INVESTIGATION AND ESTIMATION OF STORY STIFFNESS AND DAMPING RATIO OF A 4-STORY WOODEN STRUCTURE BY MICROTREMOR AND MASS SLIDING SHAKER

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ABSTRACT

In this study, the structural dynamic properties of a 4-story wooden structure were investigated. Microtremor observation was carried out to determine the first and second natural frequencies of the structure. Based on these preliminary results, mass sliding shaker was mounted on the structure to apply harmonic force vibration in order to excite the resonant behavior of the structure. Target frequencies were applied to the mass sliding shaker to obtain the responses for all four modal shapes. Signal obtained from the measurement was then mathematically converted from time domain to frequency domain using Fast Fourier Transform for better interpretation to determine all four natural frequencies. Based on these results, resonance curves for all four modal shapes were constructed. Curve fitting based on mathematical formula was also drawn by changing the values of damping ratio and peak amplitude to fit the resonance curve. From microtremor observation data and time history responses of free vibration, damping ratio of the structure was determined for comparison. Based on curve fitting of resonance curves, natural frequencies and participation functions were also identified. Using story mass in addition to these identified properties, stiffness matrix could be determined. Finally, each story's stiffness was obtained by multiplying stiffness matrix by unit displacement vector.

Keywords: Microtremor, Fourier Spectrum, Resonant curve, Damping ratio, Story stiffness.

INTRODUCTION

For the past three decades, physical characteristics of structural systems have been focused on by many researchers and this is done through the testing procedure known as system identification. To be more precise, system identification can be described as mathematical tools and algorithm representing a dynamical model from measured data. This dynamical model is then used to identify the properties of structural systems in which researchers are interested. In the field of Earthquake Engineering, the properties of structural systems are essential to be identified. These properties are related to structural response subjected to the ground motion. In order to identify the physical characteristics of a structural system, microtremor observation has been widely used for decades. Microtremor observation generally provides clear amplification ratios for first and second natural frequencies. With ambient noises, the amplification ratios for higher mode are inaccurate or sometimes misinterpreted. The force vibration technique is then introduced especially for light structure such as wooden structure. Previous studies have shown that story stiffness of the structures are commonly obtained based on the values of first and second natural frequencies. For multi-degree of freedom structures, it is desirable to obtain natural frequencies for higher modes in order to give more accurate story stiffness. The purpose of this study is to estimate damping ratio and story stiffness of a 4-story wooden structure by using force

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vibration to stimulate the resonant behavior of the structure. Force vibration test also provides in depth modal analysis of the structure to obtain dynamic properties of the structure.

THEORY AND METHODOLOGY

System Configuration of the Building

The vibration of the building/structure during earthquake and microtremor is composed of several different types such as sway, rocking, relative displacement and several modes such as translations, torsion and those from higher modes. In general, the modes considered in vibration experiment are varied depending on type of building and the height of the building. In this study, the 4-story wooden building is investigated. Several high-sensitive accelerometers are placed at various positions inside the building. Each sensor is designated to catch the vibration signal along each story of the building as shown in Figure (1).

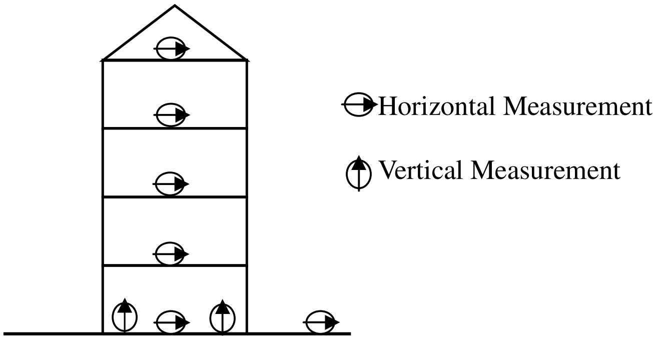

Figure 1. Sensors' location in wooden building

In general, the vibration of the building consists of rocking, sway and relative displacement. Each sensor was designated to measure the related motion. Rocking motion represents the different vertical displacement of the building. Sway motion is a relative displacement between ground and the building. Relative displacement represents the lateral deformation of each story with respect to another. For microtremor measurement, there are some assumptions needed to be made such as the floor is rigid, and ground surface is slightly separated from the building so that it is not affected by soil-structure interaction and so on.

Resonance Behavior of the Structure

The multi-degree of freedom structures possess several natural frequencies mostly corresponding to number of degrees of freedom. When the structure is subjected to the excitation, the resonance response is induced if the frequency of the excitation is close to each one of natural frequencies of the structure. In this study, a mass sliding shaker is used to implement the force function to the structure. The resonance test is carried out by changing the target shaking frequency using the oscillator generating the sinusoid function. Input and output responses are measured and data post-processing is implemented. The plots of ratios between input and output are fitted by the theoretical resonance curve. To illustrate this resonance behavior mathematically, the harmonic force at the forth story can be expressed as follows.

$$F = f \cos \omega t = (m_c a) \cos \omega t \tag{1}$$

in which m_c is the mass of the shaker and ω is frequency of input motion of the shaker. Output displacements are given by.

$$\{y\} = \sum_{s=1}^{4} {}_s u_4 \{{}_s u\} \frac{m_c a}{{}_s\omega^2 \, {}_s M} \left[A \cos_s \omega t - B \sin \omega t \right] \tag{2}$$

Therefore, the output acceleration is expressed as $\{\ddot{y}\} = -\omega^2 \{y\}$.

If only *s*th modal properties at *i*th story are extracted, response amplification in acceleration can be given as follows.

$$\left|\frac{\ddot{y}_i}{\ddot{x}_c}\right| = {}_s u_4\, {}_s u_i \left(\frac{\omega}{{}_s\omega}\right)^2 \frac{m_c}{{}_s M}\sqrt{{}_s A^2 + {}_s B^2} \approx \left(\frac{\omega}{{}_s\omega}\right)^2 {}_s\beta\, {}_s u_i \frac{m_c}{m_4}\sqrt{{}_s A^2 + {}_s B^2} \tag{3}$$

Therefore,

$$\left|\frac{\ddot{y}_i}{\ddot{x}_c}\right| = \left(\frac{\omega}{{}_s\omega}\right)^2 {}_s\beta\, {}_s u_i \frac{m_c}{m_4} \frac{1}{\sqrt{\left\{1-\left(\omega/{}_s\omega\right)^2\right\}^2 + 4\,{}_s h^2 \left(\omega/{}_s\omega\right)^2}} \tag{4}$$

where $\ddot{x}_c$, ${}_s\omega$, ${}_s\beta_s u_i$, and m_4 are input acceleration, natural frequency of the structure, participation function, and the 4th-story mass respectively.

Force Vibration Test Using Mass Sliding Shaker

In this study, mass sliding shaker or alternatively called Linear Shaker Seismic Simulation (LSSS) was used to induce the vibration on the structure. The installation of the shaker was done by fully attached the shaker at the middle of the roof floor using bolts and nuts. The reason is that the shaker can be considered as a part of the structure. All nine accelerometers were wired to the measuring device to measure the response at designated locations on the structure. One of nine sensors was attached on the shaker to capture the input motion given by the amplifier and function generator. The overview of the system configuration is shown in Figure 2.

The function generator (shown in Figure 2) generates the motion of the shaker at specified frequency and the function used was the sinusoidal function. The amplifier was used to increase the magnitude of the input signal by selecting appropriate bandwidth for better performance of the shaker.

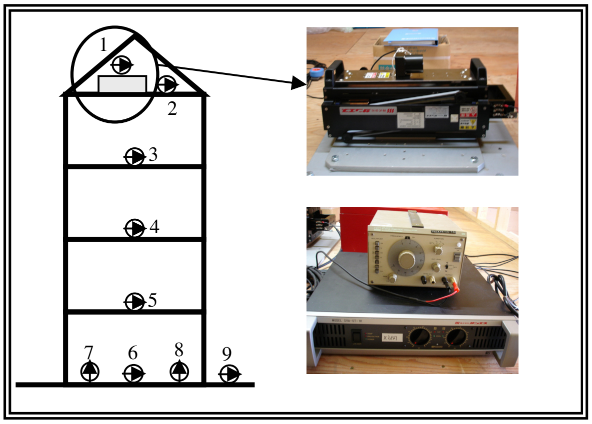

Figure 2. Overview of Shaker System Configuration

DATA ANALYSIS AND RESULTS

Microtremor observation was carried out initially to investigate the fundamental natural frequencies of the structure. The measurement was preferably done at midnight to minimize the unwanted ambient noise especially from the traffic. The measurement was recorded for 30 minutes to ensure the stable response of the structure. The sampling frequency was 200 Hz so the total number of data was 360,000. Once the raw data was converted to readable format, the response from each story can be displayed in time domain and frequency domain as shown in Figure 3 and 4.

From Figure 4 shown above, two obvious peaks of response can be observed. This implies that the first and second natural frequencies of the structure are most likely able to be spotted from the graph by projecting these two peaks on the x-axis. By doing so, these natural frequencies were found to be approximately 3.510 Hz and 11.730 Hz (dotted circles in Figure 4) for first and second natural frequencies respectively. These two frequencies were then used for the primary setup for the shaking frequency to determine the resonance frequencies. Range of target frequencies was applied to the shaker to identify all 4 natural frequencies. Fourier spectrum for each frequency was determined and

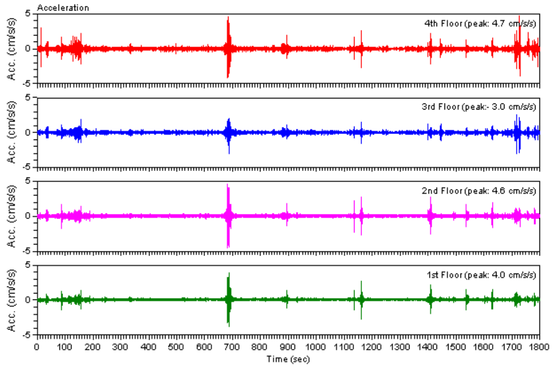

Figure 3. Time history response for each story

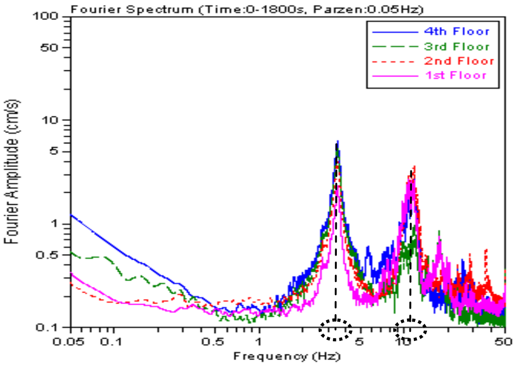

Figure 4. Fourier spectrum for each story

ratio between response of each story and shaker was plotted to form the resonance curve for each story. Then, by using Eq. (4), curve fitting can be done by selecting the values of damping ration and peak amplitude for the best fit to all resonance curves as shown in Figure 5.

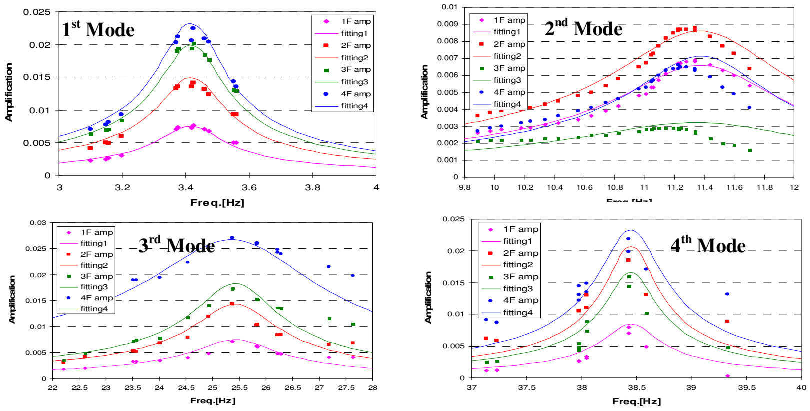

Figure 5. Resonance curves for all 4 modes

Based on the curve fitting method, damping ratio and all of the participation functions were obtained corresponding to each story for each mode. Phase lag of the response for each story was also determined to identify the modal shapes of the structure. The values of participation functions were used to plot the modal shapes as shown in Figure 6.

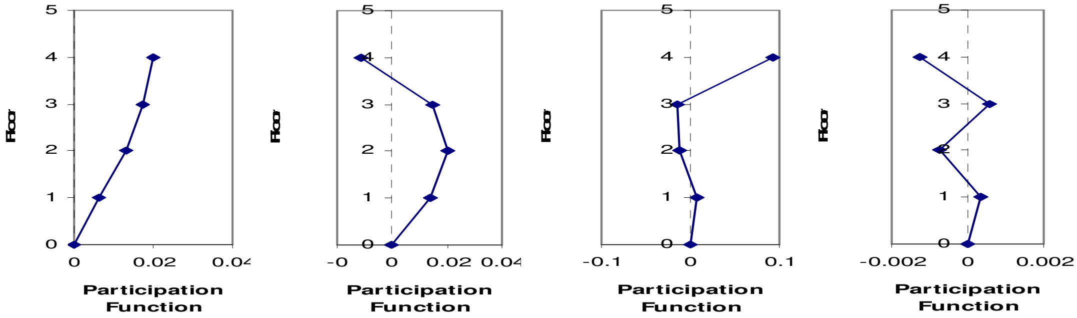

Figure 6. Participation Functions and Mode shapes

DAMPING RATIO AND STORY STIFFNESS

Half Power Method

In general, transfer function has a similar shape like a resonance curve. As described earlier, transfer function can be acquired from input and output data of microtremor observation. The frequency at the highest peak is natural frequency, f_0. .

The simplest method to estimate a damping ratio of a structure is called "Half Power Method".

In Figure 7, f_1 and f_2 can be acquired by projecting the value of $1/\sqrt{2}$ amplitude of the highest peak on the frequency axis. Then damping ratio can be estimated by

$$h = \frac{f_2 - f_1}{2f_0} = \frac{\omega_2 - \omega_1}{2\omega_0} = \frac{\Delta\omega}{2\omega_0} \tag{5}$$

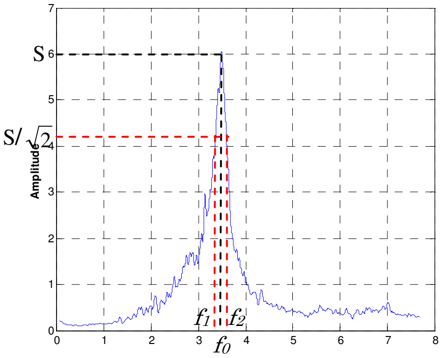

Figure 7. Transfer function

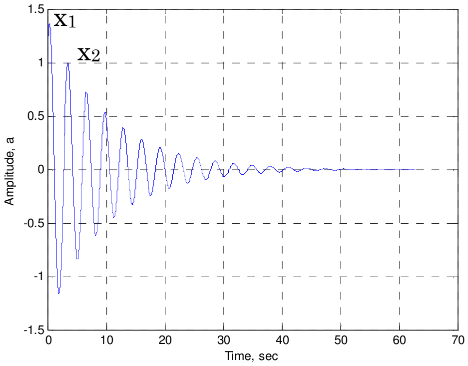

Figure 8. Free vibration

Random Decrement Technique

Curve fitting method and half power method are based on frequency domain, whereas time history domain methods are also used in several aspects. Considering two peak values (Figure 8) measured at two different positions with time interval T_d, the ratio of these two peaks x_1 and x_2 can be obtained as follows

$$\frac{x_2}{x_1} = \exp(-h\omega_0 T_d) = \exp(-h\omega_0 \frac{2\pi}{\omega_0\sqrt{1-h_2}}) = \exp(-h\frac{2\pi}{\sqrt{1-h_2}}) \tag{6}$$

Alternatively the damping ratio h can be approximately related to the logarithmic decrement as

$$h = \frac{\sqrt{1-h_2}}{2\pi}\ln\left(\frac{x_1}{x_2}\right) \approx \frac{1}{2\pi}\ln\left(\frac{x_1}{x_2}\right) \tag{7}$$

In most practical cases, several sets of logarithmic decrements are used for averaging. In general, time history domain may not be in the form of what is shown in Figure 8. The random signal is most likely presented for the structural response. In this method, the input signal can be formed by many sinusoidal waveforms. Then by superposing many part of output waveforms, only free vibration decrement can be acquired.

In this study, free vibration was obtained by inducing force vibration using the shaker and then the input motion is suddenly stopped providing free vibration afterward. This free vibration can be analyzed based on the concept of logarithm decrement. However, for the higher mode, free vibration response cannot be identified due to the ambient noise from the traffic. Therefore, in this section, 1st and 2nd mode free vibration response were considered to obtain damping ratios. Using all methods stated previously, average of damping ratios can be summarized as shown in Table 2.

Table 2. Damping ratios calculated by three methods

Story	Half Power Method		Random Decrement		Free Vibration	
	1st Mode	2nd Mode	1st Mode	2nd Mode	1st Mode	2nd Mode
Average	0.036	0.038	0.034	0.040	0.036	0.042

Estimation of Story Stiffness

By using the modal and spectral matrices, stiffness matrix can be given as follows:

$$[\mathrm{K}] = \left([\Phi]_{norm}\left[\Omega^2\right]^{-1}[\Phi]_{norm}^{\mathrm{T}}\right)^{-1} \tag{8}$$

where $[\Phi]$ is the modal matrix consisting of transfer functions obtained from previous section and Ω^2 is spectral matrix consisting of diagonal values of eigenvalues, ω_n^2 . To determine each story's stiffness, the concentrated equivalent story stiffness will be estimated by multiplying stiffness matrix [K] by the displacement vector whose elements are equal to 1. Each story's stiffness can then be obtained as shown in Figure 9.

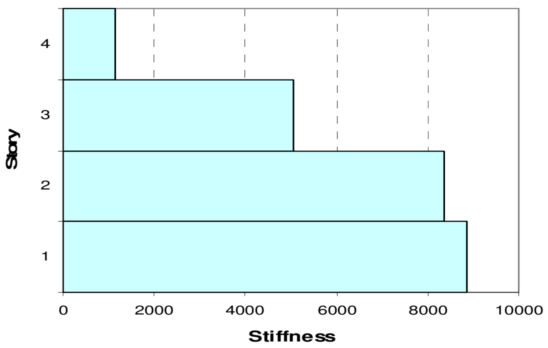

Figure 9. Story Stiffness

CONCLUSIONS

Results from this study can be summarized as follows.

- Microtremor observation provides adequate information for fundamental natural frequencies of the wooden structure. The first and second natural frequencies are found to be 3.51 Hz and 11.73 Hz.
- The target frequencies are applied to the shaker to identify the resonance behavior of the structure. The resonant frequencies are found to be 3.42 Hz, 11.4 Hz, 25.45 Hz, and 38.45 Hz for first, second, third and forth modes respectively. It is noted that the resonant frequencies are slightly smaller than those identified by microtremor observation.
- Damping ratios are obtained based on several methods to compare the values. Results from these methods show that damping ratios do not vary corresponding to each mode.
- The forth-story stiffness calculated is quite small comparing with the others. The reason is that only shear deformation is considered in this study, whereas bending and torsional deformations are ignored.

Story stiffness and damping ratio can then be analyzed to provide the dynamic properties of the structure. These dynamic properties are used as the guideline to justify how sufficient the structure is and how the structure will perform during the earthquake.

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