

Earthquake Focal Mechanism

Yuji Yagi

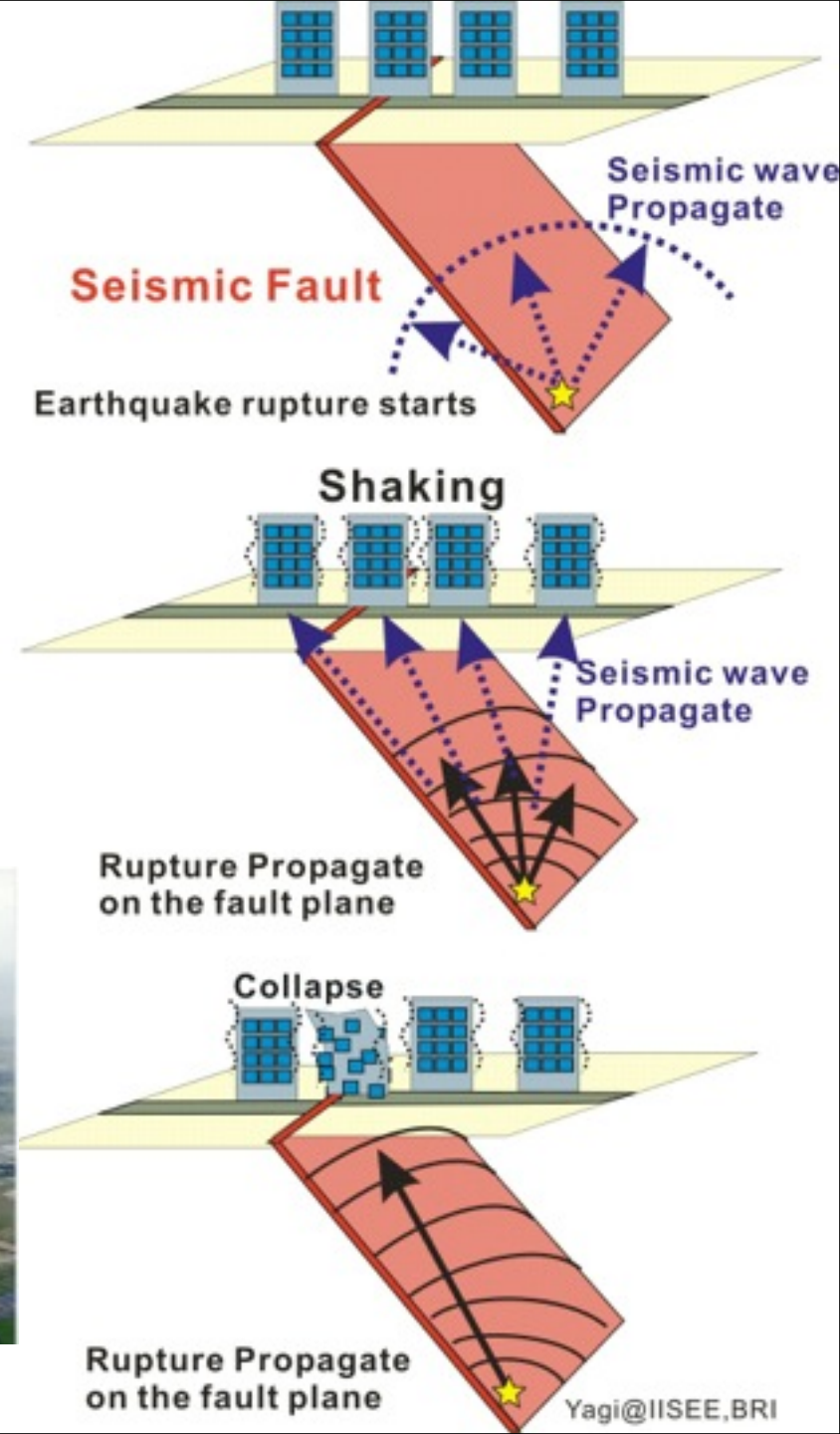
(University of Tsukuba)

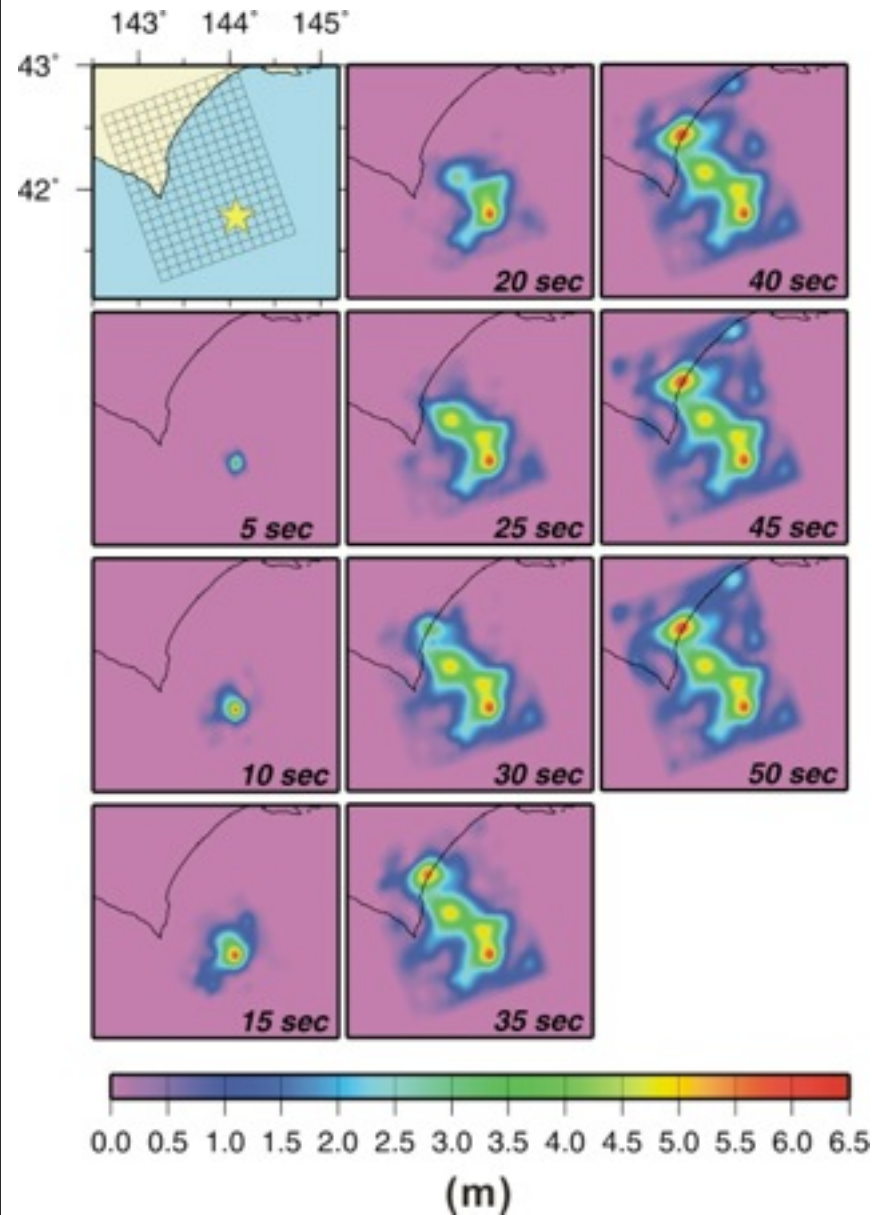
Earthquake

Earthquake is a term used to describe both **failure process** along a fault zone, and the resulting **ground shaking** and radiated seismic energy caused by the slip, or by volcanic or magmatic activity, or other sudden stress changes in the earth.



Surface rupture (Taken by Prof.Abe, the University of Tokyo)



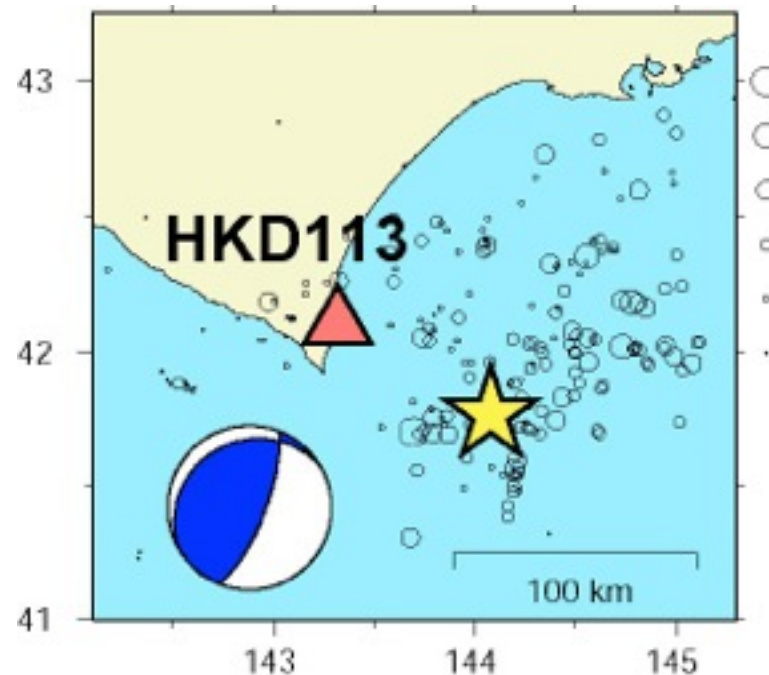


2003 Tokachi-oki, Japan earthquake

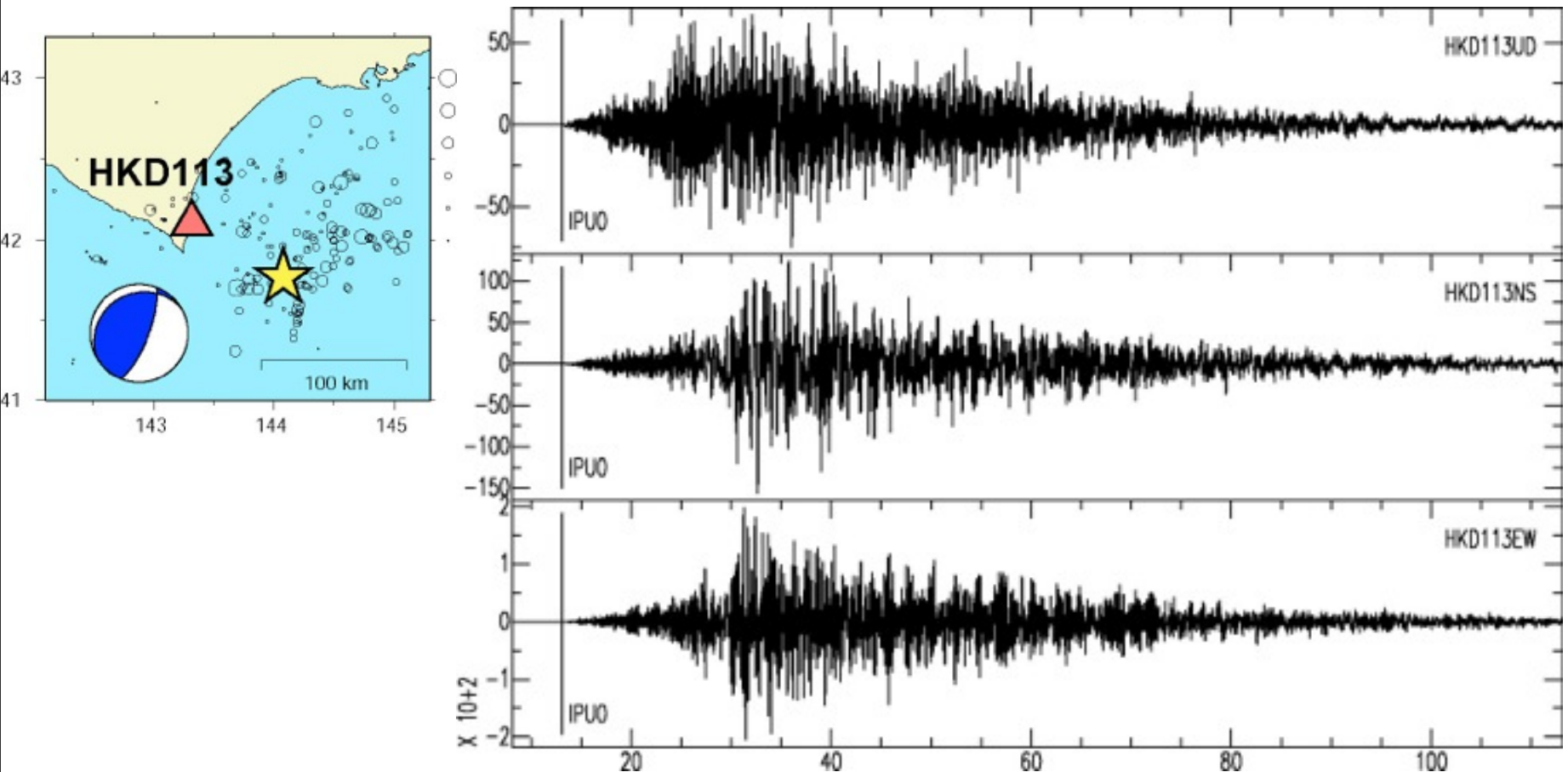
Rupture Process of Large Earthquake

Ground Motion

When an earthquake occurs, the ground shakes. The motion of ground is given by displacement $u(t)$, velocity $v(t)$, acceleration $a(t)$, as a function of time, t , in 3 directions, usually, **UD**, **NS**, and **EW**.



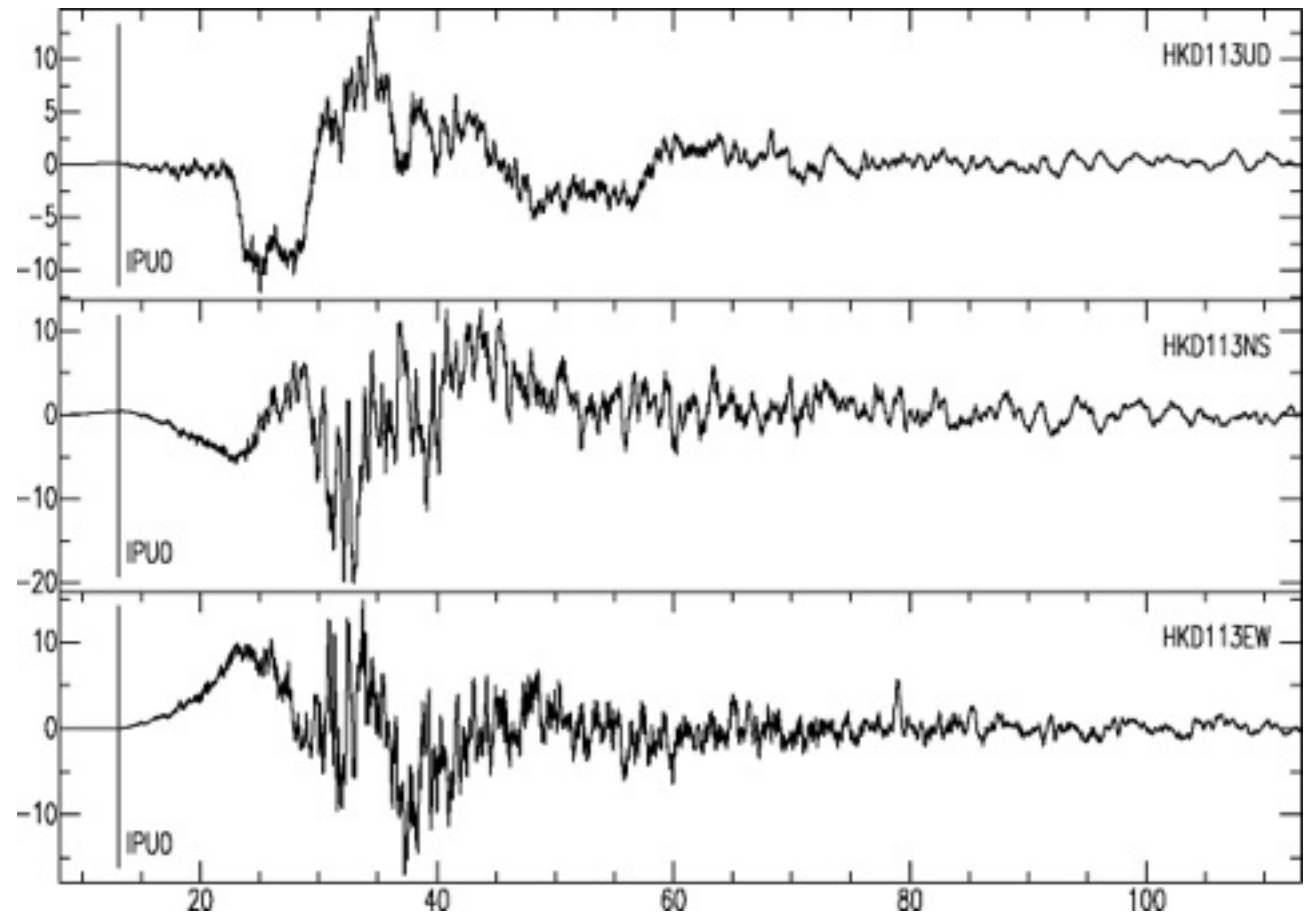
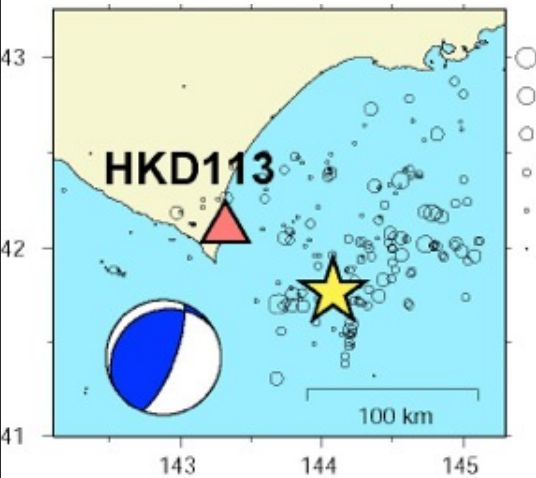
Acceleration



(unit: gal = cm/s/s)

2003 Tokachi-oki, Japan earthquake
station HKD113 (K-net)

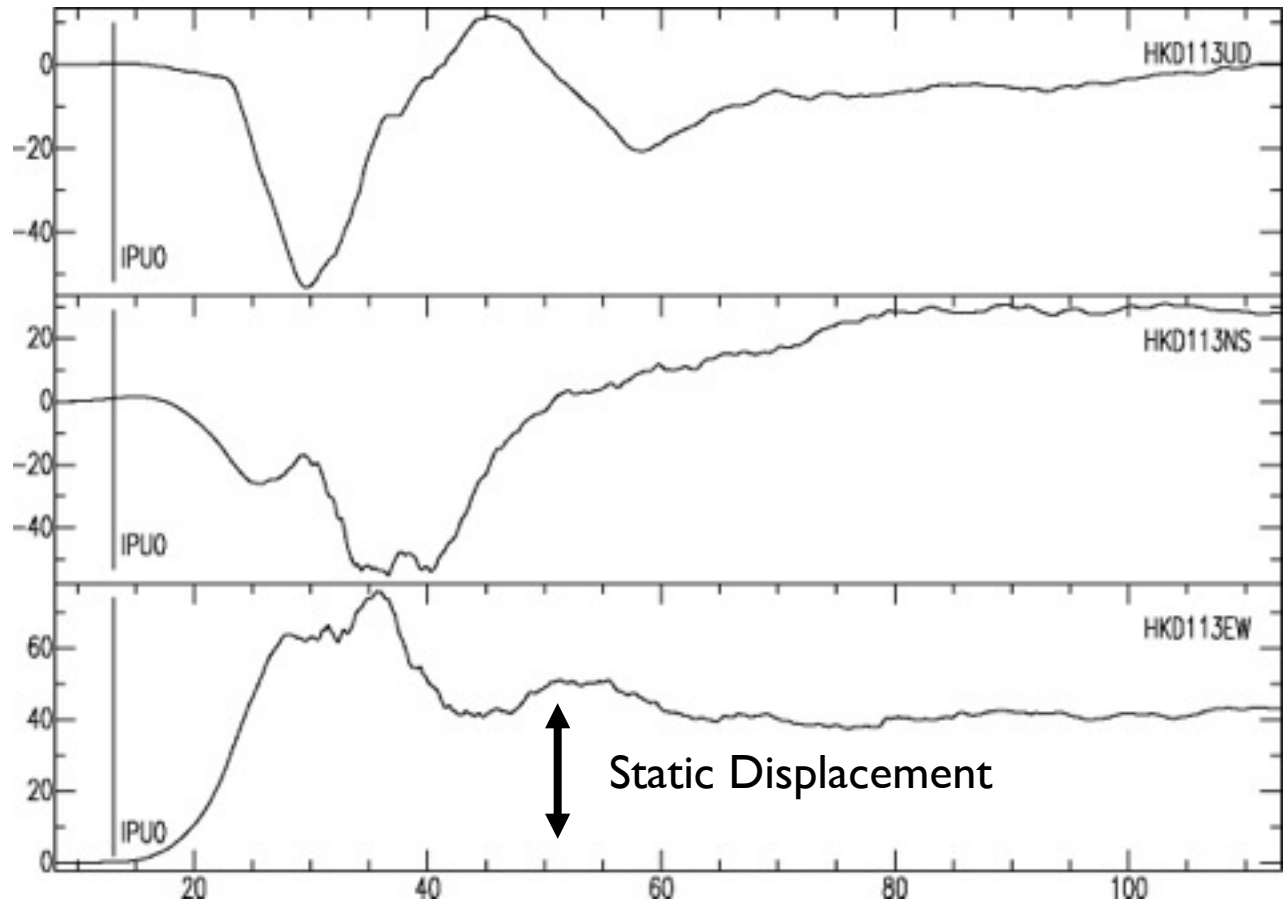
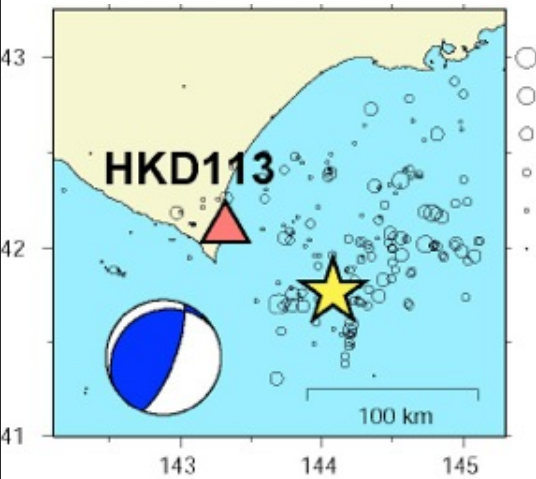
Velocity



(unit: cm/s)

2003 Tokachi-oki, Japan earthquake
station HKD113 (K-net)

Displacement

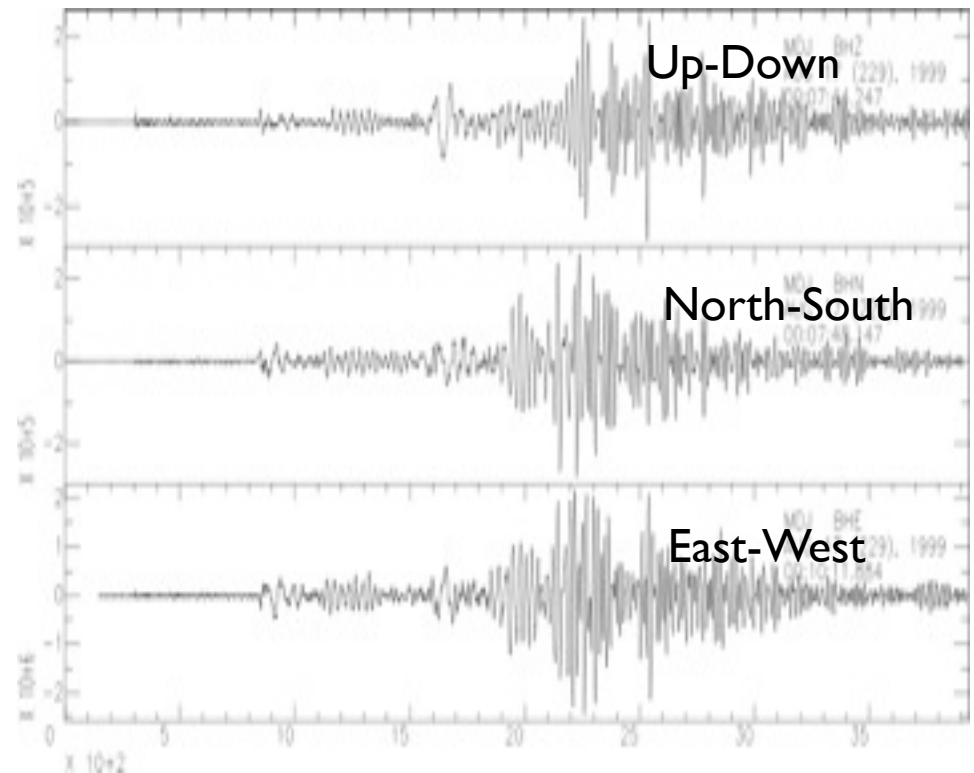
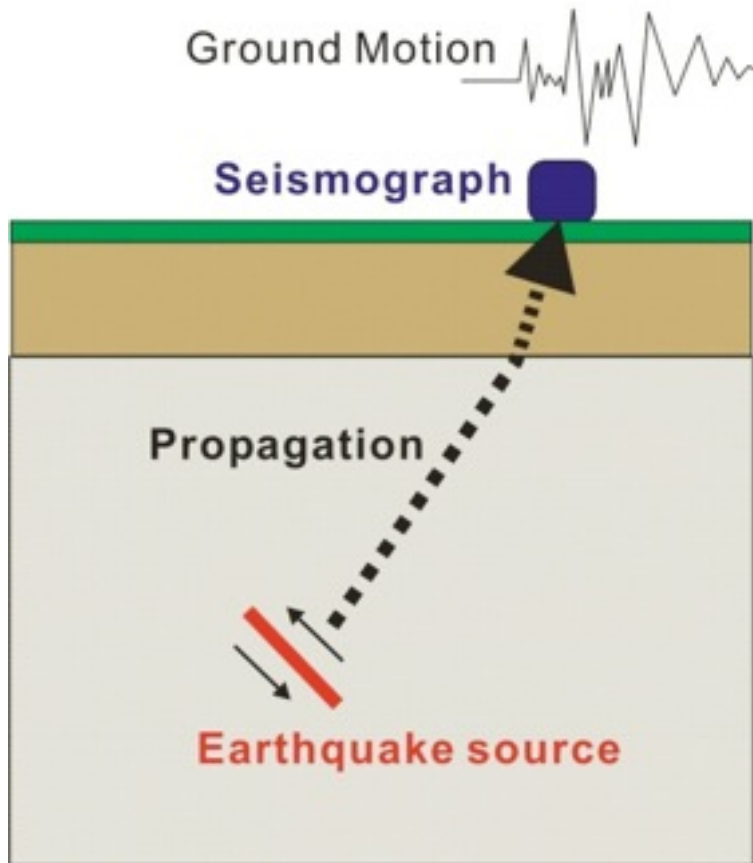


(unit: cm)

2003 Tokachi-oki, Japan earthquake
station HKD113 (K-net)

Seismology

Seismology is the study of earthquakes and the Earth using seismic waves.



E.G.) MDJ station for 1999 Turkey earthquake

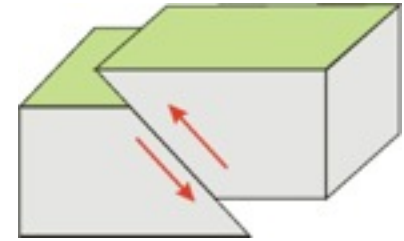
Seismology for source

From recordings of earthquake-generated waves, information about the earthquake source may be derived, including its magnitude, location, time of occurrence, its orientation, and movement on the fault.



Surface rupture in 1999 Taiwan, Chi-Chi earthquake (Taken by Prof.Abe, the University of Tokyo)

Earthquake Source



When earthquake occur, sudden rupture propagate along faults.

Since rupture velocity and slip acceleration rate are high, the large earthquake destroy near cities.

To assess at damage of earthquakes, it is important to understand the nature of earthquake.

If we can provide damage distribution with government office, it is easy to work out countermeasures.



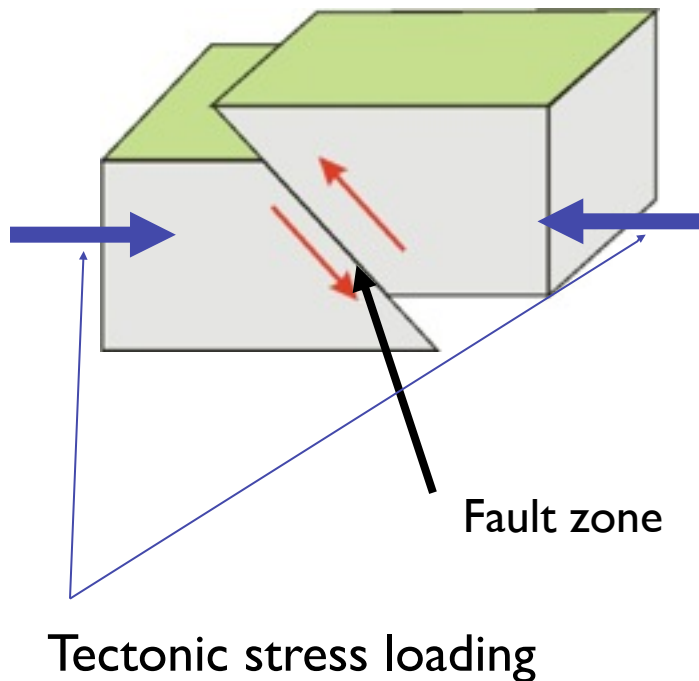
Surface rupture in 1999 Taiwan, Chi-Chi earthquake (Taken by Prof. Abe, the University of Tokyo)

Source Parameters

- Hypocenter (Latitude, Longitude, Depth)
- Origin Time (Start time of earthquake)
- Magnitude (Size of earthquake)
- Faulting Type (focal mechanism)
- Faulting Size (Length, Wide and Dislocation)
- Seismic moment (Size of earthquake)
- Stress Drop (Shear Stress Change)
- Source Process (Rupture Process)

Why source mechanism?

The source mechanism also guide you in the state of the tectonic stress field and location of the weak zone (fault zone).



Surface rupture in 1999 Taiwan, Chi-Chi earthquake (Taken by Prof. Abe, the University of Tokyo)

this lecture

Earthquake



Collection seismic waveform



Determination of hypocenter and origin time



Determination the Focal mechanism



Determination the source process



The determination method of focal mechanism

Using polarity of P-wave fast motion

Using waveform inversion

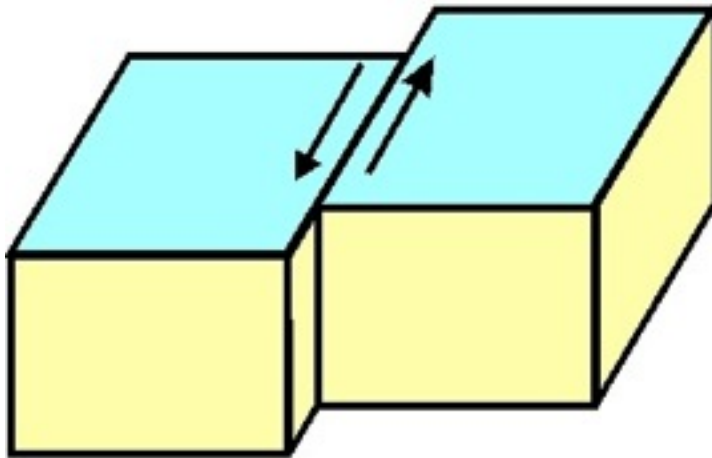
Terms and Fault Plane Parameters

Faulting can be classified three type.

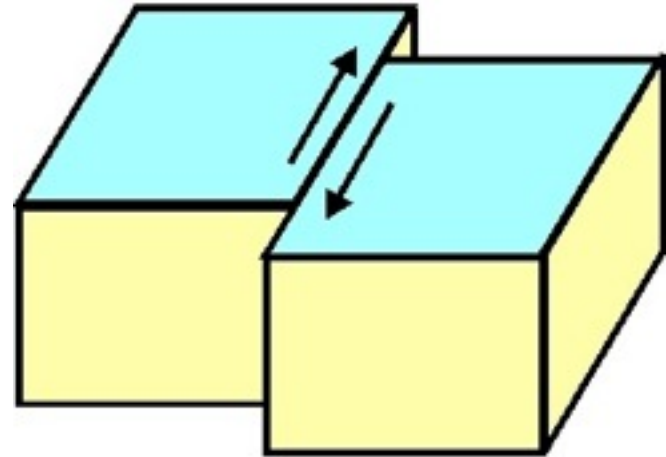
- 1) Strike Slip faulting (right- and left-lateral)
- 2) Normal faulting
- 3) Reverse faulting

Strike slip faulting

Left-lateral strike slip faulting



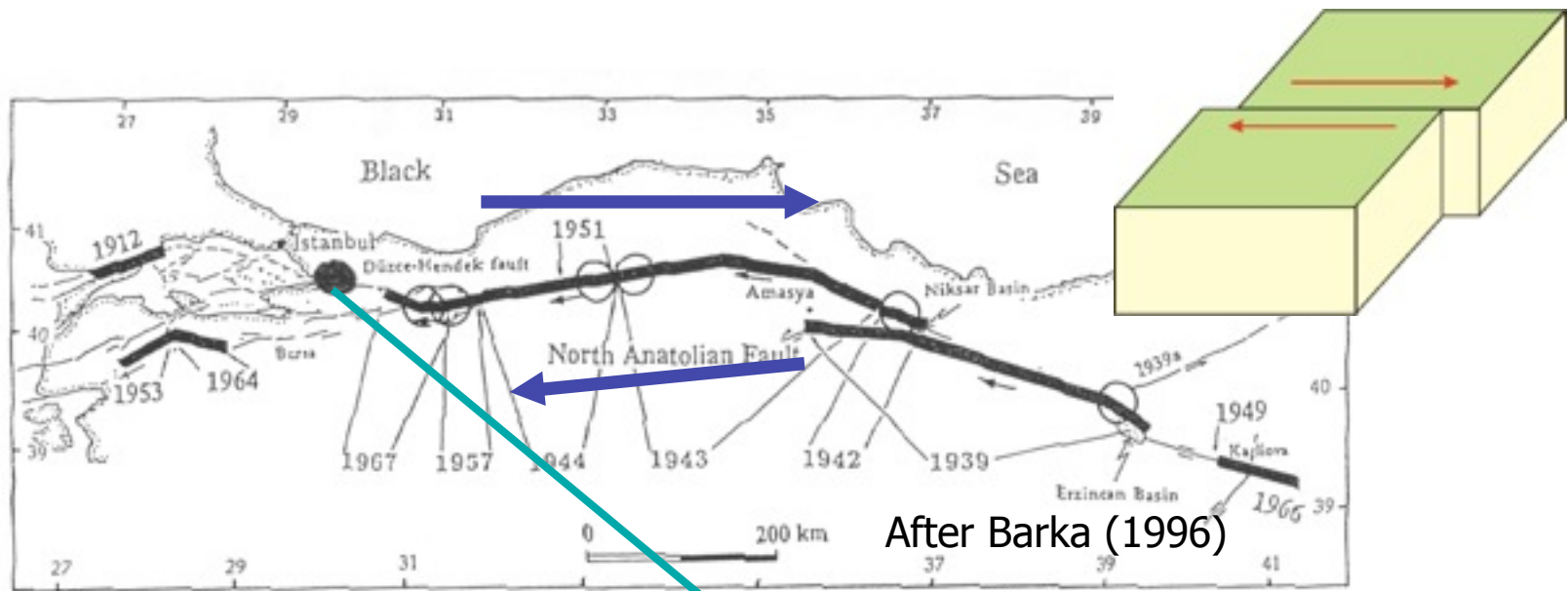
Right-lateral strike slip faulting



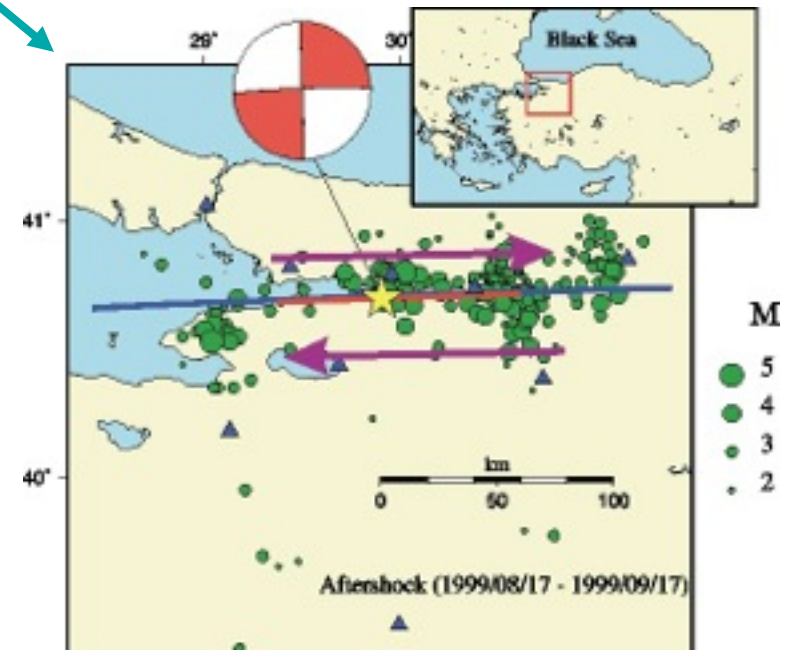
Pull left-side block -> Left-lateral strike slip faulting

Pull right-side block -> Right-lateral strike slip faulting

Strike slip faulting is often observed in intra-plate and transform faulting zone (e.g. the North Anatolian fault zone)



After Barka (1996)

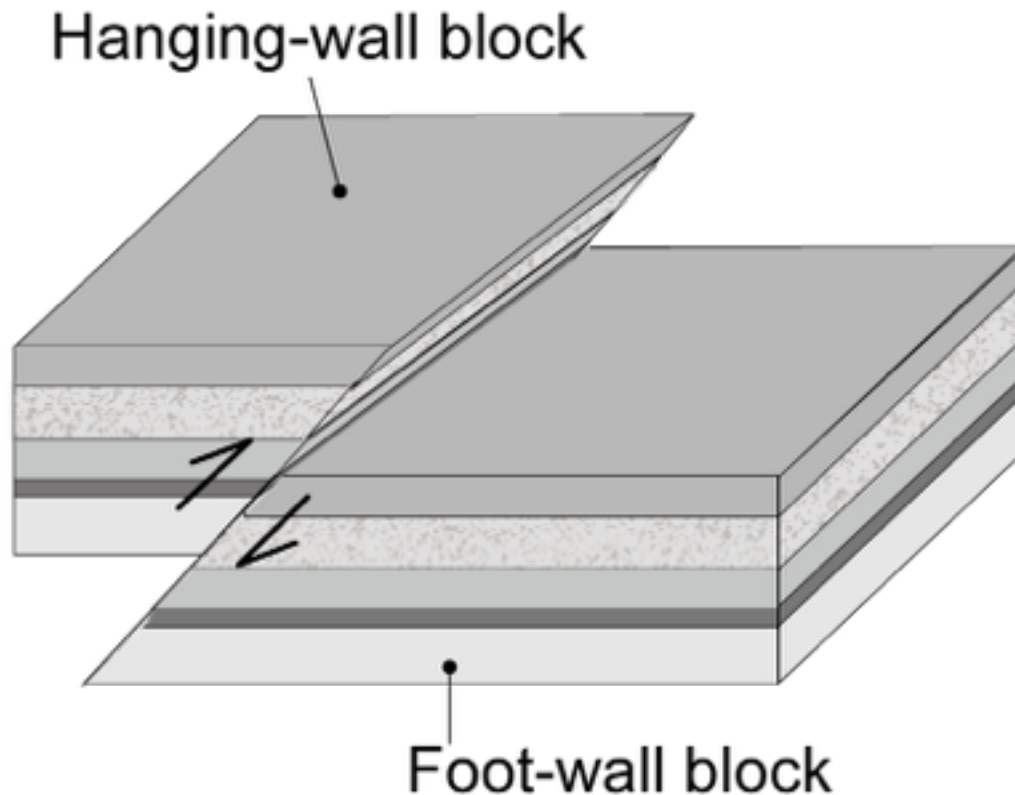


1999 Golcuk-Kocaeli, Turkey, Earthquake

(example)

The right-lateral strike-slip
North Anatolian fault zone

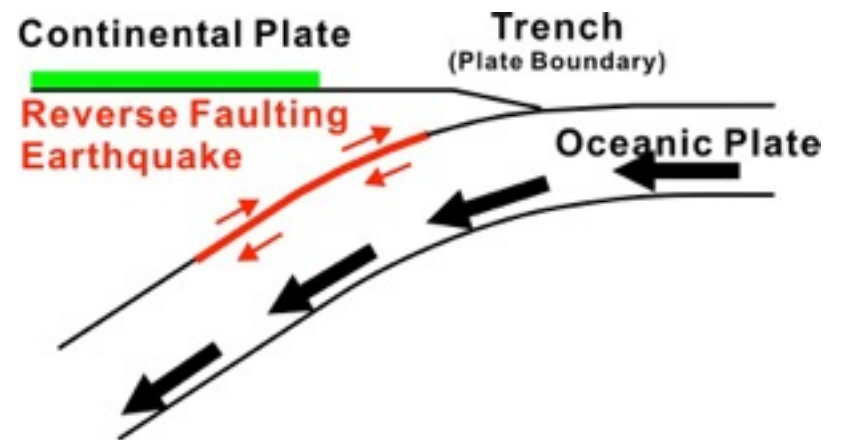
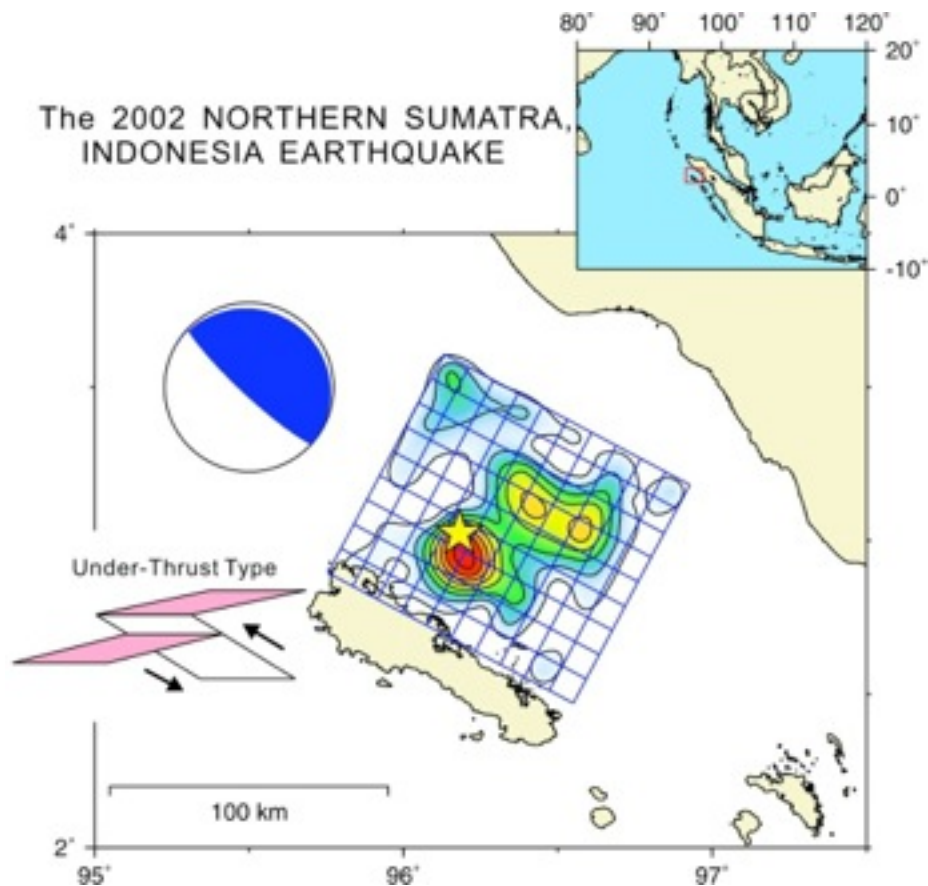
Reverse faulting



The hanging-wall block of the fault slips move upward in relation to the foot-wall block.

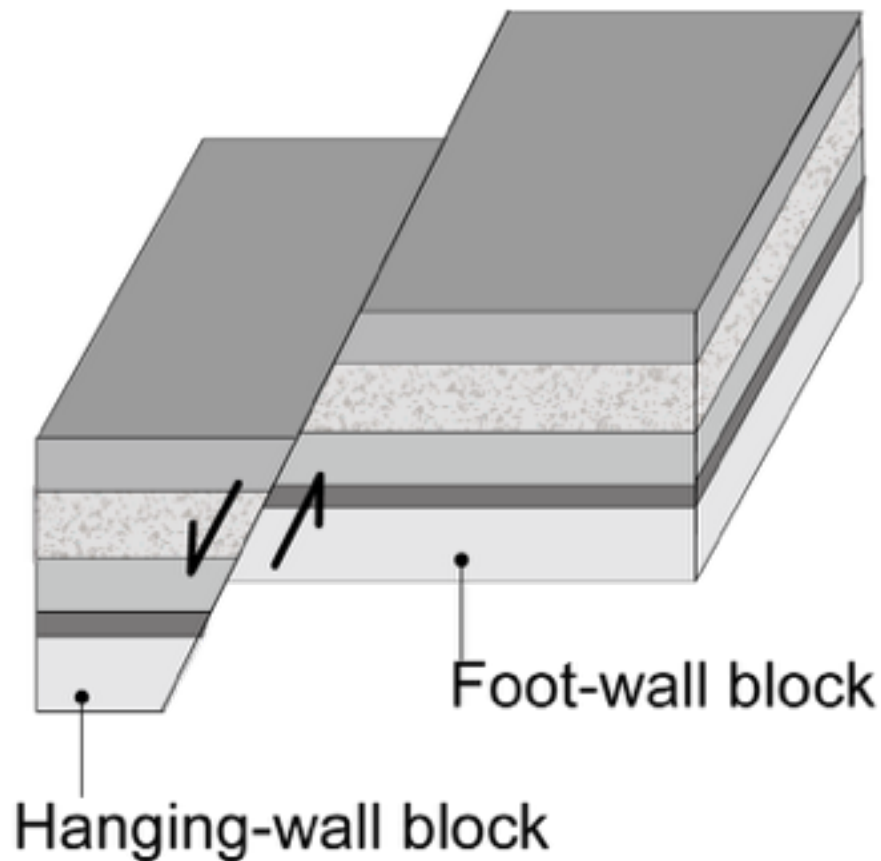
Reveres faulting is often observed in subduction zone (e.g. Japan, Sumatra ..etc).

We often call the reveres faulting with low angle “thrust faulting”).



2002 Sumatra, Indonesia, earthquake

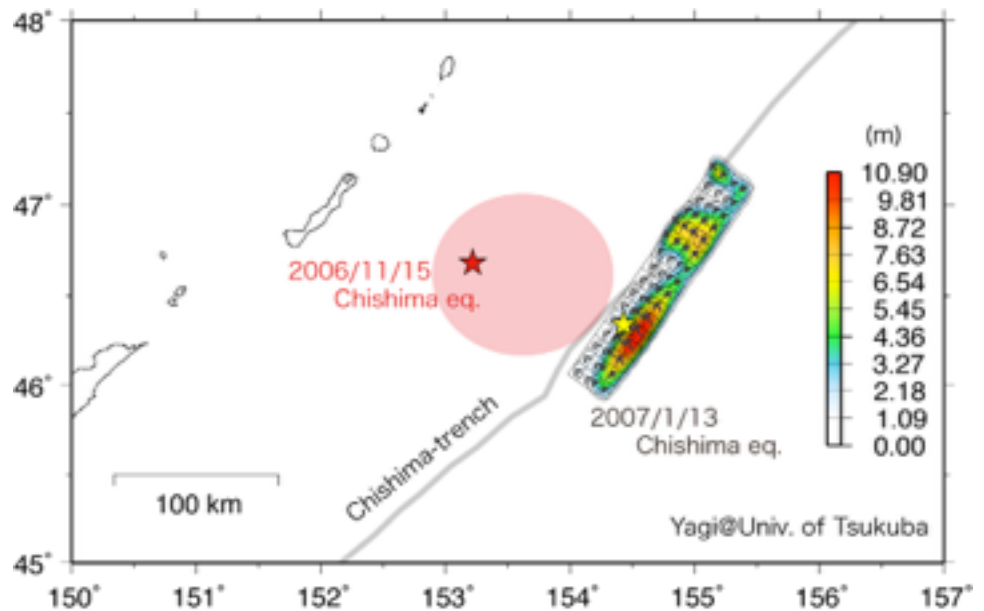
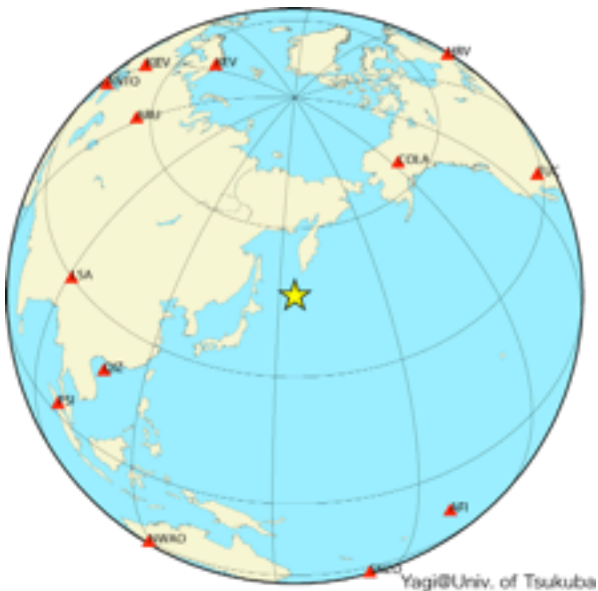
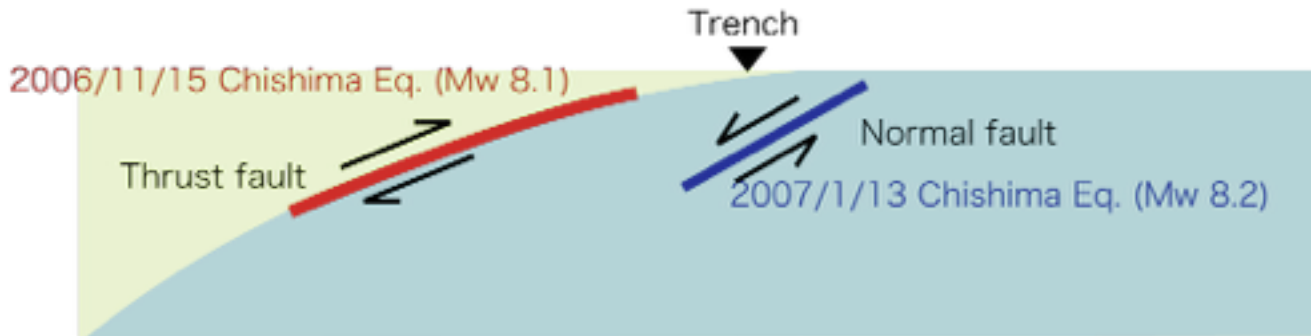
Normal Faulting



The hanging-wall block of the fault slips move downward below the foot-wall block.

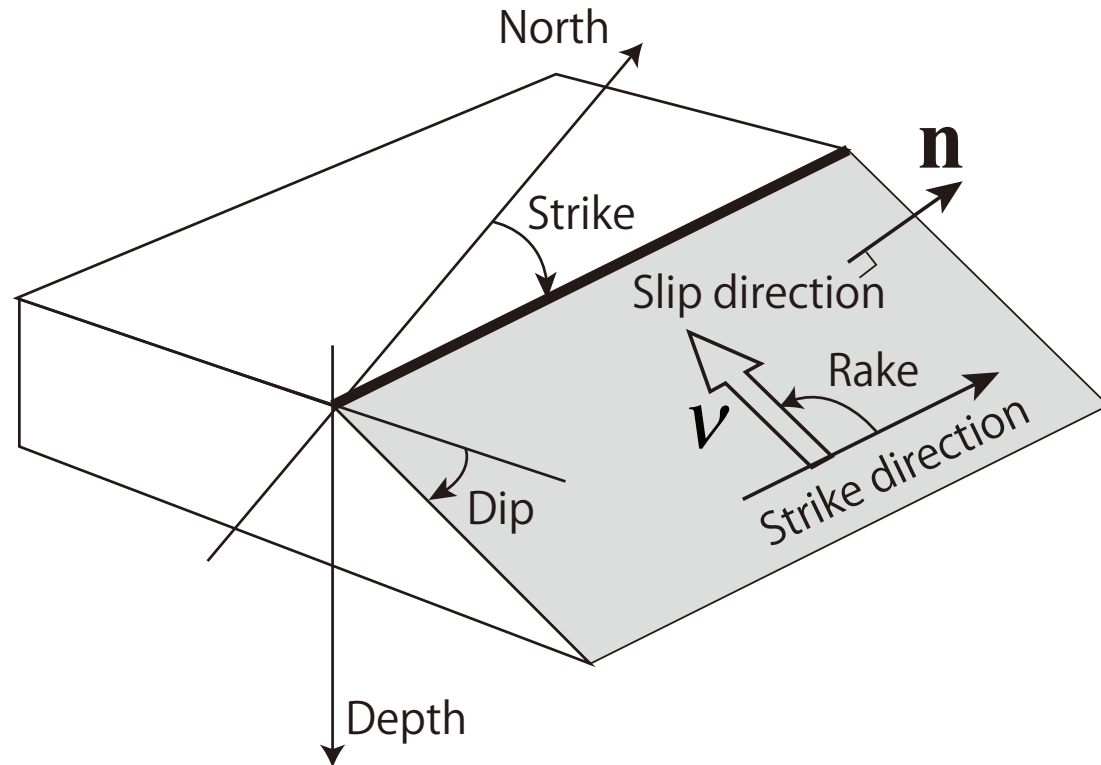
Normal faulting is often observed in intra-plate (e.g. in-slab)

Normal fault (eq. 2007/1/13 Chishima Eq.)



Fault plane parameters

To represent *fault plane* and *fault slip*, we used (*strike, dip*) and (*rake*), respectively.



Strike: The direction of the surface intersection of the fault measured clockwise from north (0 ~ 360)

Dip: A slop angle of the foot-wall block measured clockwise from horizontal (0 ~ 90)

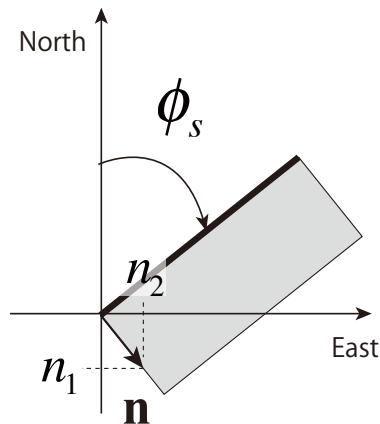
Rake (or Slip): The direction of fault movement measured counterclockwise from strike and slip direction (-180 ~ 180)

Fault plane parameters

Strike

$$\tan[180 - (\phi_s + 90)] = -\frac{n_2}{n_1}$$

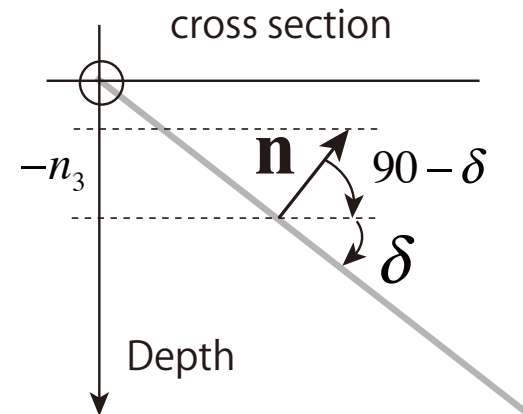
$$\therefore \phi_s = \arctan\left(-\frac{n_1}{n_2}\right)$$



Dip

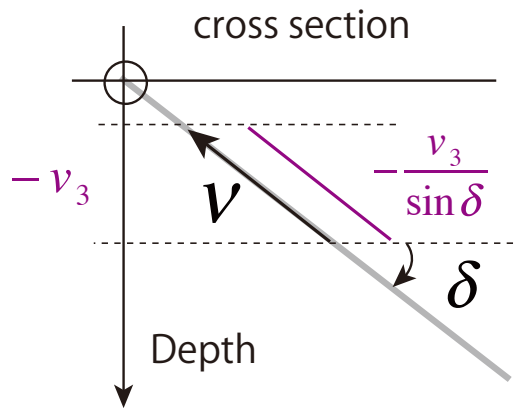
$$\sin(90 - \delta) = \sin(\delta - 90) = \cos \delta = -\frac{n_3}{|\mathbf{n}|}$$

$$\therefore \delta = \arccos(-n_3)$$

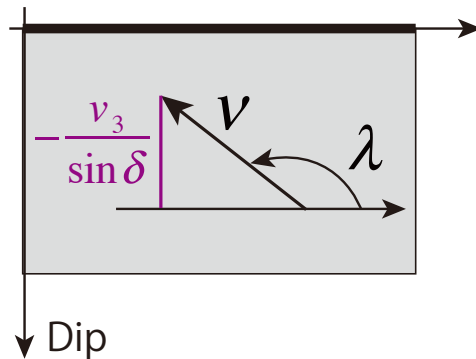


Fault plane parameters

Rake



Strike



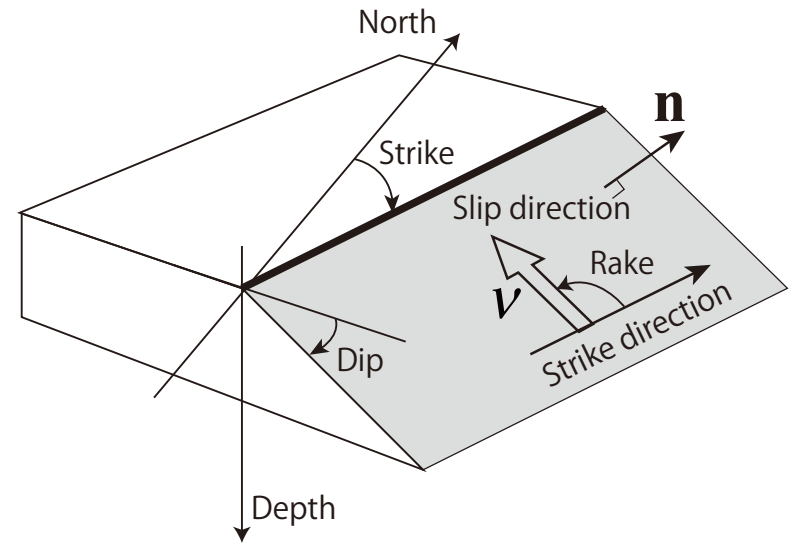
$$\sin(180 - \lambda) = -\sin(\lambda - 180) = \frac{-v_3 / \sin \delta}{|\mathbf{d}|}$$

$$\therefore \lambda = \arcsin\left(-\frac{v_3}{\sin \delta}\right)$$

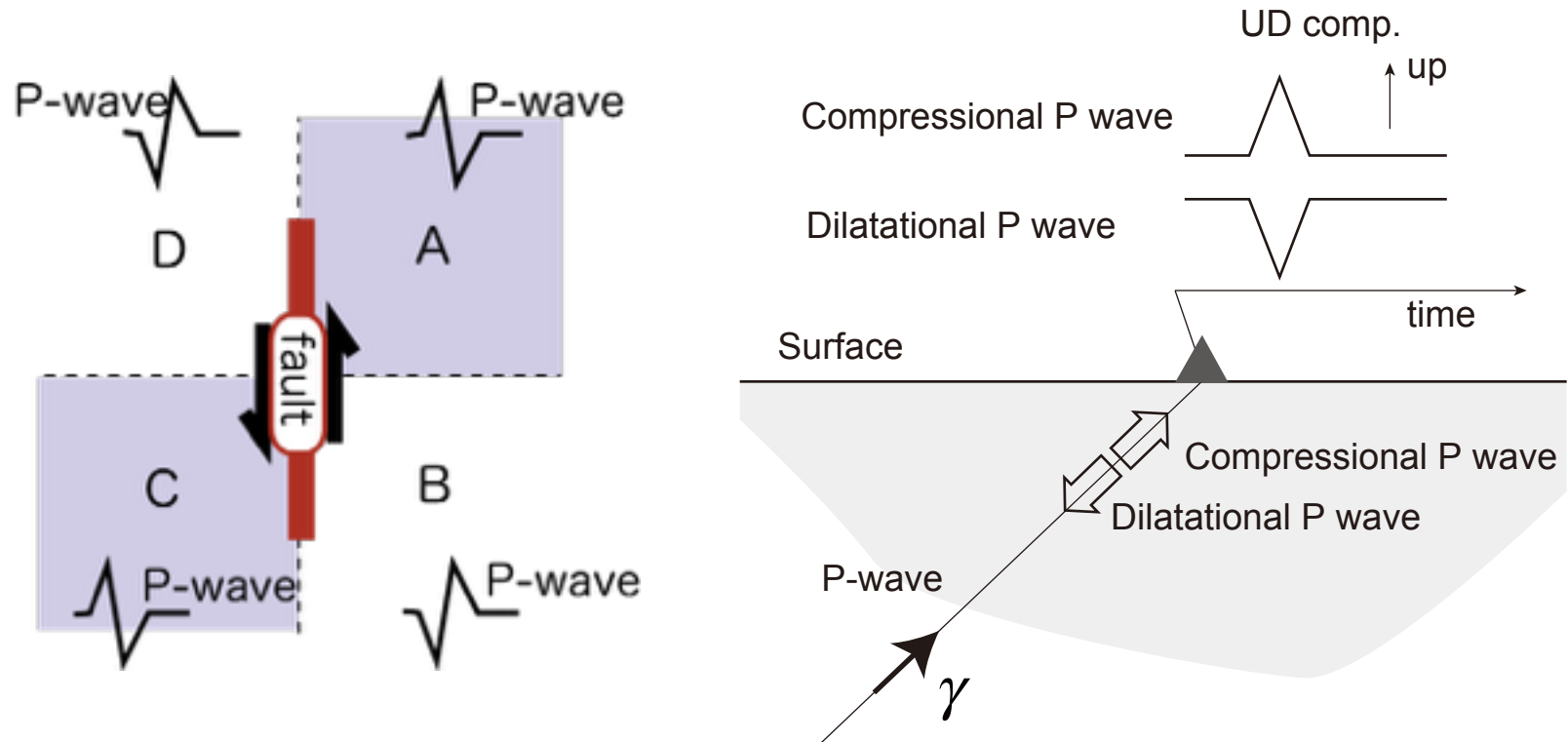
Fault plane parameters

$$\mathbf{n} = \begin{pmatrix} -\sin \delta \sin \phi_s \\ \sin \delta \cos \phi_s \\ -\cos \delta \end{pmatrix}$$

$$\mathbf{v} = \begin{pmatrix} \cos \lambda \cos \phi_s + \cos \delta \sin \lambda \sin \phi_s \\ \cos \lambda \sin \phi_s - \cos \delta \sin \lambda \cos \phi_s \\ -\sin \delta \sin \lambda \end{pmatrix}$$

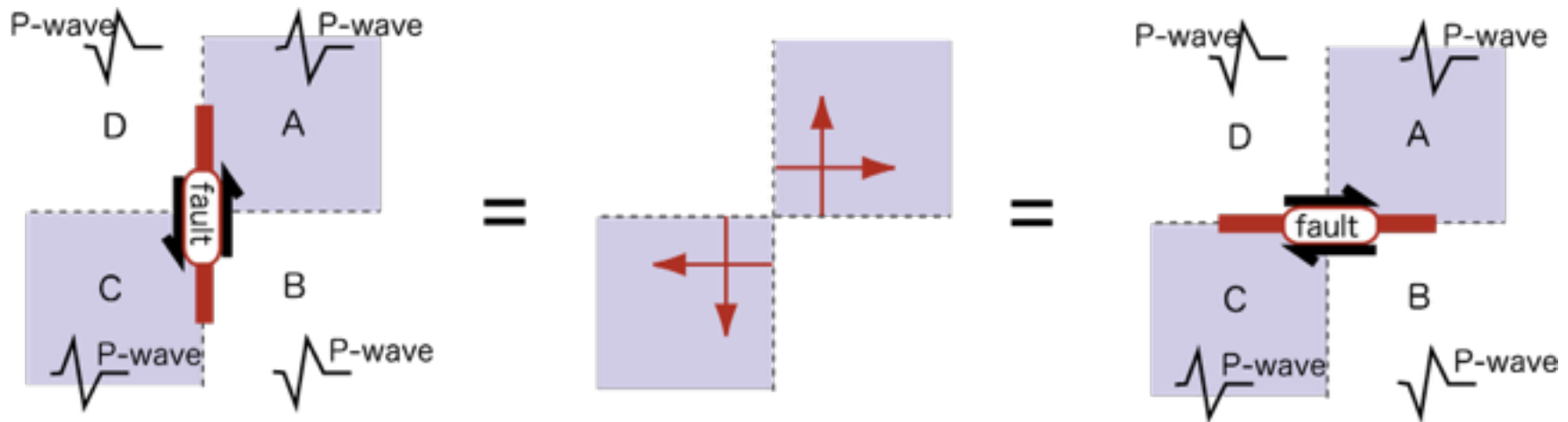


Shear Faulting and P-wave



The polarity of initial P-wave pulse from an earthquake takes either of two opposite polarities: compressional (up or pushed) and dilatational (down or pulled)

Double Couple Model



The double couple model has two pairs of single couple.

This model don't have total force and torque. Thus this model have net balanced the moment.

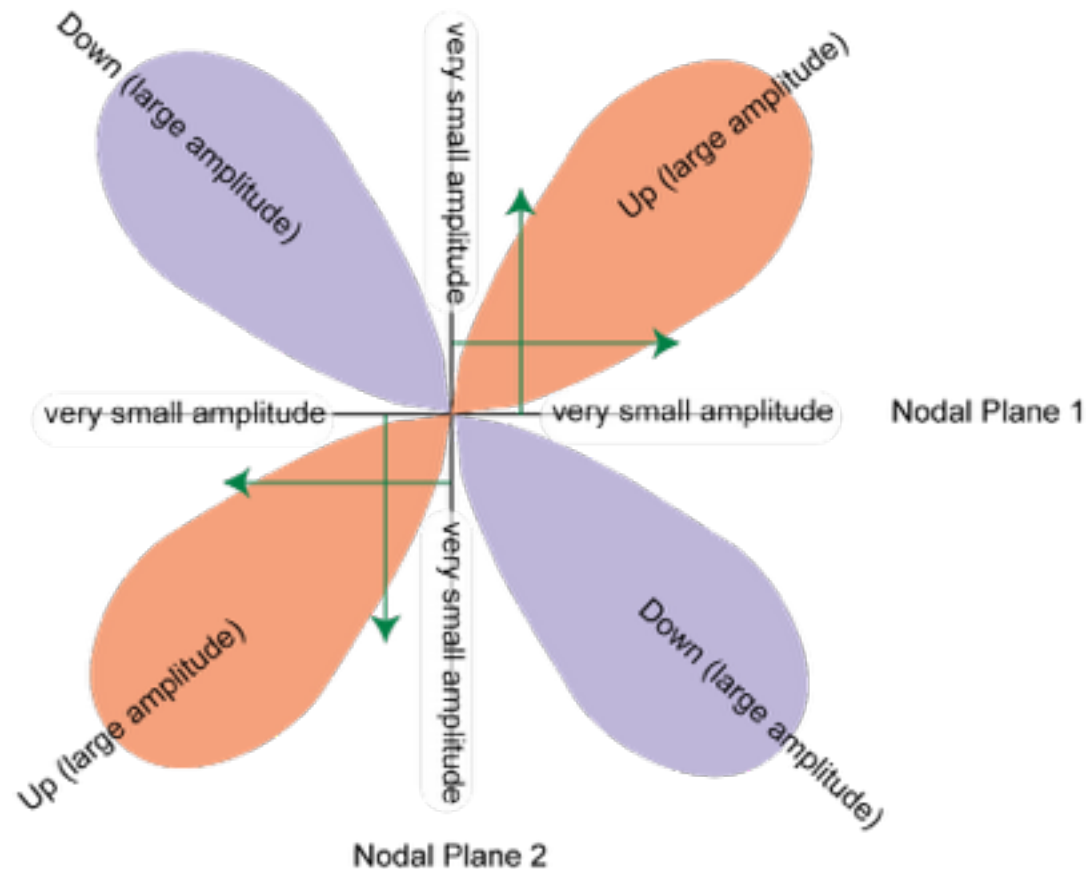
Intrinsic problem

Any time, we can not choose the actual fault plane of two nodal planes with point source. If we want to determine one fault plane, we need to refer to aftershock distribution or tectonic setting.

Radiation Pattern of P-wave

The P-wave radiation from a source have four-lobed pattern.

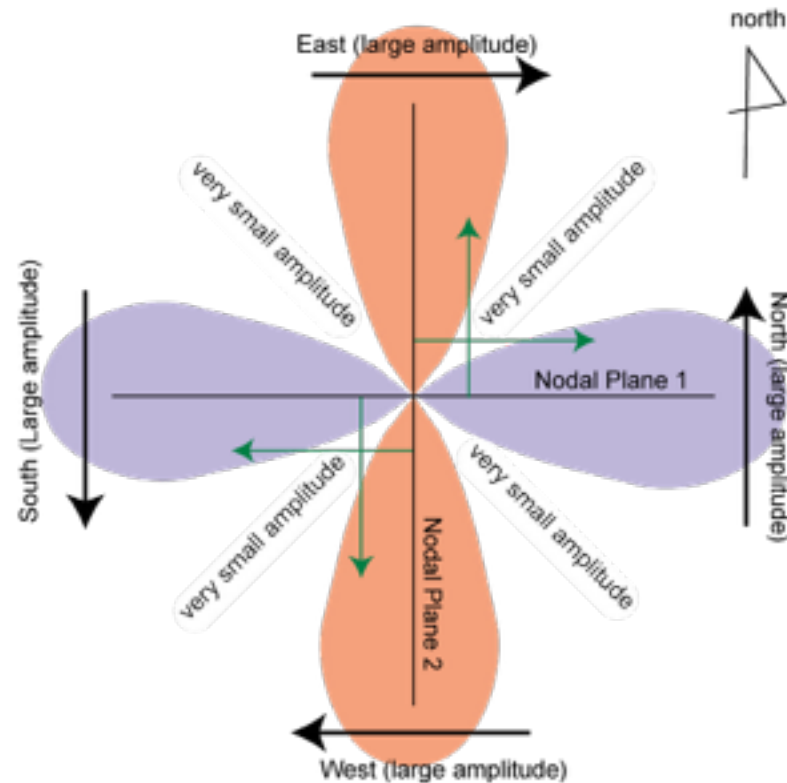
We can estimate nodal plane 1,2 using P-wave polarity and/or amplitude.



Radiation Pattern of S-wave

The S-wave radiation from a source have four-lobed pattern, while the orientation of the pattern is different form that of P-wave polarity.

Since the S-wave polarity is more difficult to identify than the P-wave, seismologist analyze P-wave much more often S-wave motion to determine the focal mechanism.

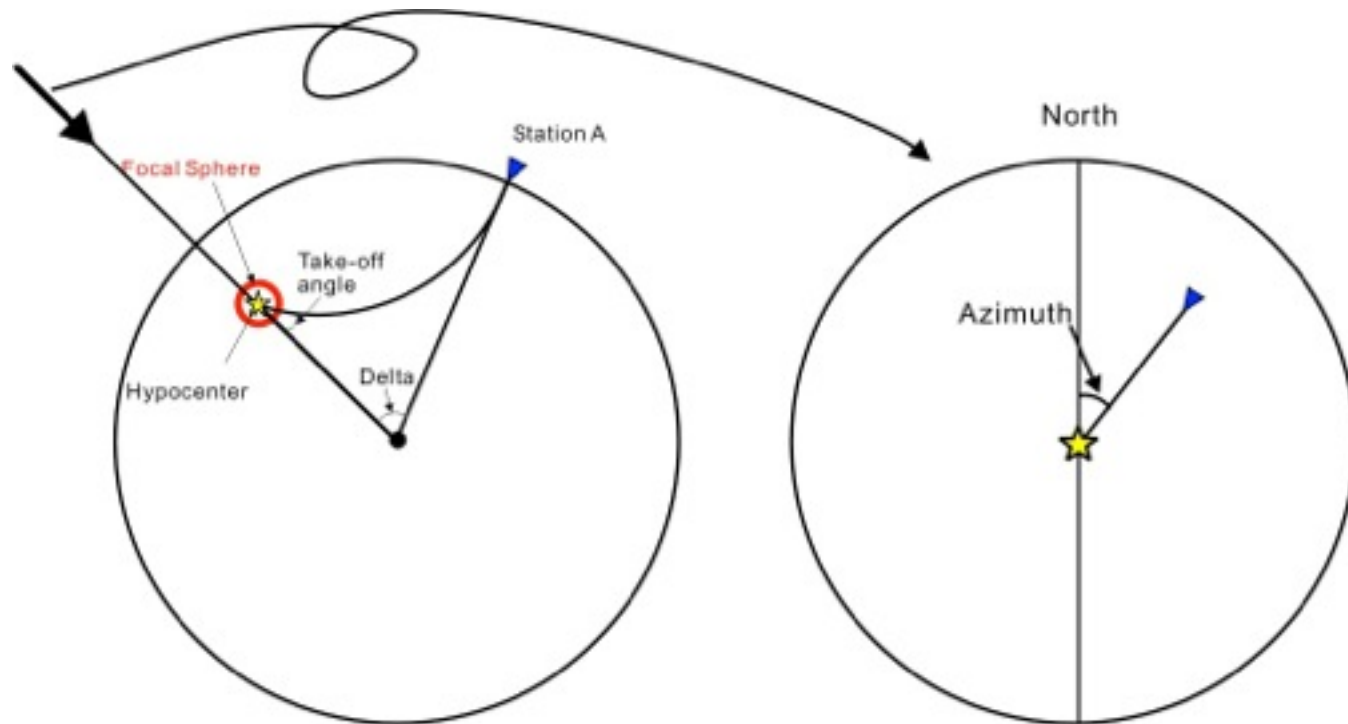


Focal Mechanism Diagram

We use a graphical procedure which enables us to represent the global distribution of polarity data and the two nodal planes on a figure. We call this figure “Focal Mechanism Diagram” or “Focal Mechanism”.

Image

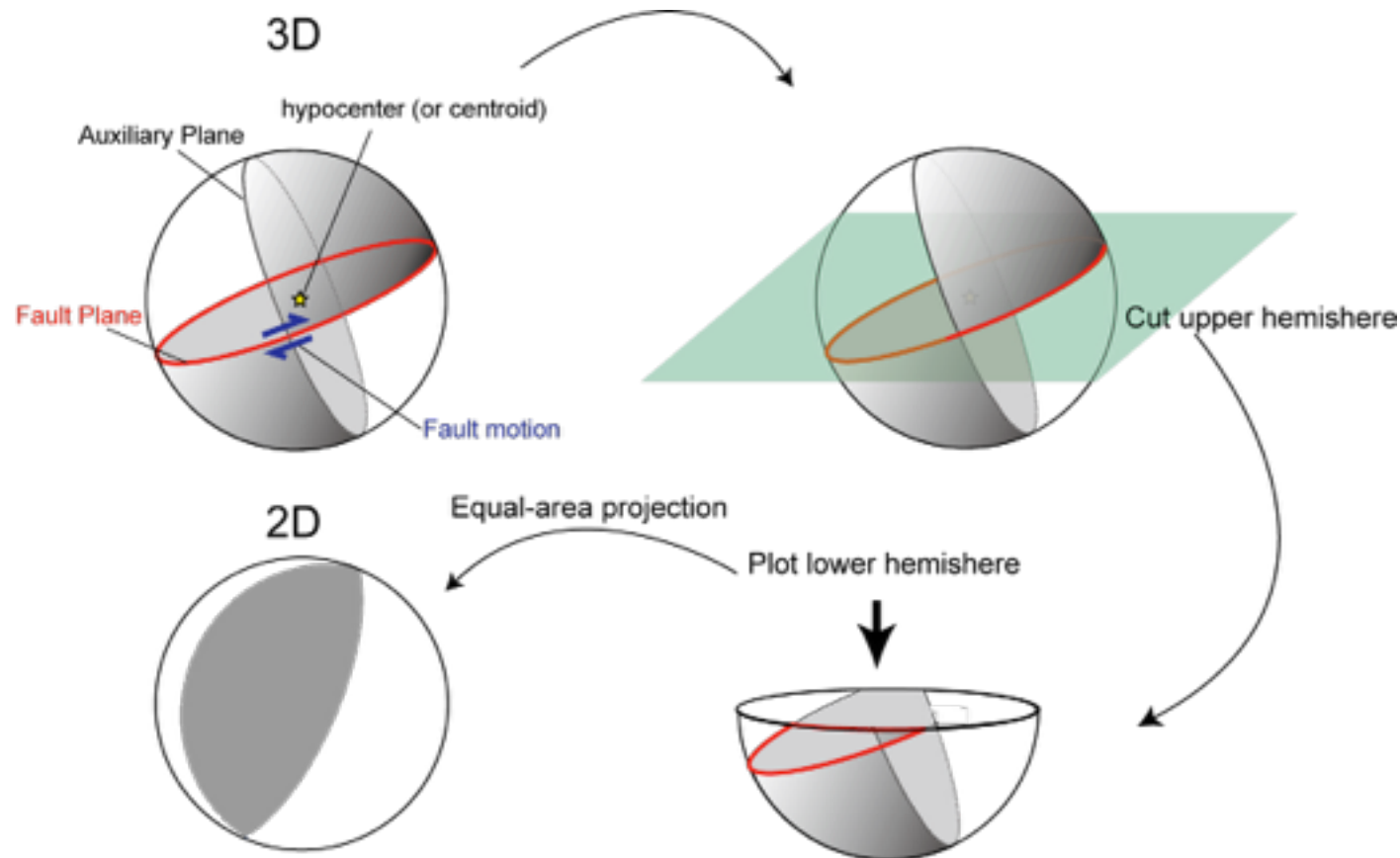
Surround the hypocenter with small sphere (focal sphere), and project polarity of data for each stations on surface of the focal sphere. Location of each stations can be obtained azimuth and take-off angle. If we have enough data, we can divide two area (up-area and down-area), and write two nodal plane.



Focal Mechanism Diagram

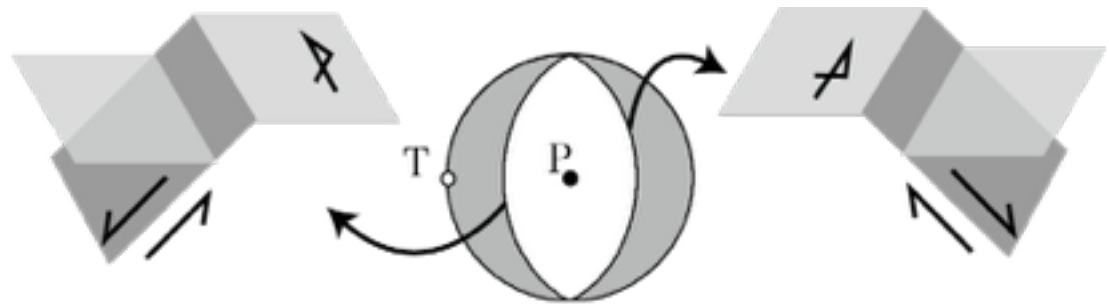
The focal sphere is three-dimensional body, so the polarity data on the sphere is three dimensional data on a two-dimensional diagram.

We projection of focal sphere onto a equatorial plane.

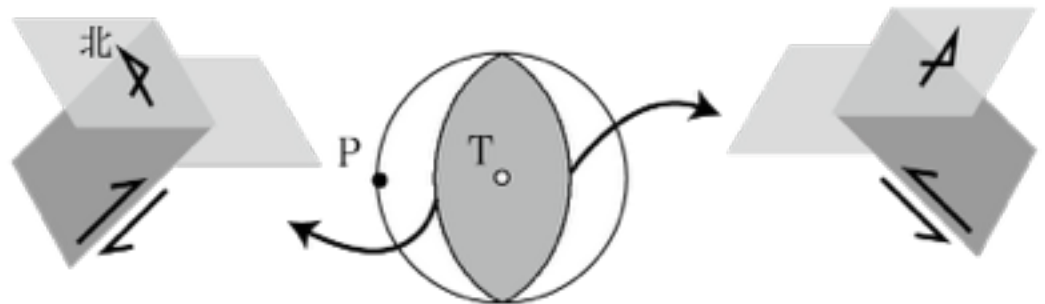


Focal mechanism and faulting type

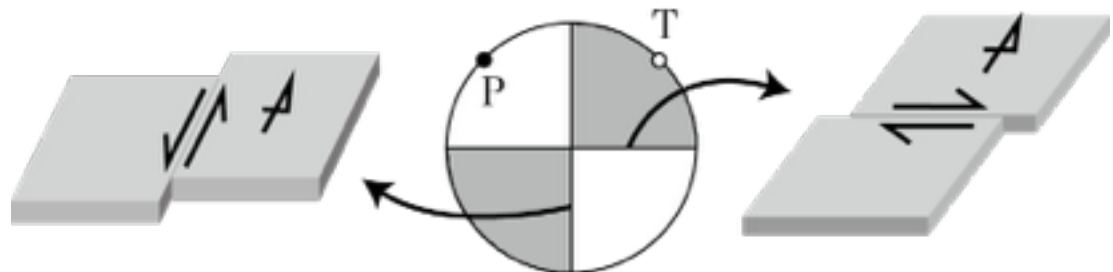
Normal faulting



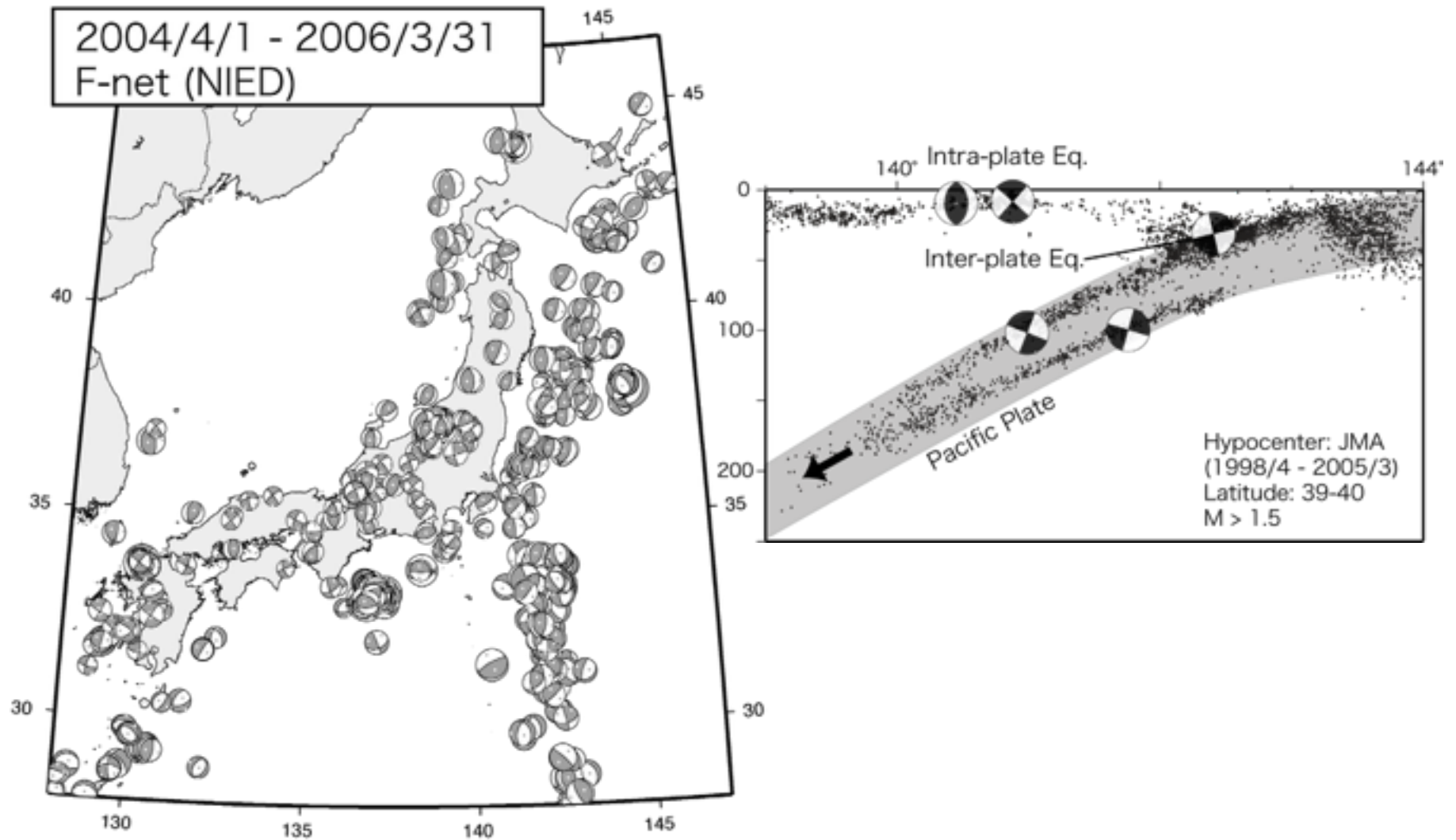
Reverse faulting



Strike Slip faulting



Focal Mechanism In Japan



Interpreting focal mechanism diagrams

Strike, Dip

Strike: The direction of the surface intersection of the fault measured clockwise from north (0~360)

Dip: A slop angle of the foot-wall block measured clockwise from horizontal (0-90)

Focal mechanism Diagram



3D image

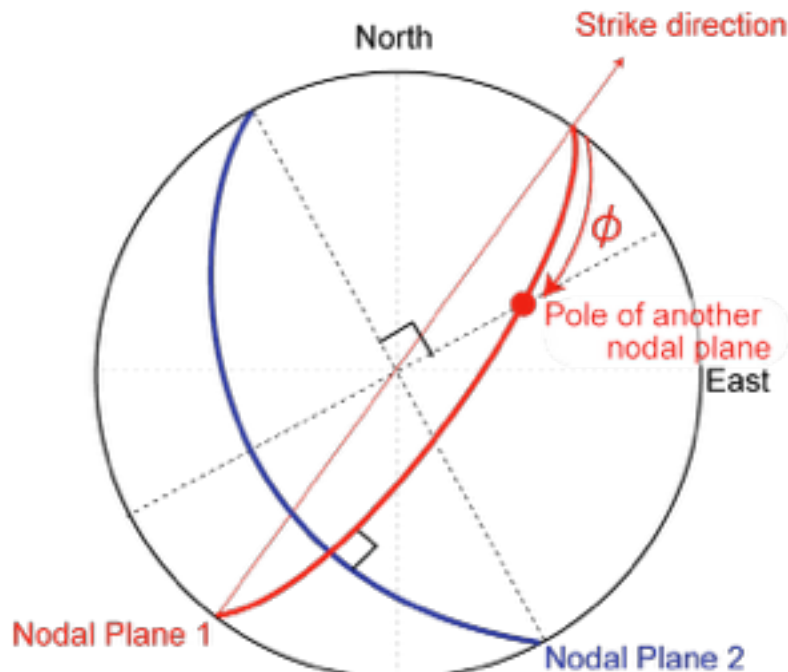


Interpreting focal mechanism diagrams

Rake

The direction of fault movement measured counterclockwise from strike and slip direction of hanging-wall (from -180 to 180)

Focal mechanism Diagram



3D image



If normal faulting, rake = $-\phi$
In reverse faulting, rake = $180-\phi$

Method of focal mechanism determination

For Single event

- Using polarity of first P-wave.

Requirement: many stations (depend on station coverage)

- Using polarity and amplitude of first P-wave.

Requirement: Five or more P-wave components

- Using waveform of P-wave and S-wave.

Requirement: Five or more waveform components

- Using overall waveform.

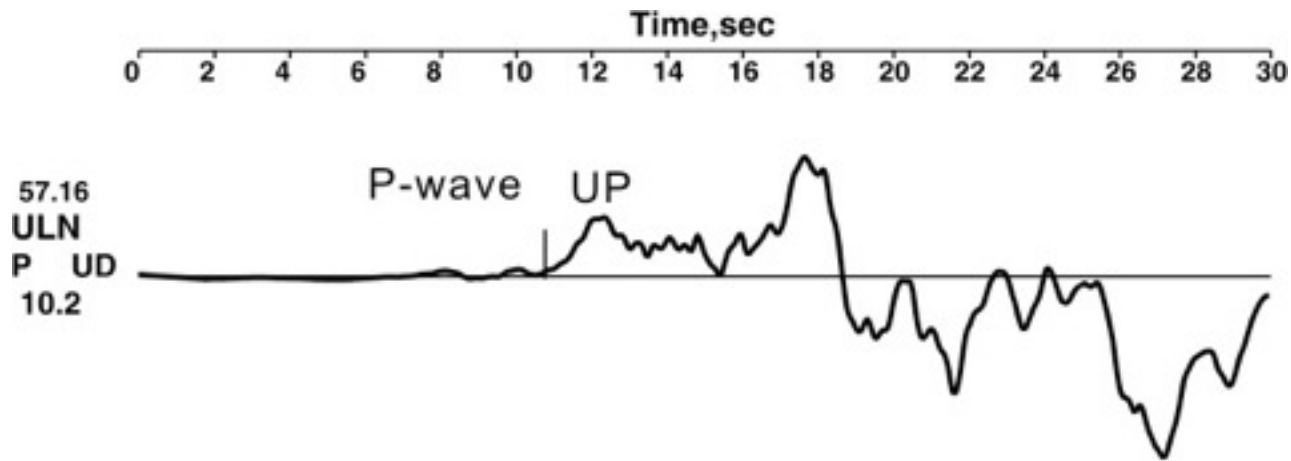
Requirement: One or more waveform stations.

Determining Fault Plane Solution using polarity of P-wave

To determine fault plane solution with manual work ,

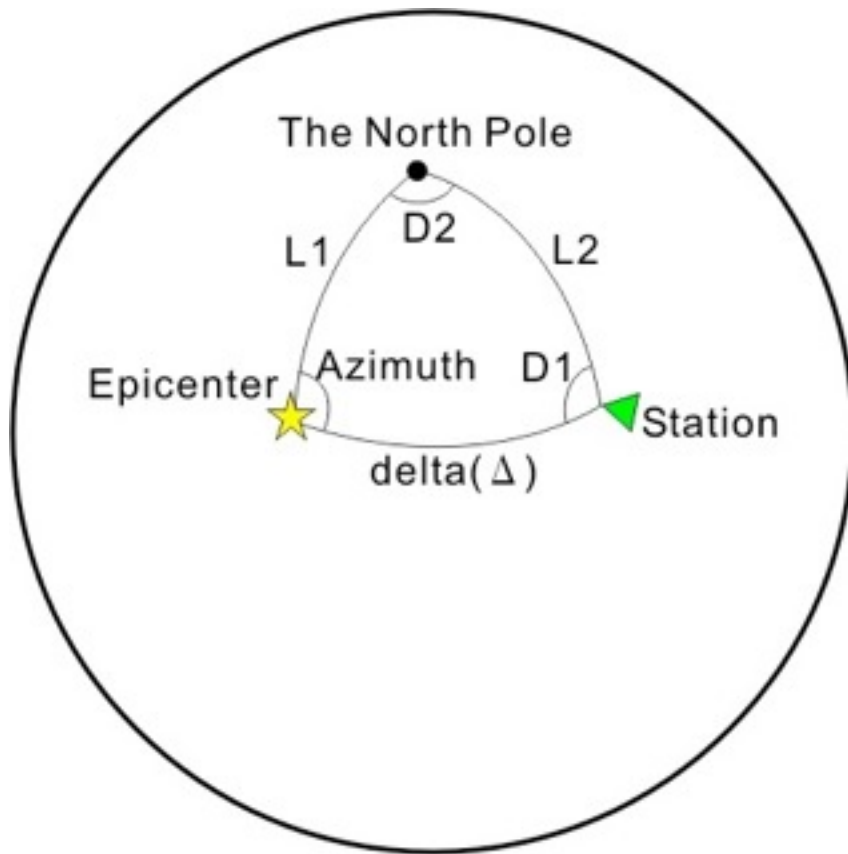
1. Determining the polarity of P-wave first motion.
2. Calculating azimuth and take-off angle for each stations.
3. Plotting the information of polarity of P-wave for each stations on the equal-area projection chart.
4. Selecting one nodal plane (A)
5. Calculating pole of nodal plane (A) that is cross point between the another nodal plane (B) and the additional plane.
6. Selecting the another nodal plane (B).

Determining the polarity of P-wave first motion



We can determine the polarity of P-wave first motion. (UD component)

Calculating azimuth

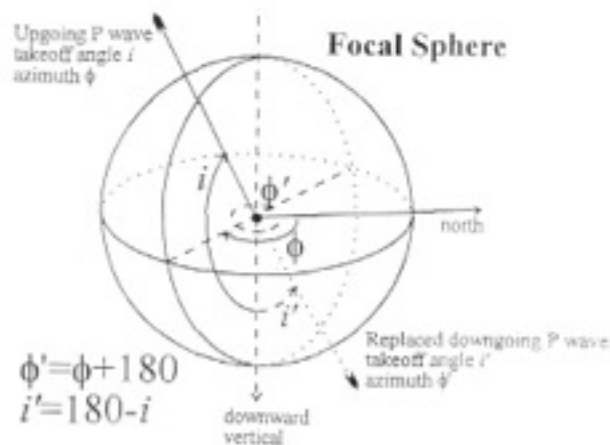
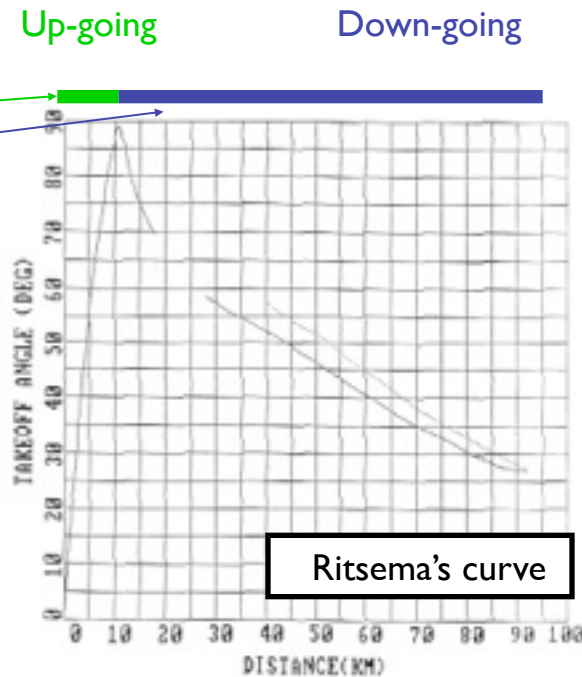
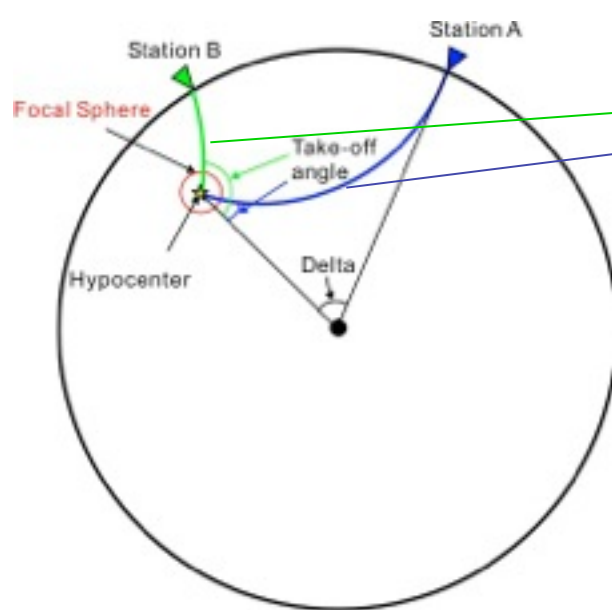


Azimuth and delta can be determined using spherical trigonometry. Consider the spherical triangle shown left plate. $D1$, $D2$, and Azimuth are the three internal angle of the spherical triangle. $L1$, $L2$ and delta are the side of the triangle in degrees measured between radii form an origin in the center of the sphere. If we know the latitude and longitude of epicenter and station, $L1$ and $L2$ and $D2$ are known parameters. Thus, we can determine delta and Azimuth using the below equation.

$$\Delta = \cos^{-1}(\cos L1 \cos L2 + \sin L1 \sin L2 \cos D2)$$

$$Azimuth = \cos^{-1}\left(\frac{\cos L2 - \cos \Delta \cos L1}{\sin \Delta \sin L2}\right)$$

Calculating take-off angle and Plotting

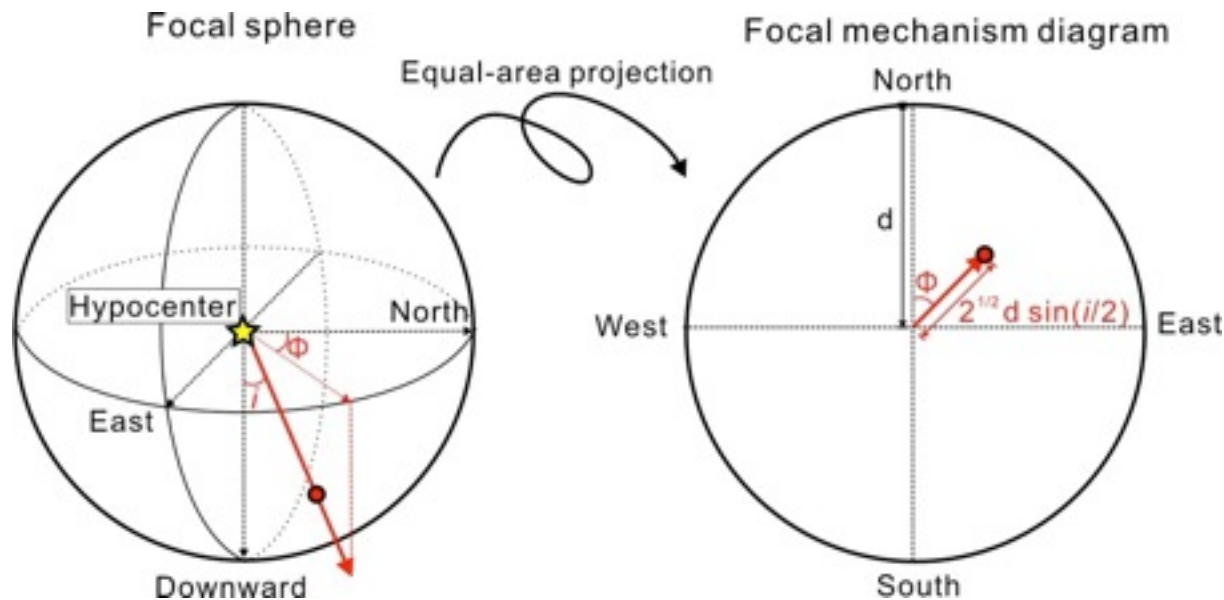


Takeoff angles can be estimated by using Ritsema's curve.

In down-going P-wave case, polarity data can be plotted on the surface of fault sphere.

In up-going P-wave case, take-off angle i and the azimuth ϕ are replace by $180-i$ and $180+\phi$, respectively.

Calculating take-off angle and Plotting



The projection rule is given by a condition of equal area. Then

$$ds = \sin i \times di \times d\phi = A \times r \times dr \times d\phi$$

Integration

$$1 - \cos i = \frac{1}{2} A r^2 + B$$

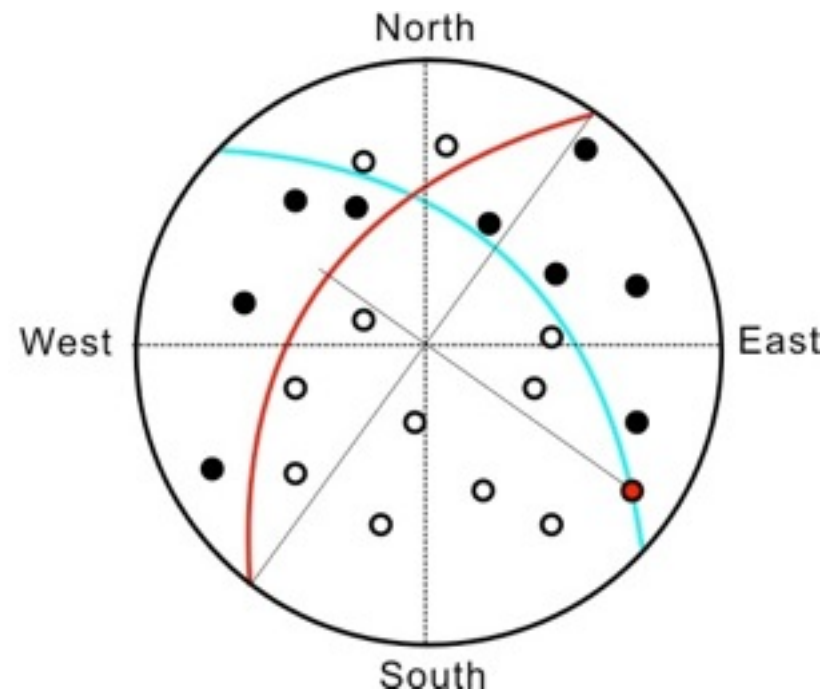
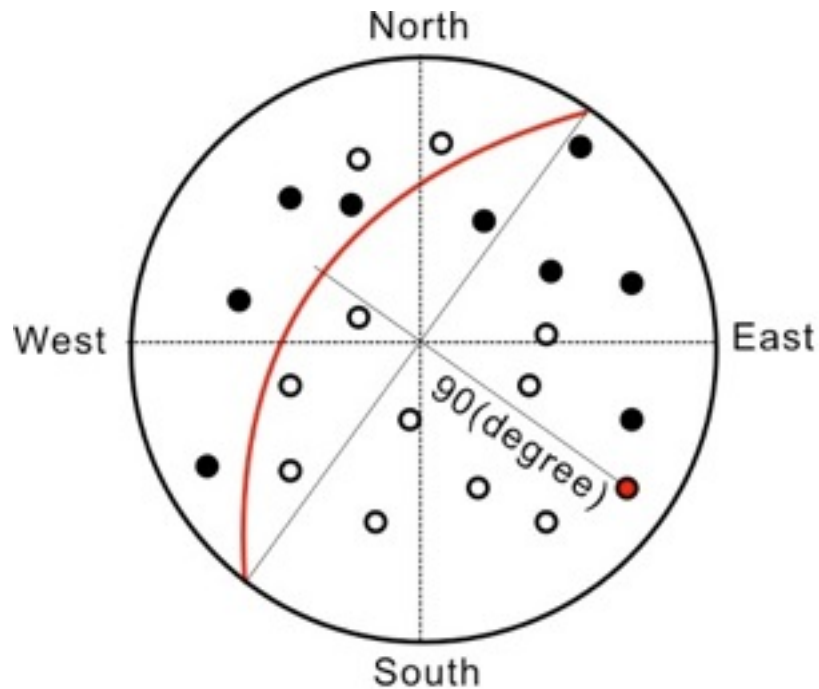
Boundary Condition

$$r = 0 \text{ for } i = 0, \quad r = d \text{ for } i = \delta/2$$

Thus

$$r = d \sqrt{1 - \cos i} = \sqrt{2} d \sin \frac{i}{2}$$

Selecting nodal plane



1. Selecting one nodal plane (A)
2. Calculating pole of nodal plane (A)
3. Selecting the another nodal plane (B)

Method of focal mechanism determination

1. Using polarity of first P-wave.
 - many stations (depend on station coverage)
- Using polarity and amplitude of first P-wave.
 - Five or more stations
- Using waveform of P-wave and S-wave.
 - Five or more components
- Using overall waveform.
 - One or more stations.

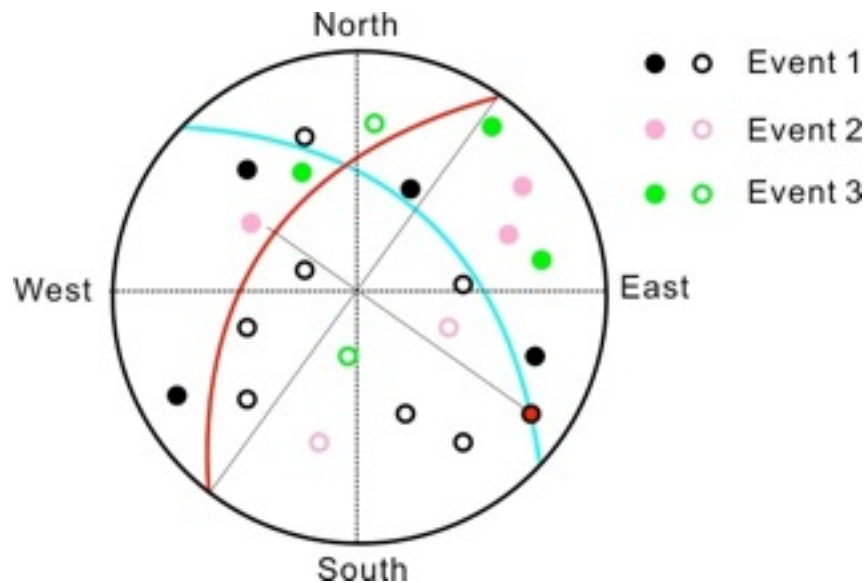
Method of focal mechanism determination

Composite P-first motion method:

In the case of a sparse local seismic network, P-first motion data may not be enough to determine reasonable solution.

If we assume that the focal mechanisms for many earthquakes in close area are identical, the up-down information for each earthquake can be drawn in same focal mechanism diagram. Using this focal mechanism diagram, we can determine the composite focal mechanism occurred in special area.

Composite P-first motion method



If we assume that the focal mechanisms for many earthquakes in close area are identical, the up-down information for each earthquake can be drawn in same focal mechanism diagram.

Using this focal mechanism diagram, we can determine the composite focal mechanism occurred in special area.

Moment Tensor Inversion

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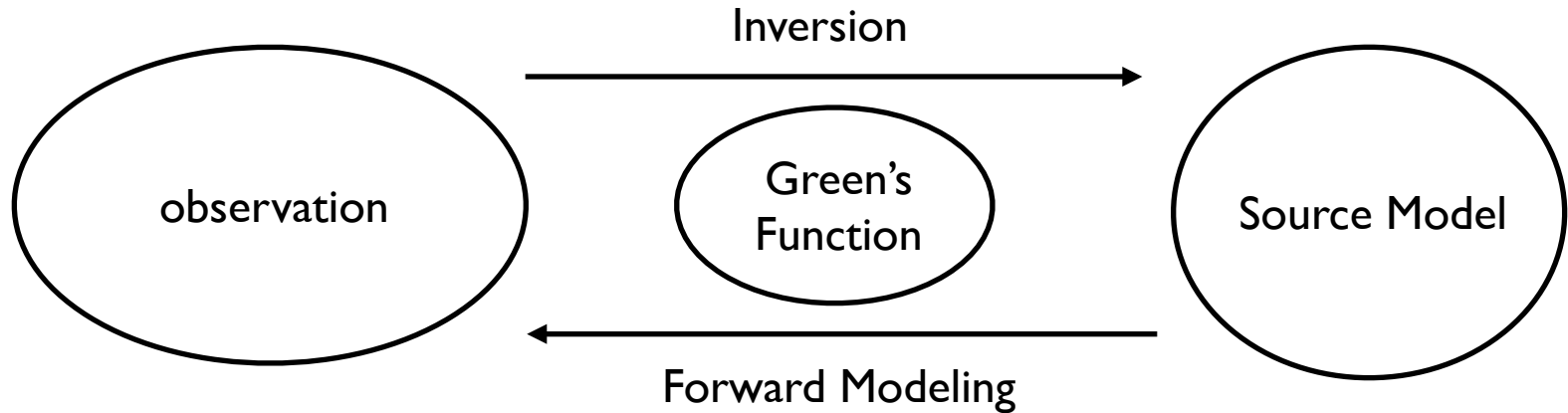
Introduction

Now, we can compute synthetic seismograms that are comparable with observed seismograms.

The seismic waveforms contain the information of the focal mechanism and seismic slip area.

If we assume earth structure, we can calculate green's function and estimate moment tensor components and location of centroid using waveform inversion scheme.

Forward modeling and Inversion



To estimate source model, we often apply two method.

Forward Modeling (try and error)

Input: source model

Output: synthetic waveform

Inversion

Input: observation data

Output: source model

Linear Inversion

Observation Equation

$$\mathbf{y} = \mathbf{f}(\mathbf{x}) \quad \mathbf{y}: \text{observation}; \mathbf{x}: \text{model}$$

Linear case

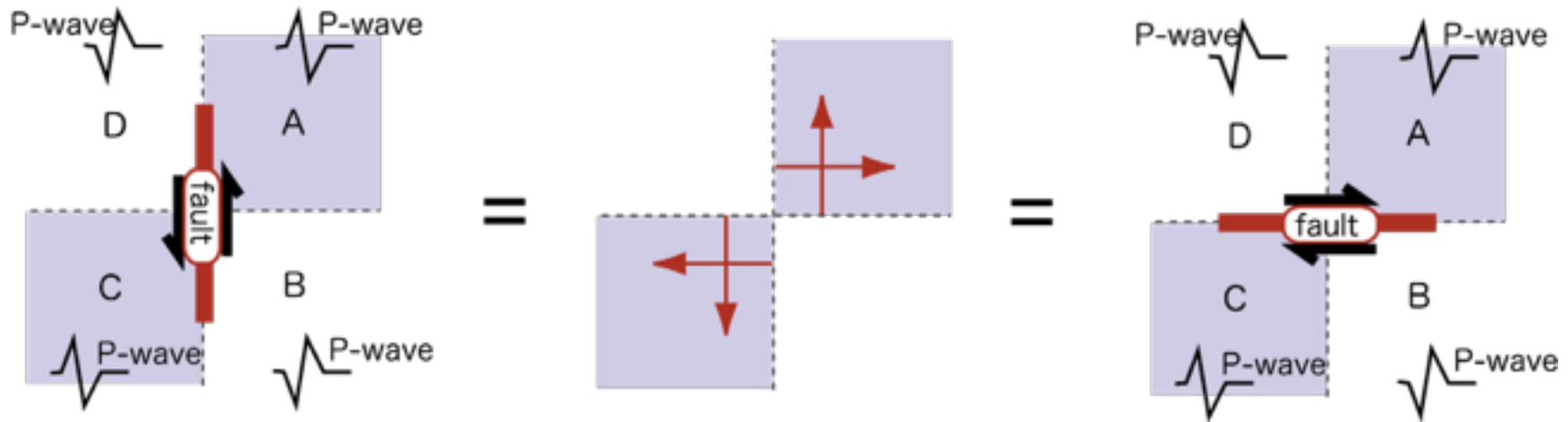
$$\mathbf{y} = \mathbf{A}\mathbf{x} \quad \mathbf{A}: \text{Kernel Matrix}$$

And then,

$$\hat{\mathbf{x}} = \left(\mathbf{A}^T \mathbf{A} \right)^{-1} \mathbf{A}^T \mathbf{y}$$

We can get model information from observation data.

Double Couple Model



The double couple model has two pairs of single couple.

This model don't have total face and torque. Thus this model have net balanced the moment.

Intrinsic problem

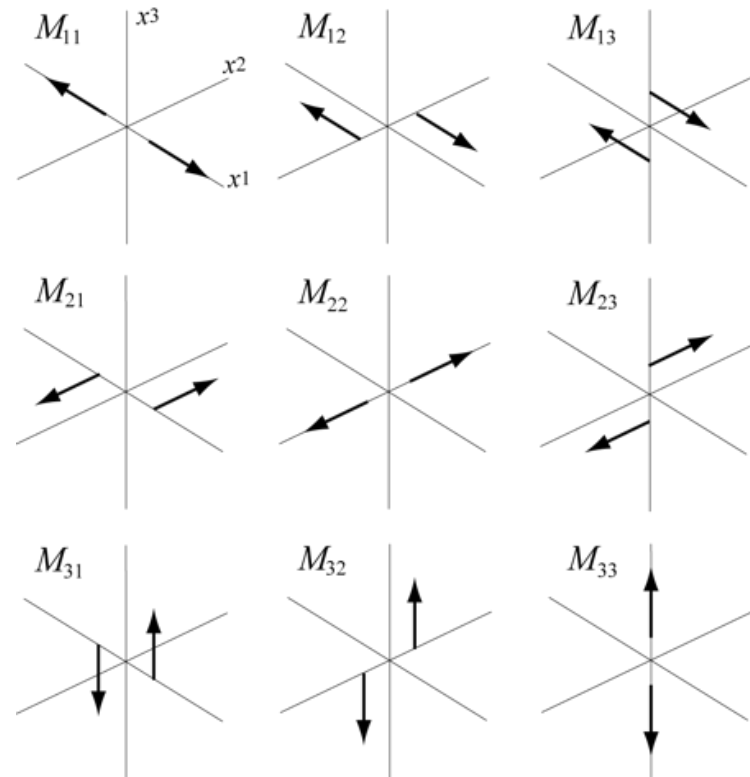
Any time, we can not choose the actual fault plane of two nodal planes with point source. If we want to determine one fault plane, we need to refer to aftershock distribution or tectonic setting.

Seismic Moment tensor

The moment tensor consists of nine single-couples force in the local-source Cartesian coordinate system.

The explosion source described can be modeled by the sum of the three dipole terms, $M_{11} + M_{22} + M_{33}$, with each having equal moment.

The seismic moment tensor is always symmetric.



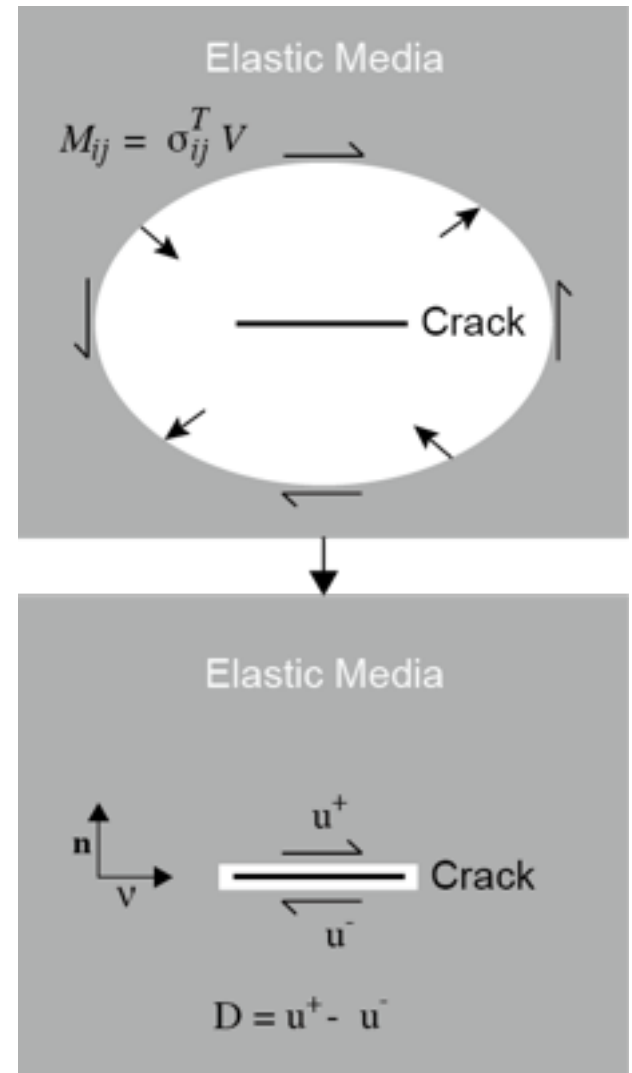
Moment tensor and fault motion

Moment tensor and crack

$$M_{ij} = \left[\lambda \left(\sum_{k=1}^3 v_k n_k \right) \delta_{ij} + \mu (v_i n_j + v_j n_i) \right] DS$$

Moment tensor and dislocation

$$M_{ij} = (v_i n_j + v_j n_i) \mu DS = (v_i n_j + v_j n_i) M_0$$



Moment Tensor and Radiation Pattern

Moment tensor and Far-field term

$$U_c(r, \gamma, t) = \frac{R_c}{4\pi\rho c^3} \frac{1}{r} \underbrace{Q(t)}_{\text{Q-effect}} * \underbrace{\dot{M}_0\left(t - \frac{r}{c}\right)}_{\text{Moment-rate function}}$$

Q-effect

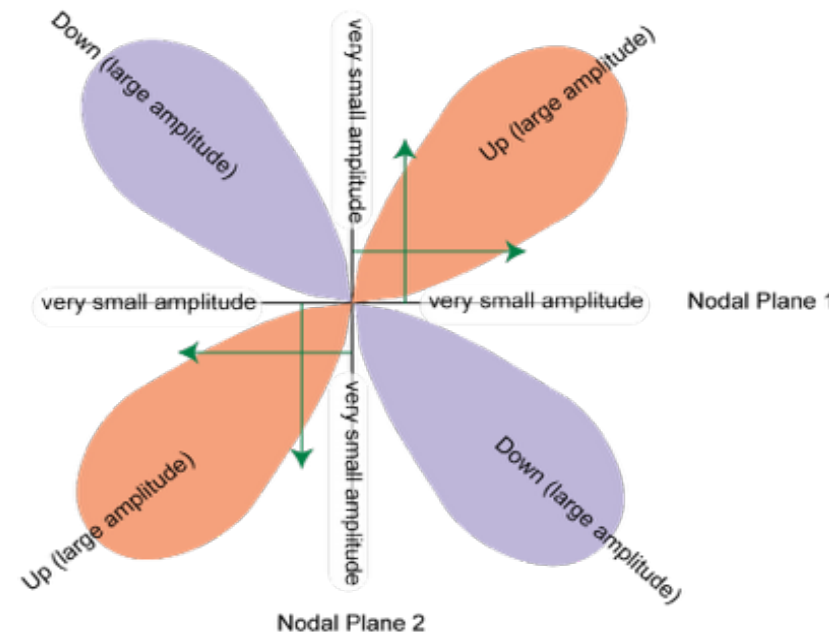
Moment-rate function

Radiation Pattern

$$R_C = \sum_{i=1}^3 \sum_{j=1}^3 m_{ij} \gamma_i e_{Cj}$$

P-wave Radiation Pattern

$$R_P = 2 \left(\sum_{i=1}^3 v_i \gamma_i \right) \left(\sum_{j=1}^3 n_j \gamma_j \right)$$



Radiation Pattern in Nuclear Weapons Testing

Moment tensor components

$$m_{ij} = \delta_{ij} \quad i, j = 1, 2, 3$$

P-wave Radiation Pattern

$$R_P = \sum_{i=1}^3 \sum_{j=1}^3 \delta_{ij} \gamma_i e_{Pj} = \sum_{i=1}^3 \sum_{j=1}^3 \delta_{ij} \gamma_i \gamma_j = \sum_{i=1}^3 \gamma_i \gamma_i = 1$$

S-wave Radiation Pattern

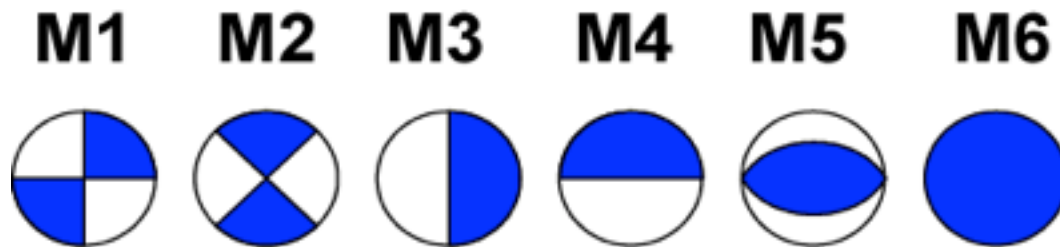
$$R_S = \sum_{i=1}^3 \sum_{j=1}^3 \delta_{ij} \gamma_i e_{Sj} = \sum_{i=1}^3 \gamma_i e_{Si} = 0$$

Basis Moment Tensor

We can treat source and propagation process as linear operators, and the moment tensor can describe double-couple at any time.

It is possible to construct observed waveform by summing the weighted the green's functions for each basis moment tensor.

We assume only double-couple model, the number of independent components of moment tensor is five.



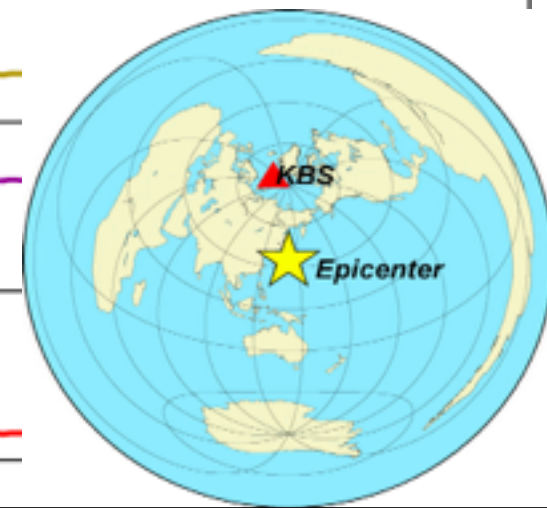
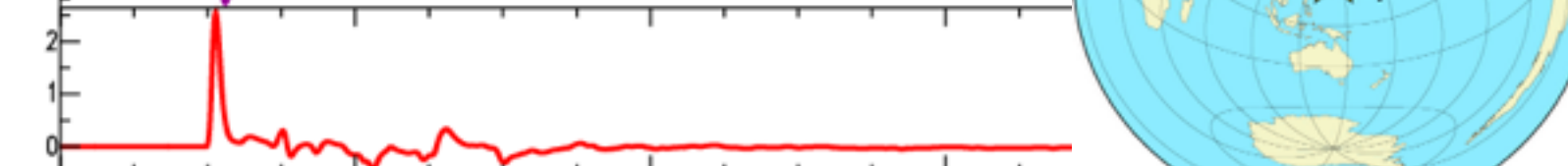
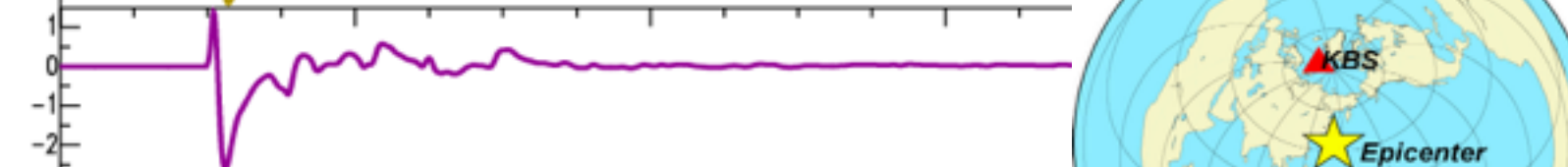
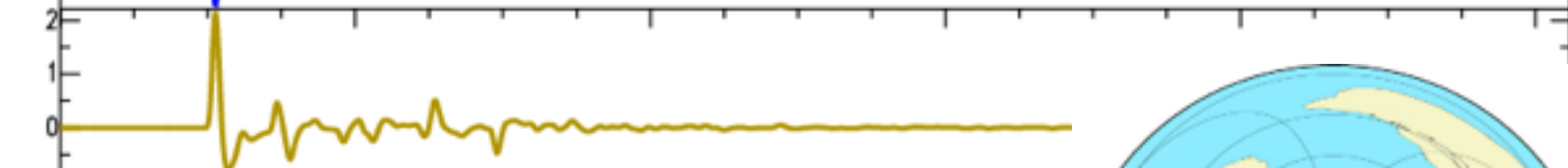
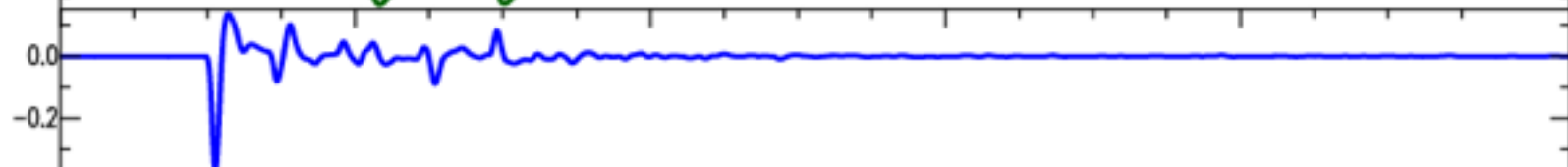
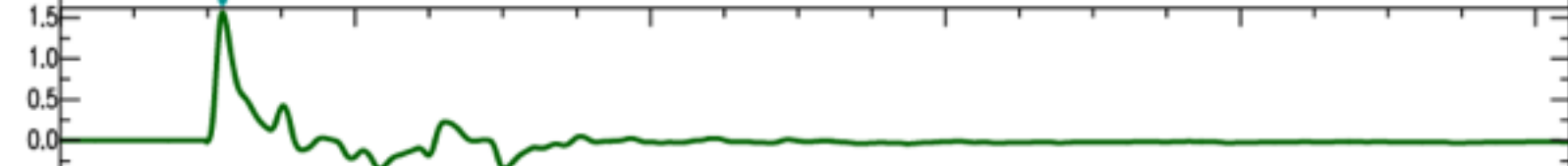
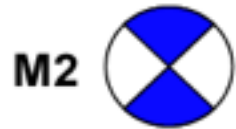
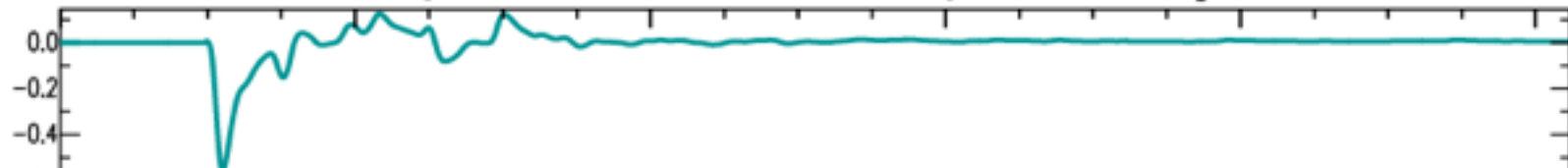
Kikuchi and Kanamori (1991, BSSA)

Green's Function for each basis moment tensor

JB structure model (Moho = 33 km)

Station KBS (Az.=350, Del. =61)

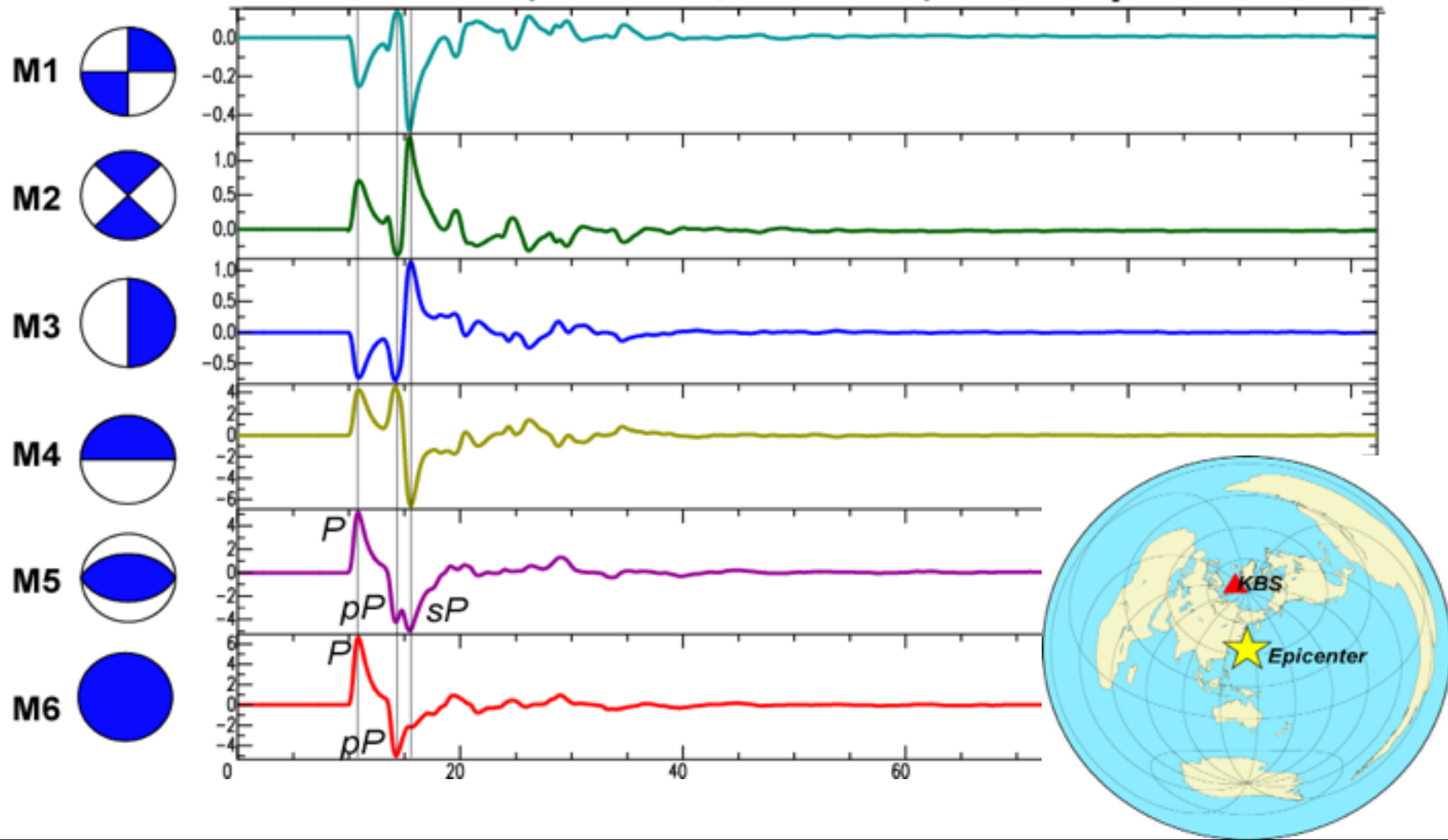
Depth = 0.5 km



Green's Function for each basis moment tensor

JB structure model (Moho = 33 km)

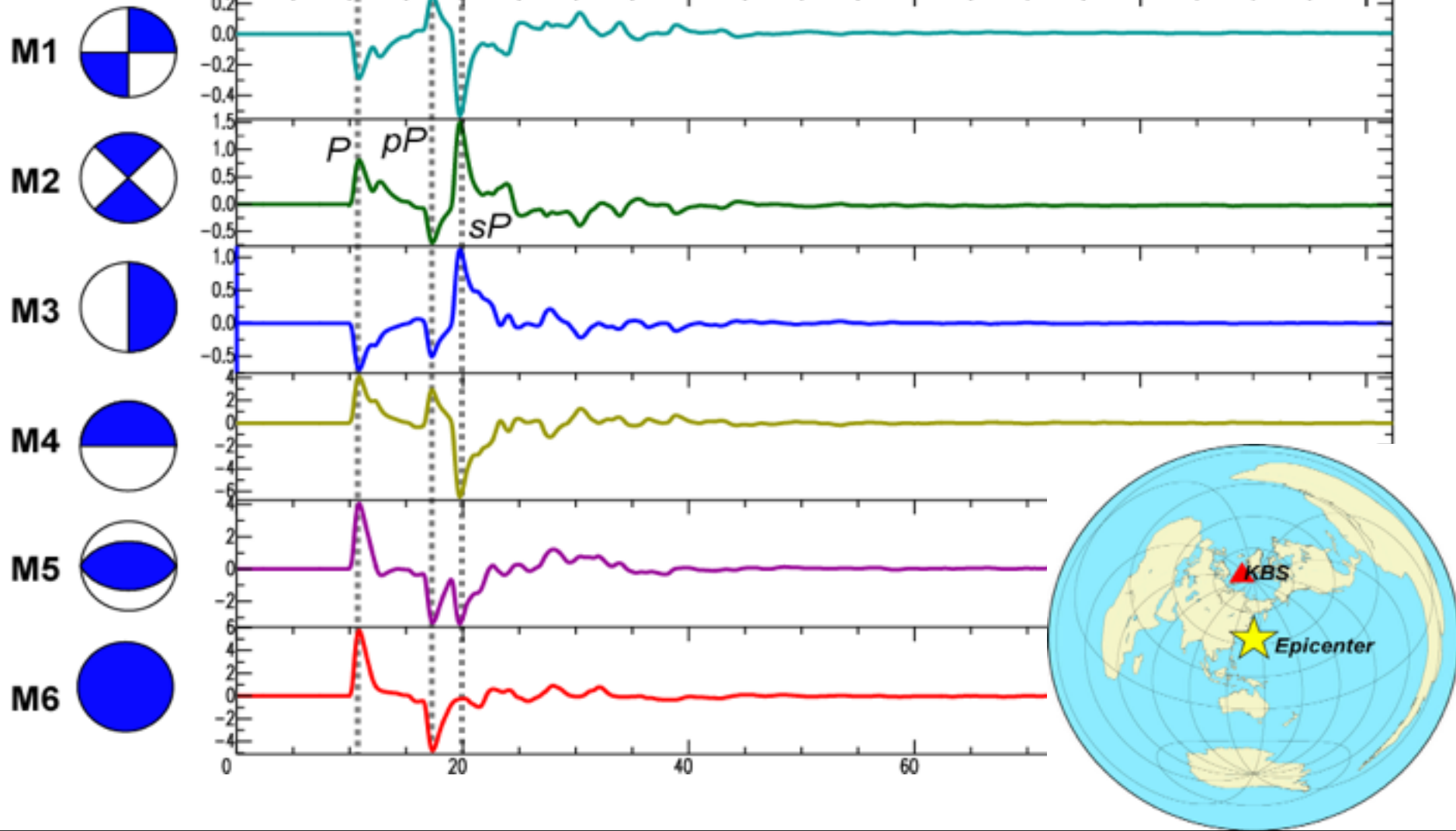
Station KBS (Az.=350, Del. =61) Depth = 10 km



Green's Function for each basis moment tensor

Station KBS (Az.=350, Del. =61)

Depth = 20 km



Moment Tensor Inversion

Observed seismic waveform of c component at a station j due to seismic moment release in a source volume V

$$\underline{u_{cj}(t)} = \sum_{q=1}^6 \iiint_V \underline{\tilde{G}_{cj q}(t, \xi)} * \underline{\tilde{M}_q(t, \xi)} d\xi + e'_{cj}(t)$$

Obs. Waveform Green's function Moment-rate function

Assumption 1: **Point source model**, in which we assume the seismic waveform to be radiated from one point.

$$u_{cj}(t) = \sum_{q=1}^6 \tilde{G}_{cj q}(t, \underline{\xi_c}) * \dot{M}_q(t) + e_{cj}(t)$$

Centroid

with

$$\dot{M}_q(t) = \iiint_V \tilde{M}_q(t, \xi) d\xi$$

Assumption 2: One earthquake has one focal mechanism.

$$\dot{M}_q(t) = m_q \times [\dot{M}_0(t)/M_0] = m_q \times S(t)$$

$$u_{cj}(t) = \sum_{q=1}^6 m_q \times S(t) * \tilde{G}_{cjq}(t, \xi_c) + e_{cj}(t)$$

Applying Low-pass filter

$$d_{cj}(t) = F(t) * u_{cj}(t) = \sum_{q=1}^6 m_q \times F(t) * S(t) * \tilde{G}_{cjq}(t, \xi_c) + e_{cj}(t)$$

$$\simeq \sum_{q=1}^6 m_q \times \underbrace{\delta(t)}_{\text{delta function}} * F(t) * \tilde{G}_{cjq}(t, \xi_c) + e_{cj}(t)$$

delta function

$$d_{cj}(t) = \sum_{q=1}^6 m_q \times G_{cjq}(t, \xi_c) + e_{cj}(t)$$

with $G_{cjq}(t, \xi_c) = \tilde{G}_{cjq}(t, \xi_c) * F(t)$

Vector Form:

$$\mathbf{d} = \mathbf{A}(\xi_c) \mathbf{m} + \mathbf{e}$$

The solution of the above matrix equation is obtained by least square approach if we assume the centroid location.

$$\hat{\mathbf{m}} = [\mathbf{A}^T(\xi_c) \mathbf{A}(\xi_c)]^{-1} \mathbf{A}^T(\xi_c) \mathbf{d}$$

We can estimate optimal the centroid using the grid-search method, which minimizes normalized L2-norm as

$$\|\mathbf{d} - \mathbf{A}(\xi_c) \hat{\mathbf{m}}\|_2 / \|\mathbf{d}\|_2 \Rightarrow \min$$

$$\text{L2-norm: } \|\mathbf{x}\|_2 = \left(\sum_{n=1}^N x_n^2 \right)^{1/2}$$

Moment Tensor to the two Fault Planes

Transformation from moment tensor to the two fault planes.

First we obtain the eigenvectors (**t**, **b**, **p**) of moment tensor.

$$\begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} = \begin{pmatrix} \mathbf{t} & \mathbf{b} & \mathbf{p} \end{pmatrix} \begin{pmatrix} M_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -M_0 \end{pmatrix} \begin{pmatrix} \mathbf{t} & \mathbf{b} & \mathbf{p} \end{pmatrix}^T$$

Next, we obtain fault vector (**n**: normal vector of fault plane, **v**: slip vector) from the eigenvectors (**t**, **b**, **p**) using the equation:

$$\text{Fault plane 1:} \quad \mathbf{n} = \frac{1}{\sqrt{2}}(\mathbf{t} + \mathbf{p}), \quad \mathbf{v} = \frac{1}{\sqrt{2}}(\mathbf{t} - \mathbf{p})$$

$$\text{Fault plane 2:} \quad \mathbf{n} = \frac{1}{\sqrt{2}}(\mathbf{t} - \mathbf{p}), \quad \mathbf{v} = \frac{1}{\sqrt{2}}(\mathbf{t} + \mathbf{p})$$

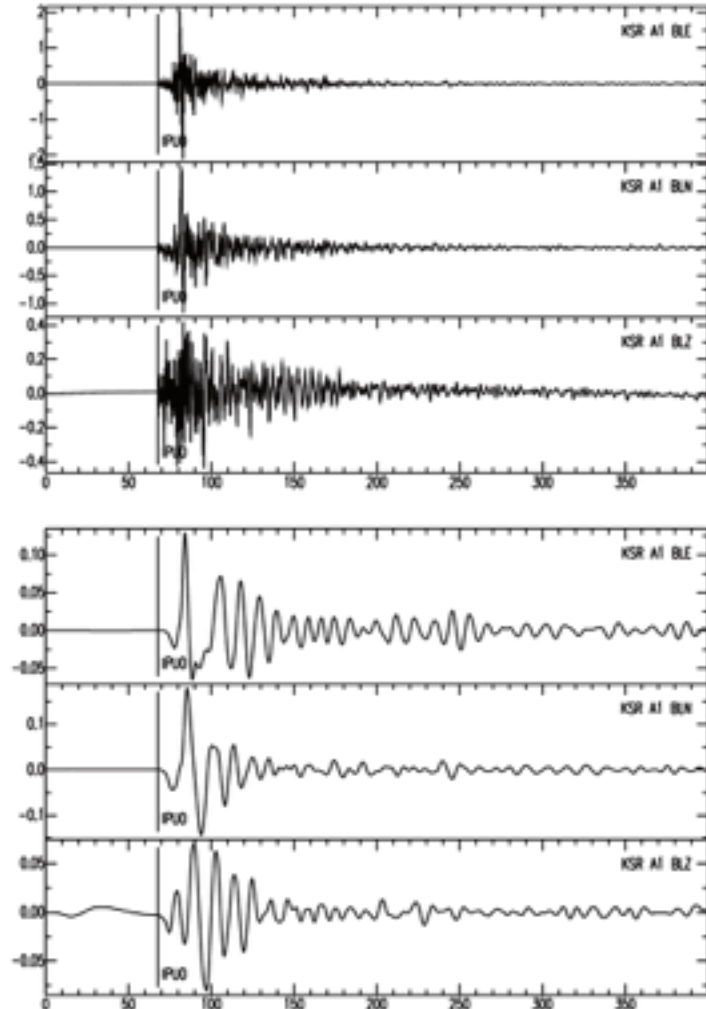
Moment tensor inversion for middle earthquake using local seismic network

Duration of source time function of middle size < 10 sec

If we apply a low pass filter, we can neglect source time function. In my program for local seismic network, we neglect effect of source time function.

Grid Search parameter:

only location of centroid



Moment tensor inversion for middle earthquake using local seismic network

Step 1:

Proceeding for near-source data set

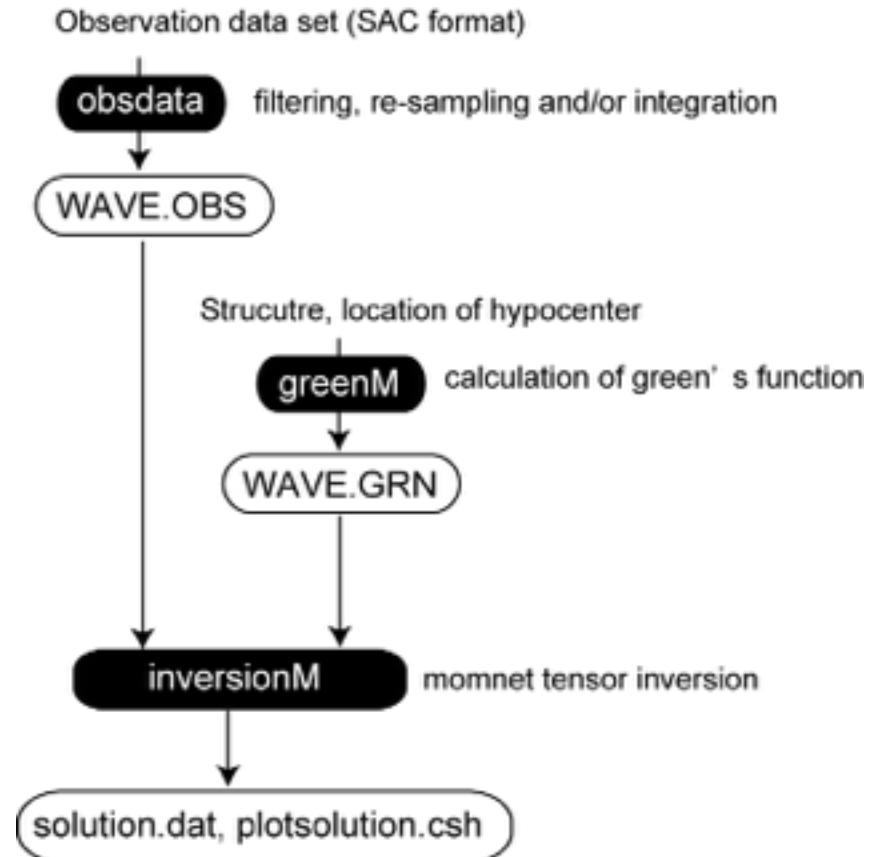
filtering, re-sampling

Step 2:

Calculation of Green's Function

Step 3:

Inversion



Moment tensor inversion for middle earthquake using local seismic network

If epicenter = horizontal location of centroid of moment release,

grid search parameter

-> only depth.

