Moment Tensor Inversion

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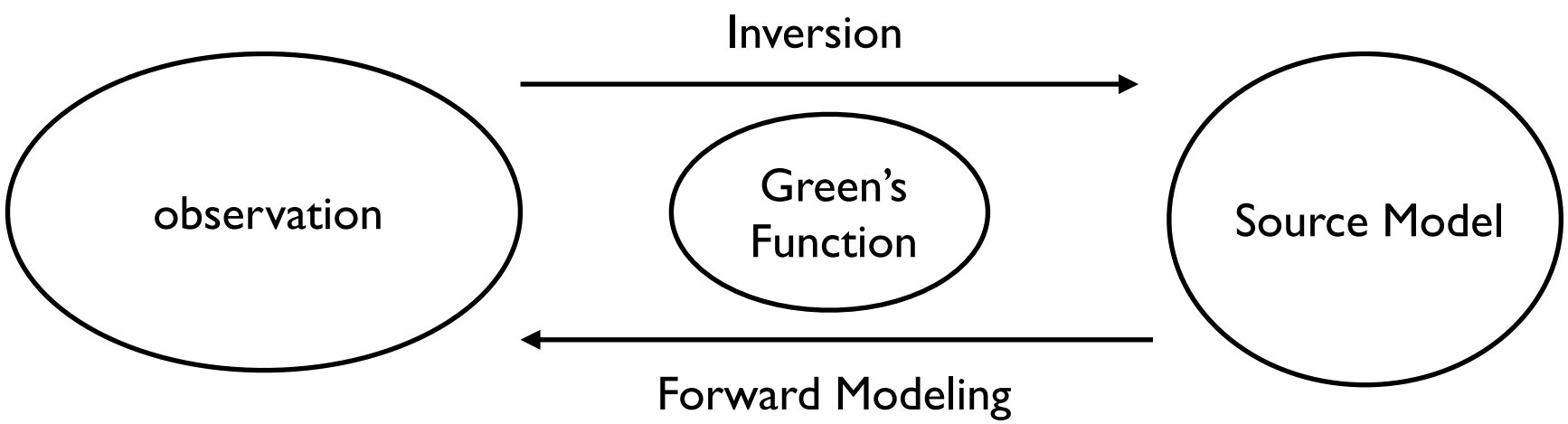
Introduction

Now, we can compute synthetic seismograms that are comparable with observed seismograms.

The seismic waveforms contain the information of the focal mechanism and seismic slip area.

If we assume earth structure, we can calculate green's function and estimate moment tensor components and location of centroid using waveform inversion scheme.

Forward modeling and Inversion



To estimate source model, we often apply two method.

- Forward Modeling (try and error)
  - Input: source model
  - Output: synthetic waveform
- Inversion
  - Input: observation data
  - Output: source model

Linear Inversion

Observation Equation

$$\mathbf{y} = \mathbf{f}\left(\mathbf{x}\right)$$

$\mathbf{y}$: observation; $\mathbf{x}$: model

Linear case

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

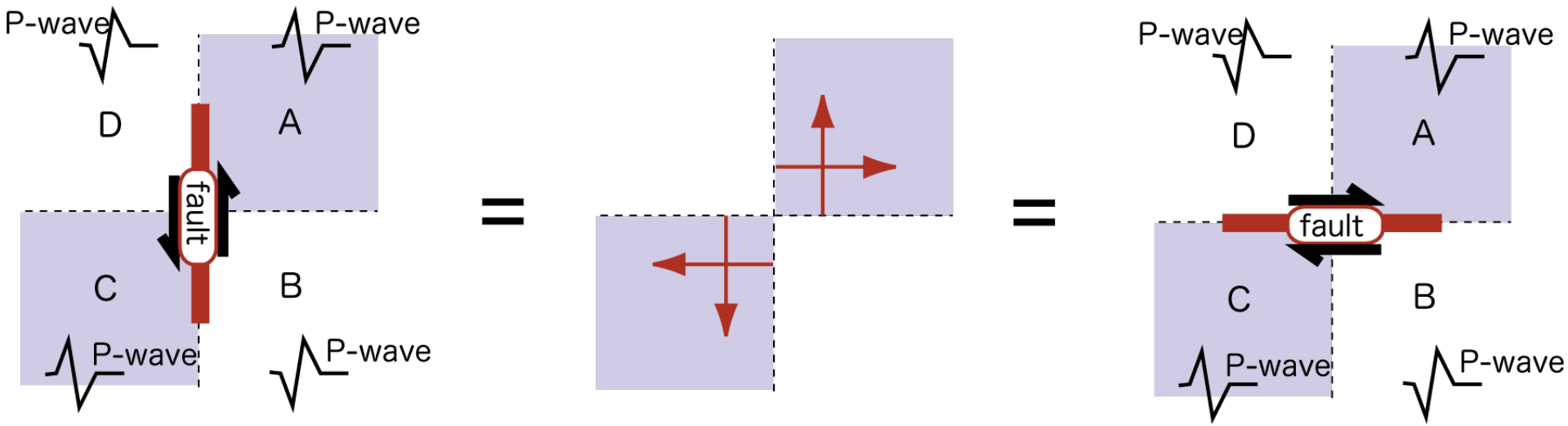
$\mathbf{A}$: Kernel Matrix

And then,

$$\hat{\mathbf{x}} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{y}$$

We can get model information from observation data.

Double Couple Model



The double couple model has two pairs of single couple.

This model don't have total face and torque. Thus this model have net balanced the moment.

Intrinsic problem

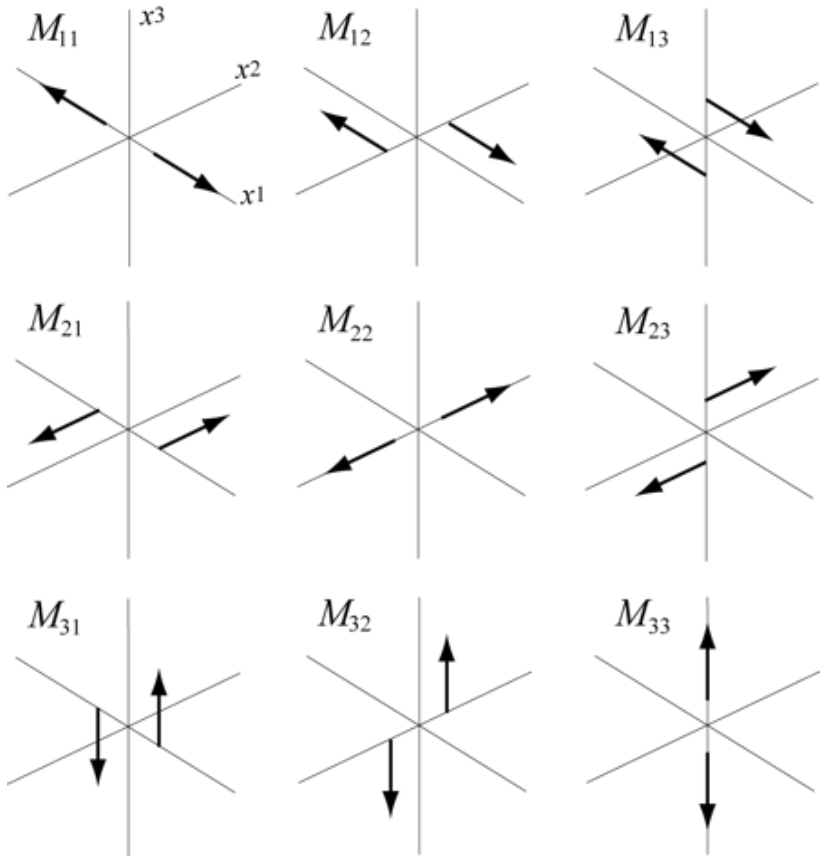
Any time, we can not choose the actual fault plane of two nodal planes with point source. If we want to determine one fault plane, we need to refer to aftershock distribution or tectonic setting.

Seismic Moment tensor

The moment tensor consists of nine single-couples force in the local-source Cartesian coordinate system.

The explosion source described can be modeled by the sum of the three dipole terms, $M_{11} + M_{22} + M_{33}$, with each having equal moment.

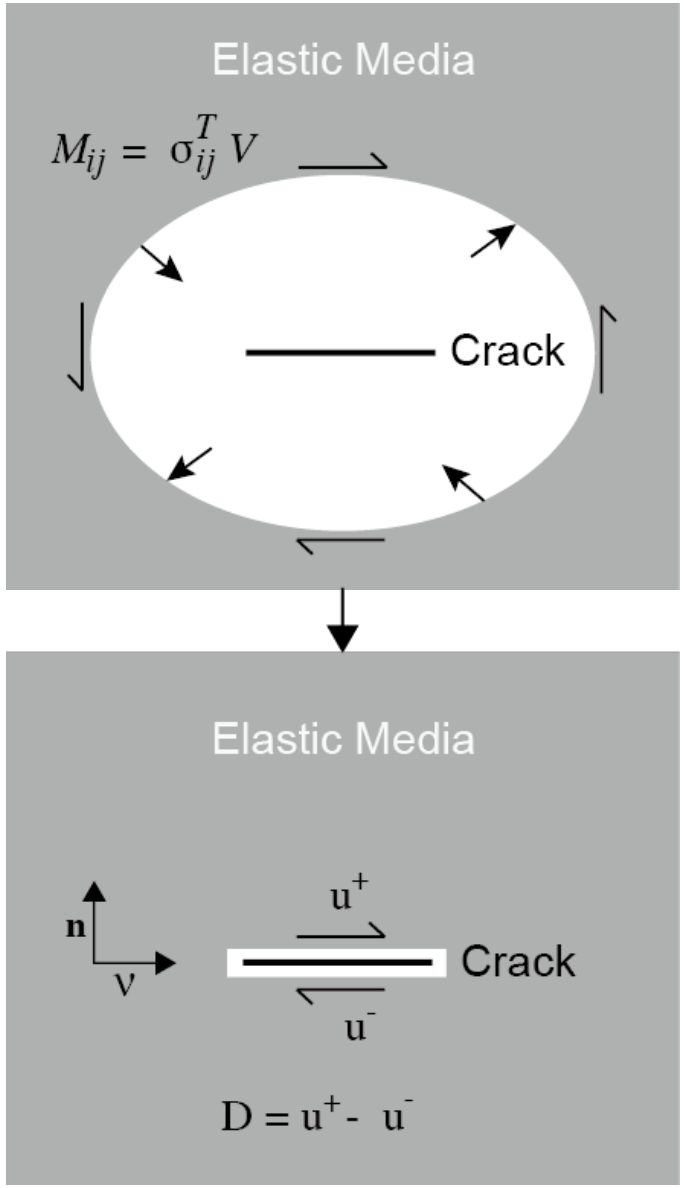
<u>The seismic moment tensor is always symmetric.</u>



Moment tensor and fault motion

Moment tensor and crack

$$M_{ij} = \left[\lambda \left(\sum_{k=1}^{3} v_k n_k \right) \delta_{ij} + \mu \left(v_i n_j + v_j n_i \right) \right] DS$$



Moment tensor and dislication

$$M_{ij} = \left(v_i n_j + v_j n_i \right) \mu DS = \left(v_i n_j + v_j n_i \right) M_0$$

Moment Tensor and Radiation Pattern

Moment tensor and Far-field term

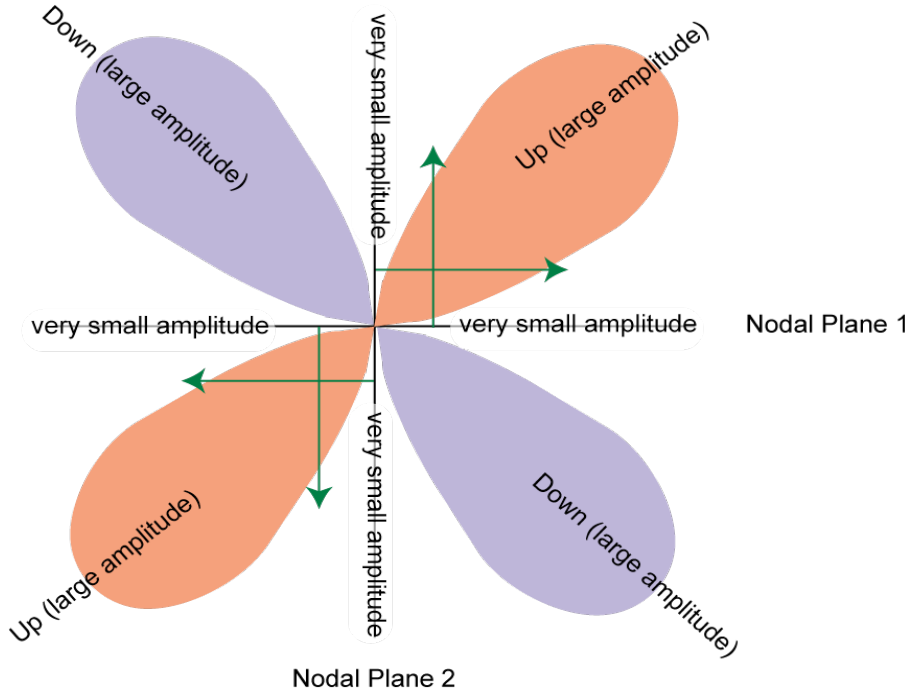
$$U_c(r,\gamma,t) = \frac{R_c}{4\pi\rho c^3}\frac{1}{r}\underbrace{Q(t)}_{\text{Q-effect}} * \underbrace{\dot{M}_0\left(t-\frac{r}{c}\right)}_{\text{Moment-rate function}}$$

Radiation Pattern

$$R_C = \sum_{i=1}^{3}\sum_{j=1}^{3} m_{ij}\gamma_i e_{Cj}$$

P-wave Radiation Pattern

$$R_P = 2\left(\sum_{i=1}^{3} v_i\gamma_i\right)\left(\sum_{j=1}^{3} n_j\gamma_j\right)$$



Radiation Pattern in Nuclear Weapons Testing

Moment tensor components

$$m_{ij} = \delta_{ij} \qquad i, j = 1,2,3$$

P-wave Radiation Pattern

$$R_P = \sum_{i=1}^{3}\sum_{j=1}^{3}\delta_{ij}\gamma_i e_{Pj} = \sum_{i=1}^{3}\sum_{j=1}^{3}\delta_{ij}\gamma_i\gamma_j = \sum_{i=1}^{3}\gamma_i\gamma_i = 1$$

S-wave Radiation Pattern

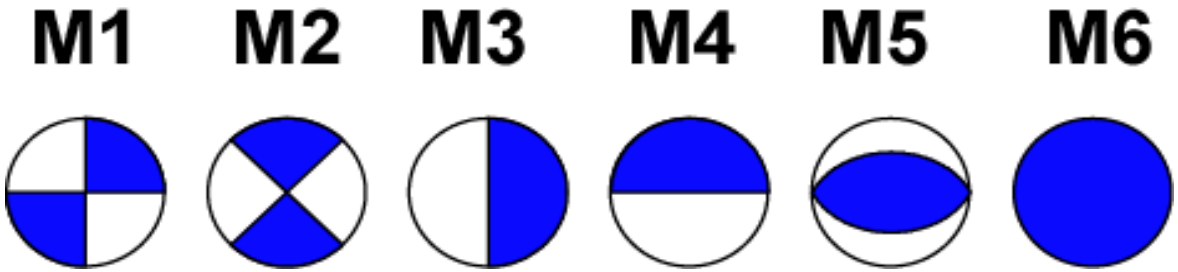
$$R_S = \sum_{i=1}^{3}\sum_{j=1}^{3}\delta_{ij}\gamma_i e_{Sj} = \sum_{i=1}^{3}\gamma_i e_{Si} = 0$$

Basis Moment Tensor

We can treat source and propagation process as linear operators, and the moment tensor can describe double-couple at any time.

It is possible to construct observed waveform by summing the weighted the green's functions for each basis moment tensor.

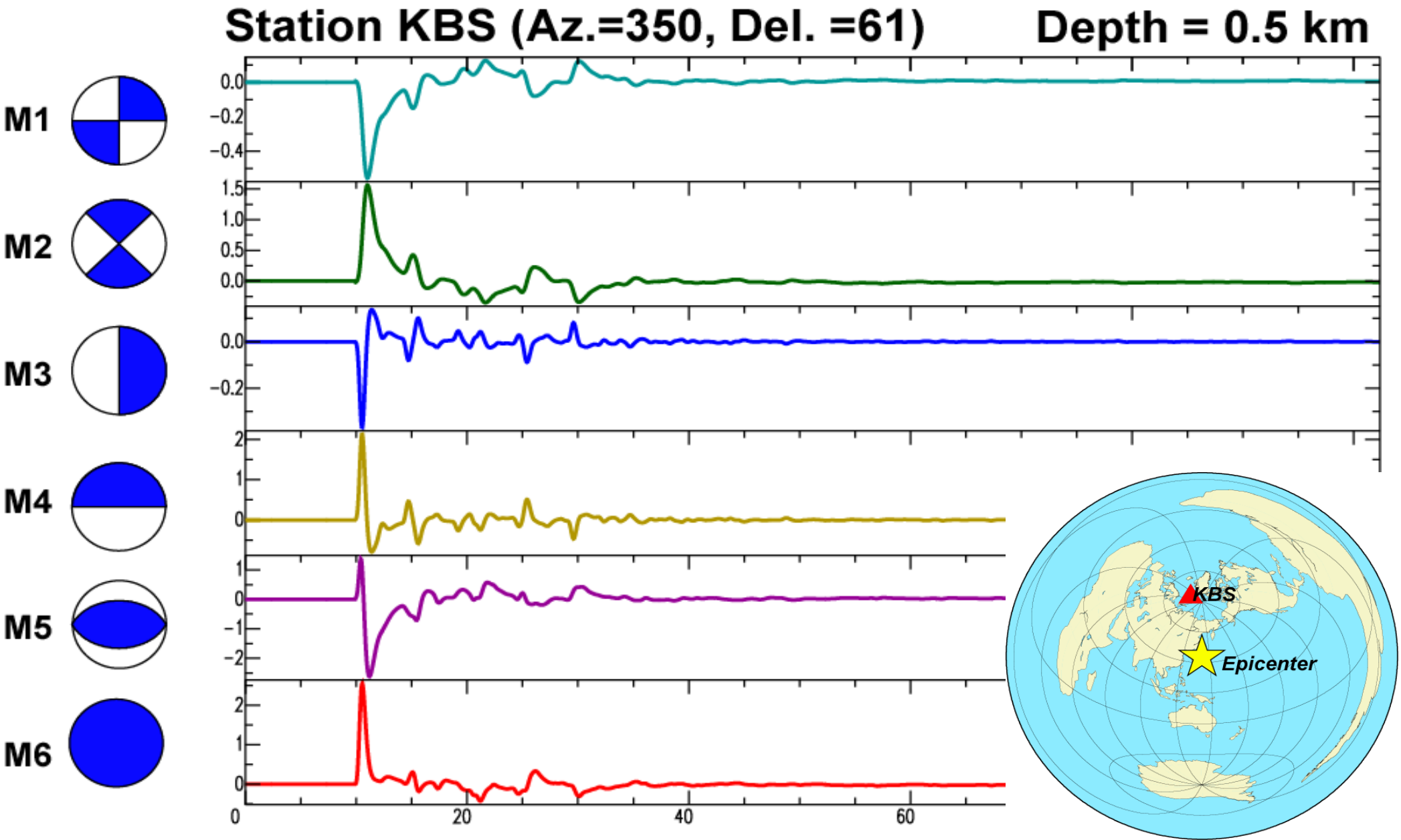
We assume only double-couple model, the number of independent components of moment tensor is five.



Kikuchi and Kanamori (1991, BSSA)

Green's Function for each basis moment tensor

JB structure model (Moho = 33 km)



Green's Function for each basis moment tensor

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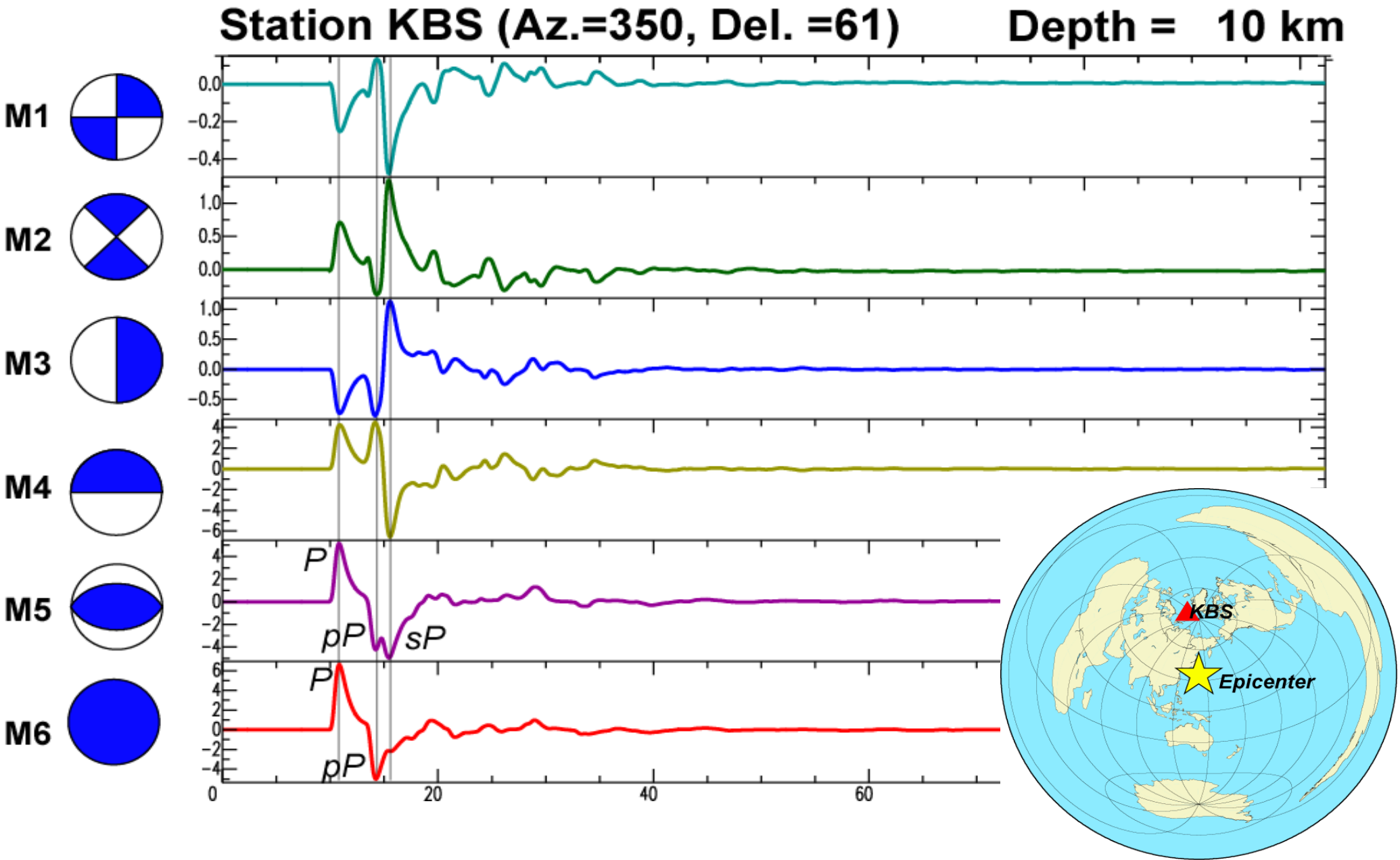
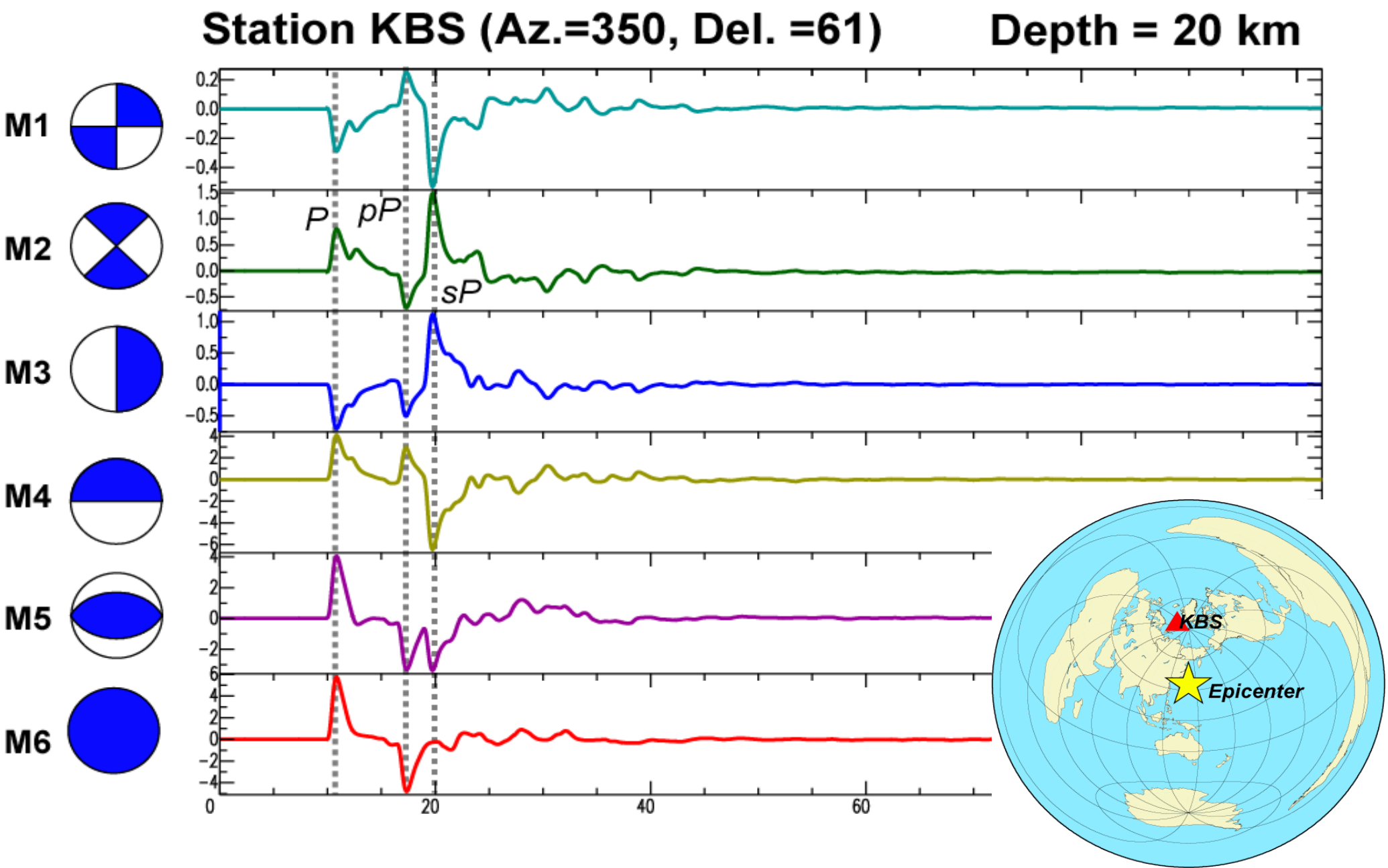


Green's Function for each basis moment tensor



Moment Tensor Inversion

Observed seismic waveform of c component at a station j due to seismic moment release in a source volume V

$$\underbrace{u_{cj}(t)}_{\text{Obs. Waveform}} = \sum_{q=1}^{6} \iiint_v \underbrace{\tilde{G}_{cjq}(t,\xi)}_{\text{Green's function}} * \underbrace{\tilde{M}_q(t,\xi)}_{\text{Moment-rate function}} d\xi + e'_{cj}(t)$$

Assumption 1: **Point source model**, in which we assume the seismic waveform to be radiated from one point.

$$u_{cj}(t) = \sum_{q=1}^{6} \tilde{G}_{cjq}(t,\underbrace{\xi_c}_{\text{Centroid}}) * M_q(t) + e_{cj}(t)$$

with

$$M_q(t) = \iiint_V \tilde{M}_q(t,\xi) d\xi$$

Assumption 2: **One earthquake has one focal mechanism.**

$$M_q(t) = m_q \times \dot{M}_0(t)$$

$$u_{cj}(t) = \sum_{q=1}^{6} m_q \times M_0(t) * \tilde{G}_{cjq}(t, \boldsymbol{\xi}_c) + e_{cj}(t)$$

Applying Low-pass filter

$$d_{cj}(t) = F(t) * u_{cj}(t) = \sum_{q=1}^{6} m_q \times \left[F(t) * M_0(t) \right] * \tilde{G}_{cjq}(t, \boldsymbol{\xi}_c) + e_{cj}(t)$$

$$\simeq \sum_{q=1}^{6} m_q \times \left[F(t) * \underline{S(t)} \right] * \tilde{G}_{cjq}(t, \boldsymbol{\xi}_c) + e_{cj}(t)$$

simple function
(e.q. delta function)

$$d_{cj}(t) = \sum_{q=1}^{6} m_q \times G_{cjq}(t, \boldsymbol{\xi}_c) + e_{cj}(t)$$

with $\quad G_{cjq}(t, \boldsymbol{\xi}_c) = \tilde{G}_{cjq}(t, \boldsymbol{\xi}_c) * F(t)$

Vector Form:

$$\mathbf{d} = \mathbf{A}\left(\xi_c\right)\mathbf{m} + \mathbf{e}$$

The solution of the above matrix equation is obtained by least square approach if we assume the centroid location.

$$\hat{\mathbf{m}} = \left[\mathbf{A}^T\left(\xi_c\right)\mathbf{A}\left(\xi_c\right)\right]^{-1}\mathbf{A}^T\left(\xi_c\right)\mathbf{d}$$

We can estimate optimal the centroid using the grid-search method, which minimizes normalized L2-norm as

$$\left\|\mathbf{d} - \mathbf{A}\left(\xi_c\right)\hat{\mathbf{m}}\right\|_2 \Big/ \left\|\mathbf{d}\right\|_2 \Rightarrow \min$$

L2-norm: $\left\|\mathbf{x}\right\|_2 = \left(\sum_{n=1}^{N} x_n^2\right)^{1/2}$

Moment Tensor to the two Fault Planes

Transformation from moment tensor to the two fault planes.

First we obtain the eigenvectors (**t**, **b**, **p**) of moment tensor.

$$\begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} = \begin{pmatrix} \mathbf{t} & \mathbf{b} & \mathbf{p} \end{pmatrix} \begin{pmatrix} M_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -M_0 \end{pmatrix} \begin{pmatrix} \mathbf{t} & \mathbf{b} & \mathbf{p} \end{pmatrix}^T$$

Nest, we obtain fault vector (**n:** unit normal vector to fault plane, **d:** unit slip vector) form the eigenvectors (**t**, **b**, **p**) using the equation:

$$\text{One fault plane model}: \mathbf{n} = \frac{1}{\sqrt{2}}(\mathbf{t}+\mathbf{p}),\ \mathbf{d} = \frac{1}{\sqrt{2}}(\mathbf{t}-\mathbf{p})$$

or

$$\text{Other fault plane model}: \mathbf{n} = \frac{1}{\sqrt{2}}(\mathbf{t}-\mathbf{p}),\ \mathbf{d} = \frac{1}{\sqrt{2}}(\mathbf{t}+\mathbf{p})$$

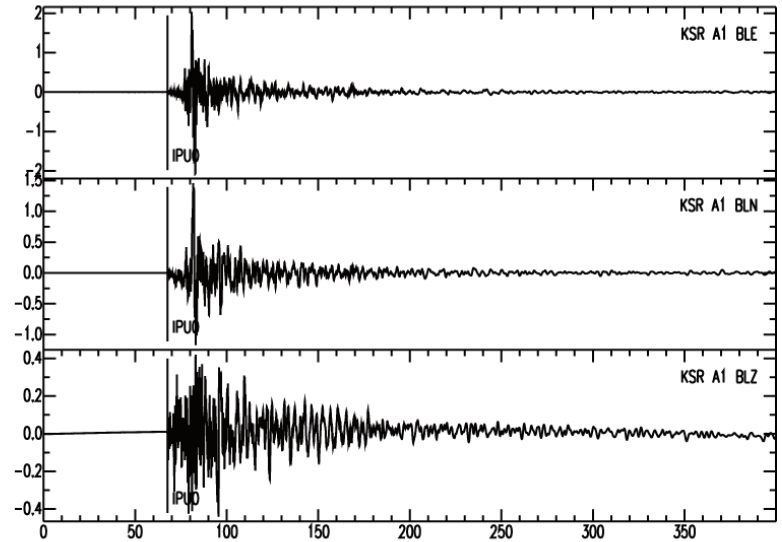
Moment tensor inversion for middle earthquake using local seismic network

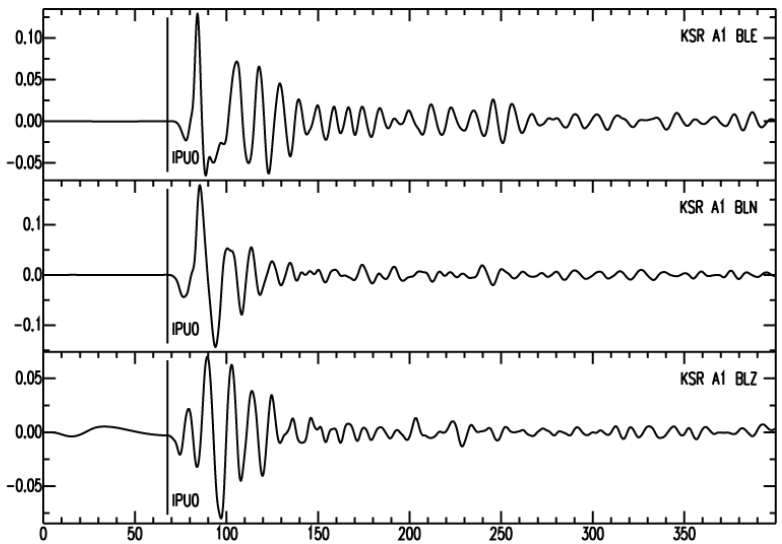
Duration of source time function of middle size < 10 sec

If we apply a low pass filter, we can neglect source time function. In my program for local seismic network, we neglect effect of source time function.

Grid Search parameter:

only location of hypocenter





Moment tensor inversion for middle earthquake using local seismic network

Step 1:

Proceeding for near-source data set

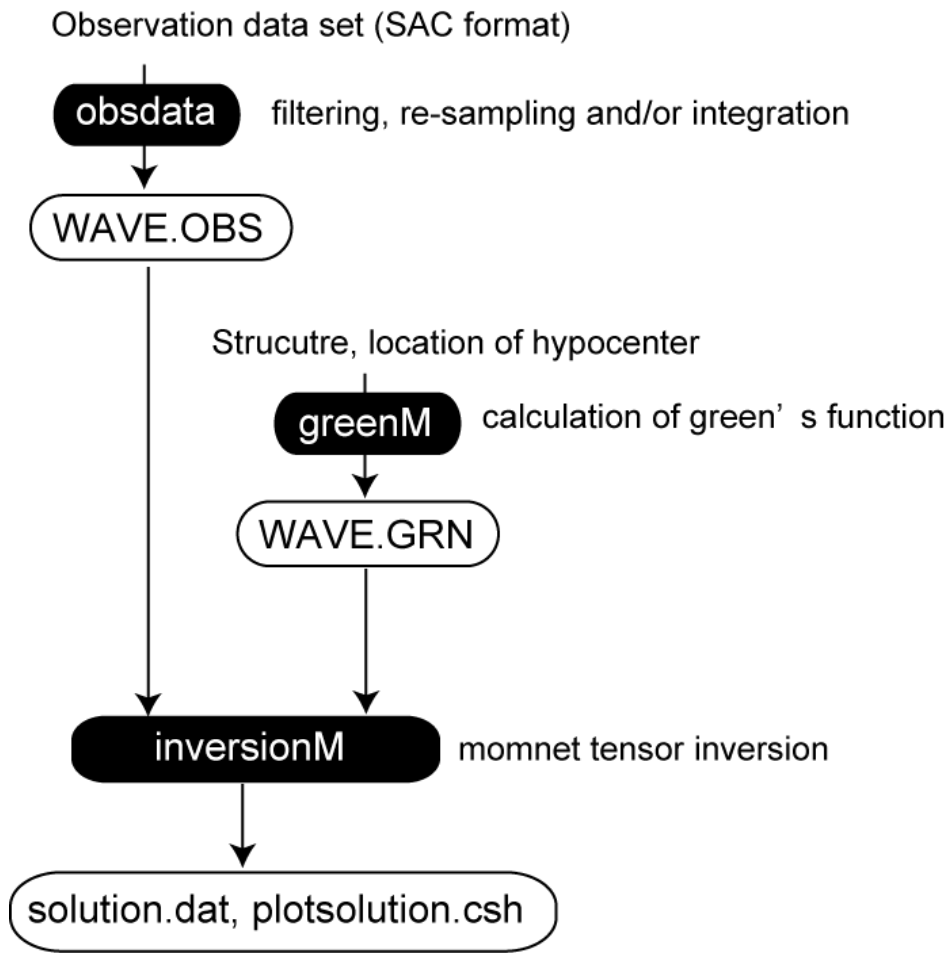
filtering, re-sampling

Step 2:

Calculation of Green's Function

Step 3:

Inversion



Moment tensor inversion for middle earthquake using local seismic network

If epicenter = horizontal location of centroid of moment release,

grid search paramter

-> only depth.