2}\right)(1+z^{-1})^2}{(1-z^{-1})^2 + 2h_1 \tan\left(\frac{\omega_1 \Delta t}{2}\right)(1-z^{-1})(1+z^{-1}) + \tan^2\left(\frac{\omega_1 \Delta t}{2}\right)(1+z^{-1})^2}$$

$$= \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{b_0 + b_1 z^{-1} + b_2 z^{-2}},$$

where,

$$a_0 = 1 + 2h_0 \tan(\omega_0 \Delta t/2) + \tan^2(\omega_0 \Delta t/2),$$

$$a_1 = -2 + 2\tan^2(\omega_0 \Delta t/2),$$

$$a_2 = 1 - 2h_0 \tan(\omega_0 \Delta t/2) + \tan^2(\omega_0 \Delta t/2),$$

$$b_0 = G\{1 + 2h_1 \tan(\omega_1 \Delta t/2) + \tan^2(\omega_1 \Delta t/2)\},$$

$$b_1 = G\{-2 + 2\tan^2(\omega_1 \Delta t/2)\},$$

$$b_2 = G(1 + 2h_1 \tan(\omega_1 \Delta t/2) + \tan^2(\omega_1 \Delta t/2)).$$

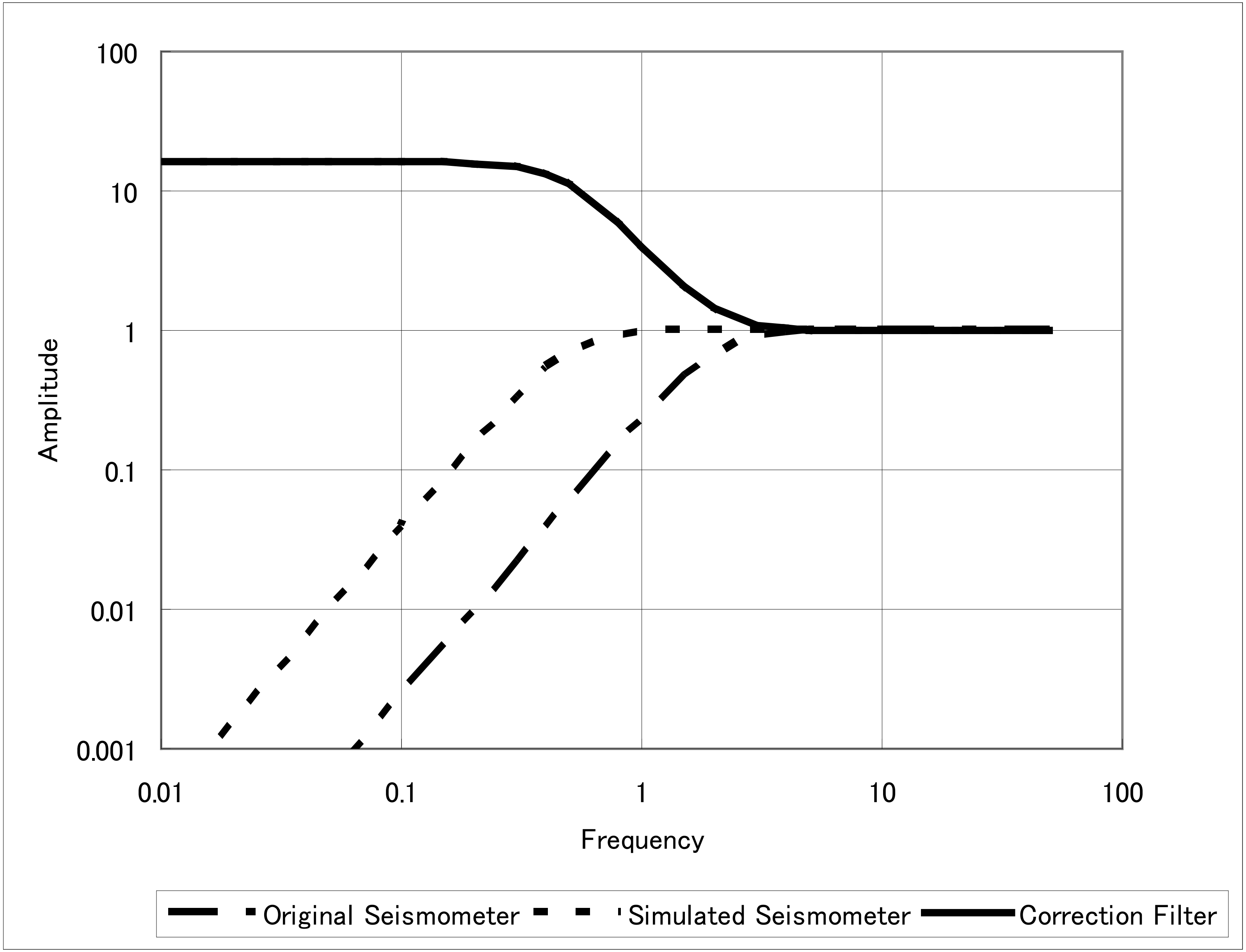
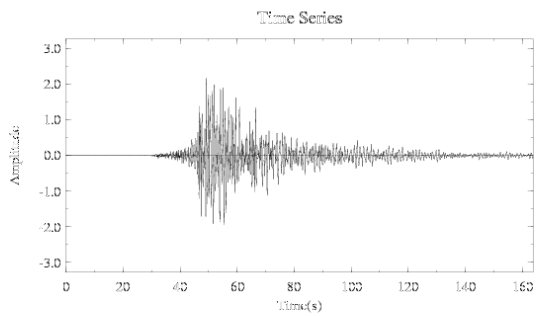
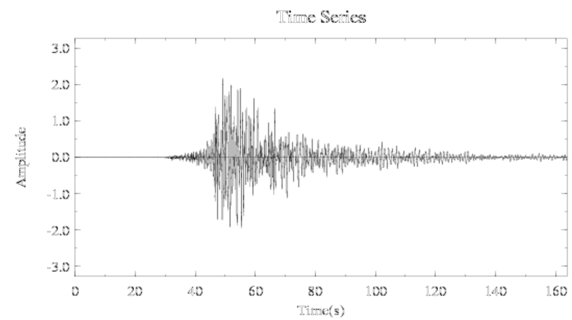


Fig.54 Amplitude Spectra of the correction filter explained above.



Ch1.dat



Output.dat

Fig.55.2 Input signal (*left*): Ch1.dat and output signal (*right*) of *CSEISMO.EXE*