Moment tensor inversion of near source seismograms

Yuji Yagi and Naoki Nishimura

ABSTRACT

We constructed a program set for estimating moment tensor solutions using near source seismograms. We take the effect of the source time function into account, which has been neglected in predominant program sets of the moment tensor analysis with near source seismograms. In our program set, we approximate the horizontal location of the centroid to be epicenter, and then estimate a seismic source duration and depth of centroid in the moment tensor inversion procedure. To evaluate our program set, we applied it to three earthquakes: an aftershock (M\text{JMA} 6.4) of the 2003 Tokachi-oki great earthquake, the 2004 Nemuro-oki earthquake (M\text{JMA} 7.1) and its aftershock (M\text{JMA} 4.2). The results show that the obtained moment tensor solutions for large earthquakes are consistent with those in other studies. We also found that the neglect of the source time duration will result in an underestimation of the seismic moment.

INTRODUCTION

The moment tensor solution is one of the most important information of earthquakes. From the moment tensor solutions, we can obtain seismic moment (M_0), moment magnitude (M_w) and fault type of earthquakes. The moment tensor solution is also necessary information for investigation of a detailed seismic source process.

The moment tensor solution has been estimated by inversion analysis of seismic waveforms observed by local and global seismic networks (e.g. Dziewonski et al., 1981; Kikuchi and Kanamori, 1991; Kawakatsu, 1995). For example, U.S. Geological Survey (USGS) and Global Centroid Moment Tensor Project have estimated the moment tensor solutions for earthquakes M > 5.5 using global seismic network, and created a catalog of earthquakes. The moment tensor information can be downloaded from their web sites: USGS web-site (http://earthquake.usgs.gov/earthquakes) and GCMT web-site (http://www.globalcmt.org). Using local seismic networks, many institutes estimated moment tensor solutions for earthquakes M > 3 and a created catalogs of earthquakes (e.g. Dreger and Helmberger, 1993; Pasyanos et al. 1996). In Japan, the National Research Institute for Earth Science and Disaster Prevention (NIED) has estimated the moment tensor solutions using a broadband seismic observation network (F-net) since 1997 (Fukuyama et al., 1998).

Since the occurrence rate of middle-size earthquakes is higher than that of large-size earthquakes, it is important to estimate moment tensor solutions of middle-size earthquakes for understanding stress field and faulting systems in local regions. So far, broadband seismic networks have been established in developing countries, we can estimate moment tensor solutions for earthquakes M > 3.5 in the developing countries and construct a catalog of moment tensor solutions. The program set of moment tensor inversion and its lecture become important for seismologists in the developing countries.

In general, the effect of source time function has been neglected in predominant program sets of the moment tensor inversion with near source seismograms, for simplicity (e.g. Dreger and Helmberger, 1993; Ito et al., 2006). The neglect of the effect of seismic source duration should result in an underestimation of the seismic moment. In this study, we constructed a formulation to estimate moment tensor solutions based on a point source model and a simple source time function, and then we applied it to local seismic data for evaluating our program set. Through application of new program set, we confirmed that the seismic moment is underestimated if we neglect the effect of the source duration. Our program set is introduced in the lecture provided by the International Institute of Seismology and Earthquake Engineering, the Building Research Institute.
In general, observed seismic waveform of \( c \) component at a station \( j \) due to seismic moment release in a source volume \( V \) is given by

\[
u_{cj}(t) = \sum_{q=1}^{s} \int \int \int \tilde{G}_{cjq}(t, \xi) \ast \tilde{M}_{q}(t, \xi) d\xi + e'_{cj}(t),
\]

where \( \tilde{G} \) is the Green’s function of basis moment tensor (Kikuchi and Kanamori, 1991), \( \tilde{M} \) is a spatio-temporal moment density function, and \( e' \) is an observed error. Since the volume change during earthquakes is too small to detect, we can neglect volume change component \( \tilde{M} \). To estimate stable moment tensor solutions, we should assume a simple source model such as the point source model, in which we assume the seismic waveform to be radiated from one point. Following the point source model, we rewrite eq. (1) as

\[
u_{cj}(t) = \sum_{q=1}^{s} \tilde{G}_{cjq}(t, \xi_c) \ast M_{q}(t) + e_{cj}(t),
\]

with

\[
M_{q}(t) = \int \int \int \tilde{M}_{q}(t, \xi) d\xi,
\]

where \( \xi_c \) is the location of the centroid, and \( e \) contains observed and modeling errors. We assumed that the focal mechanism is kept constant during an earthquake, and approximated the shape of the source time function to be an isosceles triangle with half duration of \( t_r \). Based on the assumption of the constant focal mechanism and the simple source time function, we can rewrite eq. (2) as

\[
u_{cj}(t) = \sum_{q=1}^{s} m_q \times G_{cjq}(t, t_r, \xi_c) + e_{cj}(t),
\]

with

\[
G_{cjq}(t, t_r, \xi_c) = \tilde{G}_{cjq}(t, \xi_c) \ast T(t, t_r),
\]

where \( T \) is a source time function with half duration of \( t_r \). In general, we apply low-pass filter to observed data for mitigating the aliasing effect in re-sampling procedure, the effect of the complicated seismic source process and the effect of the heterogeneity of the local velocity structure. Therefore, relationship between the observed seismic waveform and data waveform is represented by

\[
d_{cj}(t) = B(t) \ast u_{cj}(t),
\]

where \( B \) is the function of the low-pass filter. Substituting eq. (4) into (6), and then we rewrite in the following simple vector form:

\[
d = \Lambda(t_r, \xi_c)m + e,
\]

where \( d \) and \( e \) are \( N \)-dimensional data and error vectors, respectively; \( m \) is a 5-dimensional model
parameter vector; $A$ is a $N \times 5$ coefficient matrix. The solution of the above matrix equation is obtained by least square approach if we assume the duration of the source time function $T_r$ and the centroid location $\xi_o$. In the study, we assumed that horizontal location of centroid can be approximated to the epicenter, and estimated optimal depth of the centroid and half duration using the grid-search method, which minimizes normalized L2-norm as $\|d - A(t_r, \xi_0)m\|/\|d\|$. 

APPLICATION

We performed a moment tensor inversion of one large earthquake and two medium earthquakes for examining the validity of our program set.

Data and the Green’s Function

Figure 1 shows the epicenters of three earthquakes and locations of stations, which are used in this study. Two aftershocks of the 2003 Tokachi-oki great earthquake were selected for the evaluation of our program set. We retrieved near source data observed by broadband seismograph F-NET stations, NIED. 9 components at 3 stations were used to estimate moment tensor solutions. The observed raw data were corrected for seismometer responses and converted to ground velocity motions. The data were filtered in the bandpass to mitigate the effect of the heterogeneity of the local velocity structure, and re-sampled with 1 Hz. Following the NIED moment tensor analysis (Fukuyama et al., 1998), we selected filtering range 20 – 100 sec for events A and B, and 20 – 50 sec for an event C. We did not integrate observed waveform to avoid the increase of off-diagram of the data covariance components (Yagi and Fukahata, 2008). Green’s function was calculated by the discrete wave number method developed by Kohketsu (1985) with the simple J-B structure.

![Fig. 1 Location map of F-net broadband stations and earthquakes used in our analysis. The triangles and the stars represent the location of stations and the epicenters, respectively.](image_url)

Results

Now, we applied our program to an aftershock ($M_{JMA}$ 7.1; origin time: 29 Sep 2003 11:37 JST) of the 2003 Tokachi-oki great earthquake, the 2004 Nemuro-oki earthquake ($M_{JMA}$ 7.1; origin time: 29
Nov 2004 03:32 JST) and its aftershock (M_JMA 4.2; origin time: 30 Nov 2004 13:02 JST), which are labeled event A, B and C, respectively. We approximate the horizontal location of centroid to be the epicenter determined by the Japan Meteorological Agency (JMA). Figure 2 shows the focal mechanism and the comparison between the observed waveform (black line) and the synthetic waveform (gray line) for each event. As can be seen from the focal mechanisms, three earthquakes are typical thrust earthquakes along plate boundary. The estimated focal mechanisms, depths and seismic moments of three events are consistent with the F-NET MT solutions.

Table 1 shows the depth of centroid, the seismic moment, the half duration of source time function, and the normalized L2 norm for each event with our formulation and a conventional formulation that neglects the effect of source duration. As can be seen from the values of the normalized L2 norm in table 1, the waveform fitting is improved by our formulation. It should be noted that the seismic moment estimated by our formulation is always larger than that estimated by the conventional formulation. This gap should increase with the seismic moment. This result shows that the neglect of source duration is inappropriate assumption for analysis of large earthquakes, because the duration of

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**Table 1**

<table>
<thead>
<tr>
<th>Event</th>
<th>Depth (km)</th>
<th>Moment (x10**18 Nm)</th>
<th>Half Duration (sec)</th>
<th>Normalized L2 Norm</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>35.0</td>
<td>3.17 x10**18</td>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>50.0</td>
<td>0.80 x10**19</td>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>40.0</td>
<td>1.53 x10**15</td>
<td>4.0</td>
<td></td>
</tr>
</tbody>
</table>

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**Fig. 2** Results of moment tensor inversion for the event A (a), the event B (b) and the event C (c). For each case, the focal mechanism (top left-hand graph), the information of source parameters (top right-hand graph), and observed and synthetic waveforms (black lines and gray lines, respectively; bottom graph) are shown. The numbers below the station code indicate the maximum amplitude (in unit of cm/sec).
the source time function increases with the seismic moment. In fact, moment magnitude \((M_w)\) of the 2003 Tokachi-oki earthquake estimated by NIED (Ito et al., 2004) is 7.9, which is meaningfully smaller than \(M_w\ 8.1\) estimated by seismic source analysis (e.g. Yagi, 2004). The estimated half duration of event C is 4.0 sec that is somewhat large duration for M4.2 earthquake. Since the normalized L2 norm in the low frequency waveform is insensitive to value of the half duration, the value of half duration tends to have large estimation errors.

Table 1 Comparison between the source parameters obtained by our formulation and the conventional formulation.

<table>
<thead>
<tr>
<th>(a) Event A</th>
<th>Seismic moment (Nm)</th>
<th>Depth (km)</th>
<th>Half duration (sec)</th>
<th>Normalized L2 norm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our formulation</td>
<td>7.63 x 10^{18}</td>
<td>35</td>
<td>6.0</td>
<td>0.113</td>
</tr>
<tr>
<td>Conventional formulation</td>
<td>5.54 x 10^{18}</td>
<td>35</td>
<td>N/A</td>
<td>0.140</td>
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</table>

<table>
<thead>
<tr>
<th>(b) Event B</th>
<th>Seismic moment (Nm)</th>
<th>Depth (km)</th>
<th>Half duration (sec)</th>
<th>Normalized L2 norm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our formulation</td>
<td>4.05 x 10^{19}</td>
<td>50</td>
<td>6.0</td>
<td>0.129</td>
</tr>
<tr>
<td>Conventional formulation</td>
<td>3.18 x 10^{19}</td>
<td>50</td>
<td>N/A</td>
<td>0.139</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(c) Event C</th>
<th>Seismic moment (Nm)</th>
<th>Depth (km)</th>
<th>Half duration (sec)</th>
<th>Normalized L2 norm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our formulation</td>
<td>2.75 x 10^{15}</td>
<td>40</td>
<td>4.0</td>
<td>0.102</td>
</tr>
<tr>
<td>Conventional formulation</td>
<td>2.46 x 10^{15}</td>
<td>40</td>
<td>N/A</td>
<td>0.104</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

We constructed a program set for estimating moment tensor solutions using near source seismograms, which will be introduced in the lecture provided by the International Institute of Seismology and Earthquake Engineering, the Building Research Institute. We applied our program set to real data, and confirmed that the obtained moment tensor solutions for large earthquakes are well consistent with the F-NET MT solutions. We found that the neglect of the effect of seismic source duration will result in an underestimation of the seismic moment, since the neglect of seismic source duration is inappropriate assumption for analysis of large earthquakes.

**ACKNOWLEDGMENT**

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**REFERENCE**


